CS 594-004 – Homework # 5
(due March 7th)

The purpose of this homework is for you to gain a more thorough understanding of the ideas discussed in the “Projection and its Importance in Scientific Computing” lecture.

1. Using the definition of P from slide #7, prove the formula
\[ P u = (u, e_1) e_1 + \ldots + (u, e_m) e_m \]

from slide #10.

2. Prove that the CGS and MGS, as defined on slides #11 and #12, are equivalent in exact arithmetic and that the vectors resulting from the algorithms are orthonormal.

3. Show that the algorithm on slide #16 results in Q (step 3) that is orthogonal, i.e. the algorithm yields \( A = Q L^T \), a QR factorization of A.

4. Find the projection in \( \text{span}\{x, x^3, x^5\} \) of \( f(x) = \sin(x) \) on the interval \([-1,1]\) using the inner product and norm as given on slide #34.