The Landscape of High-Performance Tensor Contractions

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[1] Paul Springer et al. "Design of a high-performance GEMM-like Tensor-Tensor Multiplication"

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- Tensor contractions can be thought of as generalized GEMMs
- Three approaches to tensor contractions:
  - Nested loops
  - Loops over GEMM (LoG)
  - Transpose-Transpose-GEMM-Transpose (TTGT)

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- Three approaches to tensor contractions:
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  - Loops over GEMM (LoG)
  - Transpose-Transpose-GEMM-Transpose (TTGT)
- We propose a novel approach: GETT[1]
  - Akin to a high-performance GEMM implementation

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Outline

- Approaches to Tensor Contractions:
  - Loops over GEMM (LoG)
  - Transpose-Transpose-GEMM-Transpose (TTGT)
  - GEMM-like Tensor-Tensor Multiply (GETT)
- Tensor Contraction Code Generator[2]
- Performance Evaluation

[2] Source code available at: `https://github.com/HPAC/tccg`

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Conceptual Idea

Identify 2D subtensors and contract them via GEMM

- $\mathcal{C}_{m_1,n_1} \quad \leftarrow \sum_{k_1} \mathcal{A}_{m_1,k_1} \mathcal{B}_{k_1,n_1}$

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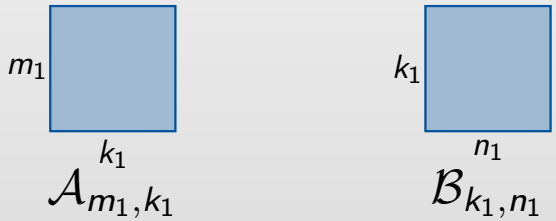


```
gemm(M1, N1, K1, A[:,:], B[:,:], C[:,:])
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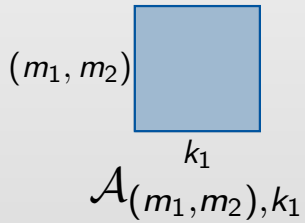

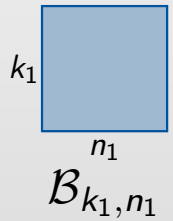

$$\texttt{gemm}(M_1 \times M_2,\ N_1,\ K_1,\ \mathcal{A}[:,:],\ \mathcal{B}[:,:],\ \mathcal{C}[:,:])$$

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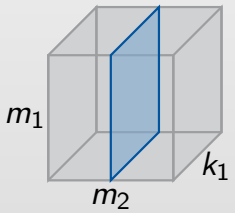
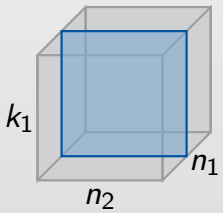


$\mathcal{A}_{m_1,m_2,k_1}$



$\mathcal{B}_{k_1,n_2,n_1}$

```
for( m2 = 0; m2 < M2; m2++ )
  for( n1 = 0; n1 < N1; n1++ )
    gemm(M1, N2, K1, A[:, m2, :], B[:, :, n1], C[:, n1, :, m2])
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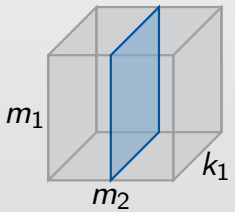

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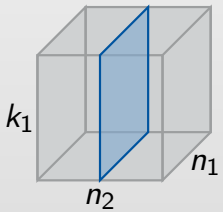

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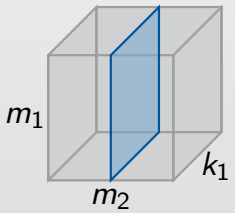

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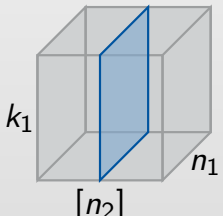

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```
for( m2 = 0; m2 < M2; m2++ )
  gemm_batch(M1, N1, K1, A[:, m2, :], B[:, n2, :], C[:, :, n2, m2], N2)
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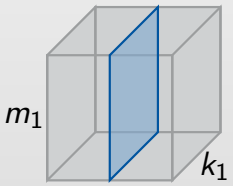

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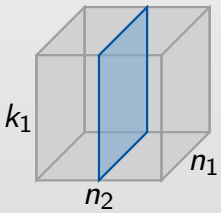

