

NAME

gjqr – Gauss-Jacobi logarithmic Quadrature Recursion Coefficients

SYNOPSIS

Fortran (77, 90, 95, HPF):

```
f77 [ flags ] file(s) ... -L/usr/local/lib -lgjl
      SUBROUTINE gjqr( a, b, s, t, alpha, beta, nquad, ierr)
      DOUBLE PRECISION a(0:MAXPTS), alpha, b(0:MAXPTS), beta
      DOUBLE PRECISION s(0:MAXPTS), t(0:MAXPTS)
      INTEGER ierr, nquad
```

C (K&R, 89, 99), C++ (98):

```
cc [ flags ] -I/usr/local/include file(s) ... -L/usr/local/lib -lgjl
```

Use

```
#include <gjl.h>
```

to get this prototype:

```
void gjqr(fortran_double_precision a[],
          fortran_double_precision b[],
          fortran_double_precision s[],
          fortran_double_precision t[],
          const fortran_double_precision * alpha_,
          const fortran_double_precision * beta_,
          const fortran_integer * nquad_,
          fortran_integer * ierr_);
```

NB: The definition of C/C++ data types **fortran_**xxx, and the mapping of Fortran external names to C/C++ external names, is handled by the C/C++ header file. That way, the same function or subroutine name can be used in C, C++, and Fortran code, independent of compiler conventions for mangling of external names in these programming languages.

DESCRIPTION

Compute the recursion coefficients and zeroth and first moments of the monic polynomials corresponding to the positive weight function

$$w(x, \alpha, \beta) = (1-x)^{\alpha} (1+x)^{\beta} (-\ln((1+x)/2))$$

with recursion relation ($n = 0, 1, 2, \dots$)

$$\begin{aligned} P_{n+1}^{\{\alpha, \beta\}}(x) = & (x - B_n^{\{\alpha, \beta\}}) * P_n^{\{\alpha, \beta\}}(x) \\ & - A_n^{\{\alpha, \beta\}} * P_{n-1}^{\{\alpha, \beta\}}(x) \end{aligned}$$

and initial conditions

$$\begin{aligned} P_{-1}^{\{\alpha, \beta\}}(x) &= 0 \\ P_0^{\{\alpha, \beta\}}(x) &= 1 \end{aligned}$$

Except in the weight function, the superscripts indicate dependence on α, β , NOT exponentiation.

The required moments are:

$$\begin{aligned} T_n^{\{\alpha, \beta\}} &= \int_0^1 w(x, \alpha, \beta) * (P_n^{\{\alpha, \beta\}}(x))^2 dx \\ S_n^{\{\alpha, \beta\}} &= \int_0^1 w(x, \alpha, \beta) * (P_n^{\{\alpha, \beta\}}(x))^2 x dx \end{aligned}$$

From these moments, the recursion coefficients are computed as:

$$\begin{aligned} A_n^{\{\alpha, \beta\}} &= T_n^{\{\alpha, \beta\}} / T_{n-1}^{\{\alpha, \beta\}} \\ B_n^{\{\alpha, \beta\}} &= S_n^{\{\alpha, \beta\}} / T_n^{\{\alpha, \beta\}} \end{aligned}$$

On entry:

alpha Power of $(1-x)$ in the integrand (**alpha** > -1).

beta Power of $(1+x)$ in the integrand (**beta** > -1).

nquad Number of quadrature points to compute. It must be less than the limit MAXPTS defined in the header file, *maxpts.inc*. The default value chosen there should be large enough for any realistic application.

On return:

a(0..nquad) Recursion coefficients: $\mathbf{a}(n) = A_n^{\{\alpha, \beta\}}$.

b(0..nquad) Recursion coefficients: $\mathbf{b}(n) = B_n^{\{\alpha, \beta\}}$.

s(0..nquad) First moments: $\mathbf{s}(n) = S_n^{\{\alpha, \beta\}}$.

t(0..nquad) Zeroth moments: $\mathbf{t}(n) = T_n^{\{\alpha, \beta\}}$.

ierr Error indicator:
 = 0 (success),
 1 (eigensolution could not be obtained),
 2 (destructive overflow),
 3 (**nquad** out of range),
 4 (**alpha** out of range).

AUTHORS

The algorithms and code are described in detail in the paper

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