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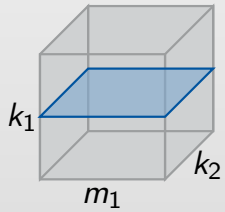

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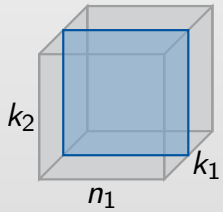

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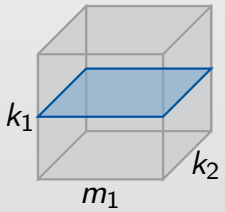

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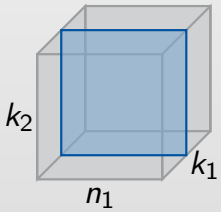

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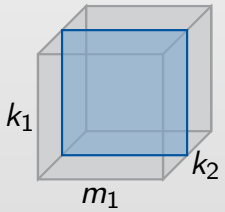

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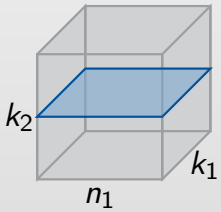

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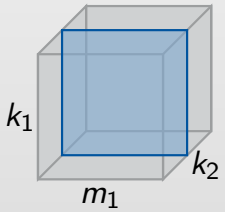

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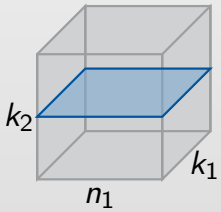

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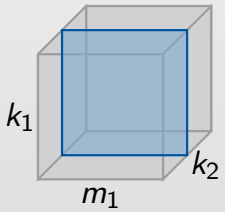

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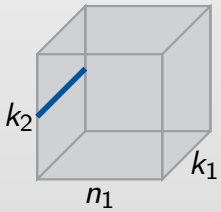

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```
for( n = 0; n1 < N1; n1++ )
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    gemv(M1, K1, A[:,:,k2]^T, B[k2,n1,:], C[:,n])
```

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  - GEMM's arithmetic intensity can be suboptimal

Map Tensor Contractions to BLAS

- Free indices of $\mathcal{A}$
  - $I_m := \{m_1, m_2, ..., m_\gamma\} = I_\mathcal{A} \cap I_\mathcal{C}$

[3] Di Napoli et al. "Towards an Efficient Use of the BLAS Library for Multilinear Tensor Contractions"

[4] Yang Shi et al. "Tensor Contractions with Extended BLAS Kernels on CPU and GPU"

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- Free indices of $\mathcal{B}$
  - $I_n := \{n_1, n_2, ..., n_\zeta\} = I_\mathcal{B} \cap I_\mathcal{C}$

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- Contracted indices
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[4] Yang Shi et al. "Tensor Contractions with Extended BLAS Kernels on CPU and GPU"

Map Tensor Contractions to BLAS

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... and more.

Transpose-Transpose-GEMM-Transpose (TTGT)

- Search space:

[5] Paul Springer et al. "TTC: A high-performance Compiler for Tensor Transpositions"

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[5] Paul Springer et al. "TTC: A high-performance Compiler for Tensor Transpositions"

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- Eliminate explicit transpositions
- Pack-and-transpose while moving data into the caches[5]
  - $\Rightarrow$ Complexity offloaded into packing routines

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```
1  // N-Loop
2  for n = 1 : nc : S_{I_n}
3     // K-Loop (contracted)
4     for k = 1 : kc : S_{I_k}
5        B̂ = identify_subtensor(B, n, k)
6        // pack B̂ into B̃ (L3 cache)
7        B̃ = packB(B̂)
8        // M-Loop
9        for m = 1 : mc : S_{I_m}
10          Â = identify_subtensor(A, m, k)
11          // pack Â into Ã (L2 cache)
12          Ã = packA(Â)
13          Ĉ = identify_subtensor(C, m, n)
14          // compute matrix-matrix product of ÃB̃
15          macroKernel(Ã, B̃, Ĉ, α, β)
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Tensor Contraction Code Generator (TCCG)

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- **Input:** Mathematical description of TC
  - e.g., `C[a,b,i,j] = A[i,k,a] * B[k,j,b];`
- **Output:** High-Performance C++ code

AICES

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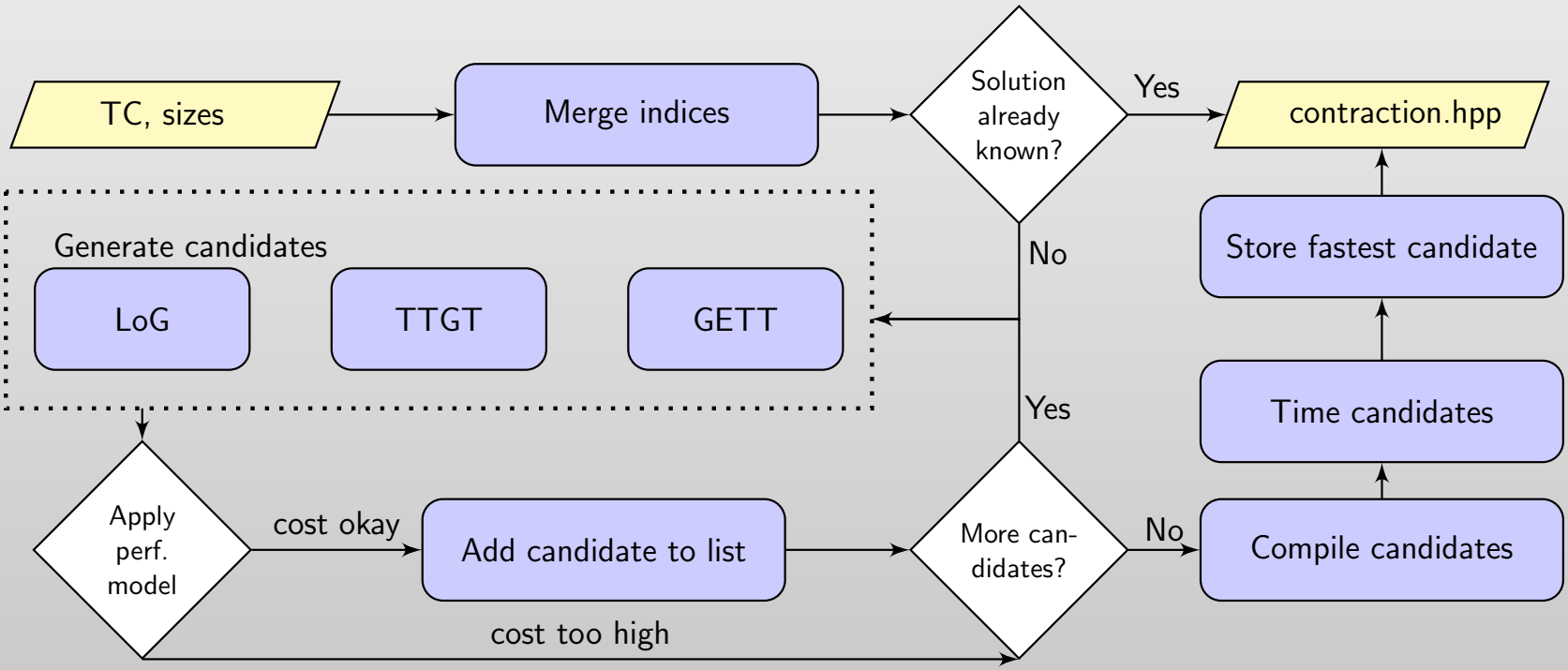
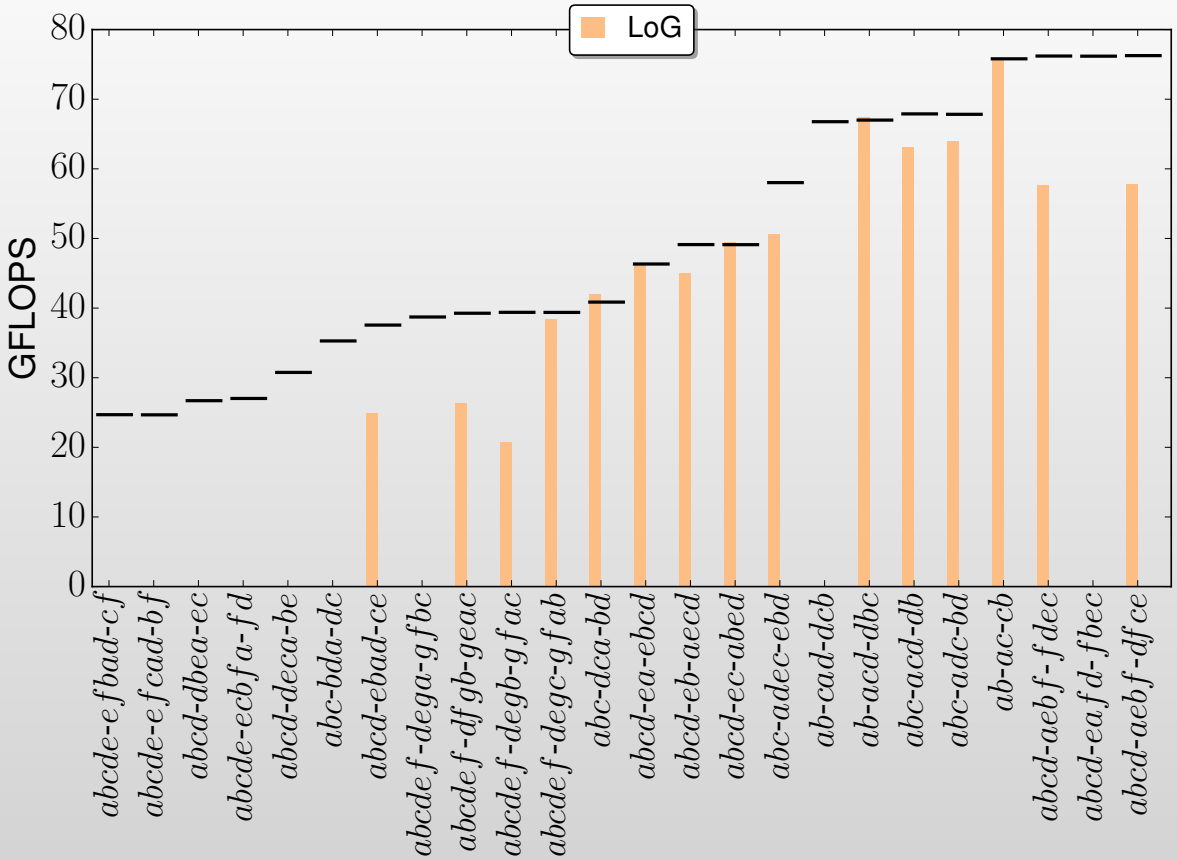


Figure: Schematic overview of TCCG.

Performance — Haswell (single core)



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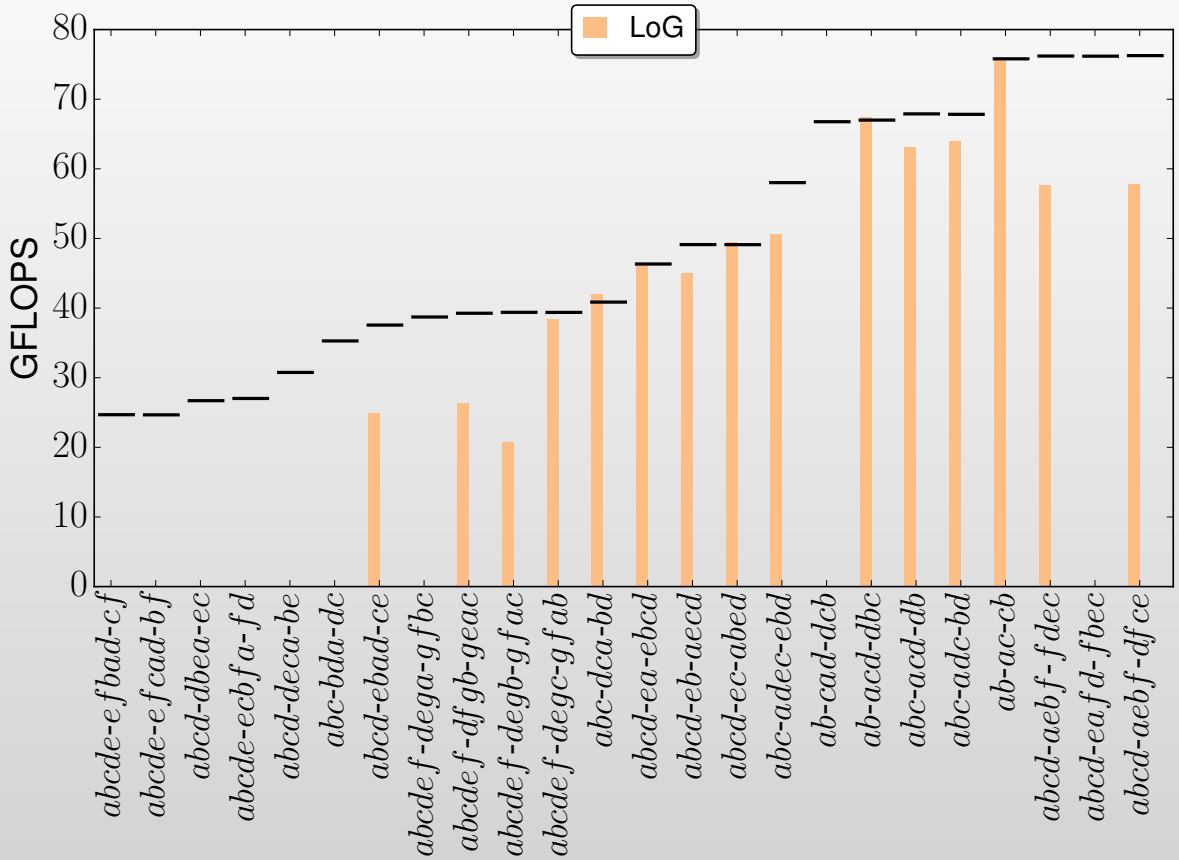

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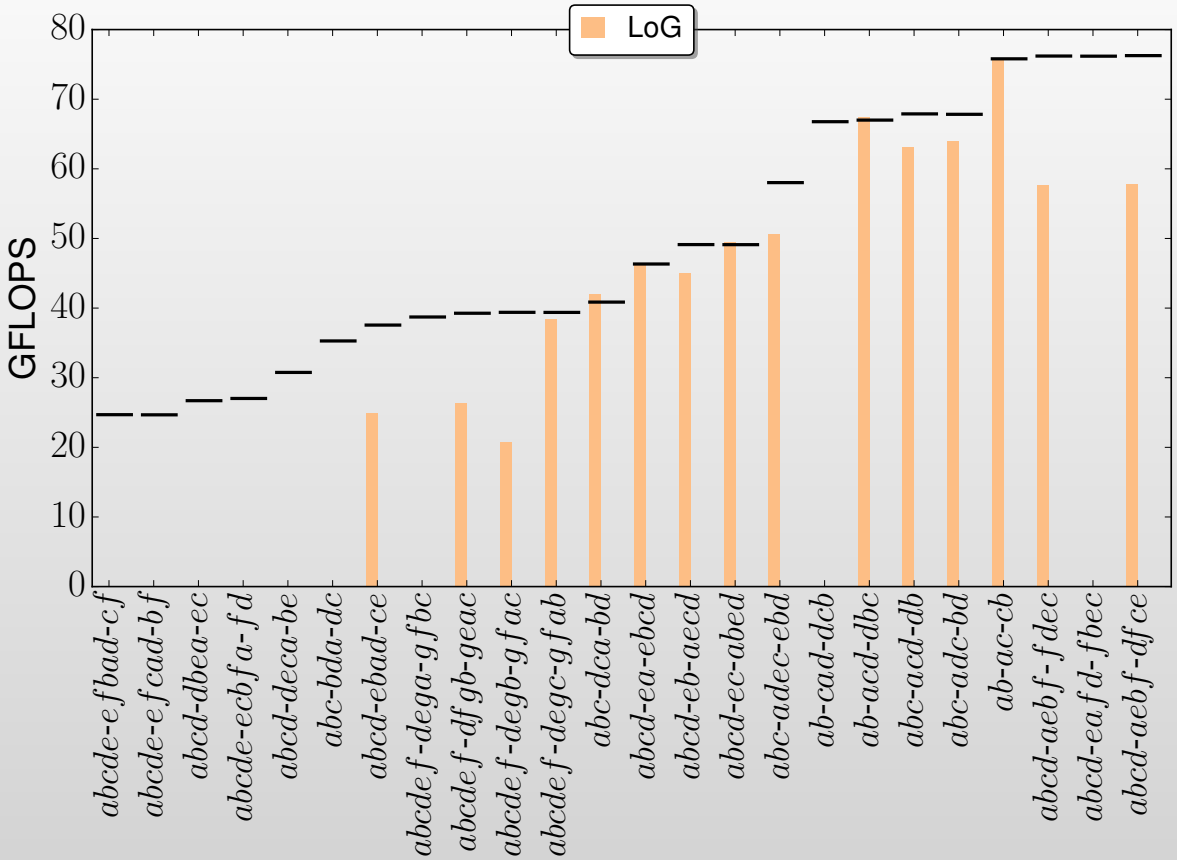

- Not all TCs can be implemented via LoG
- Mixed performance

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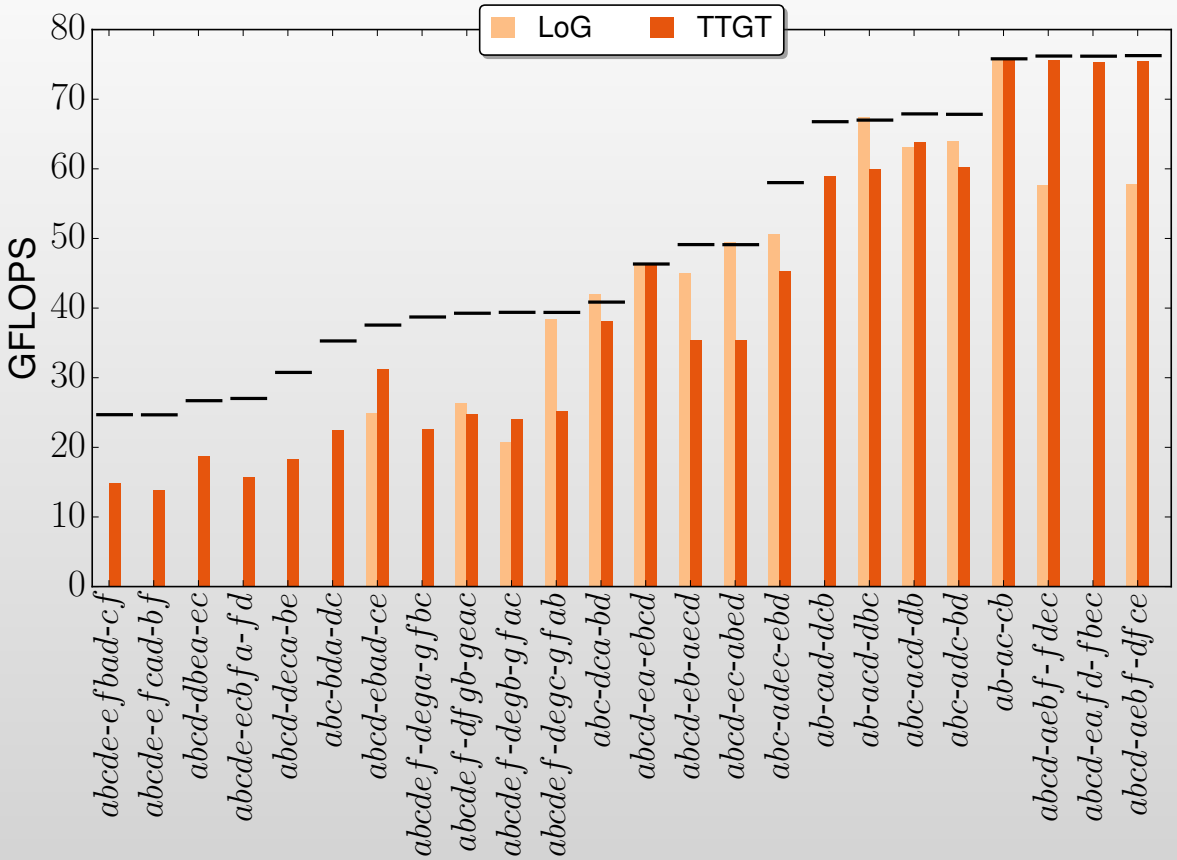

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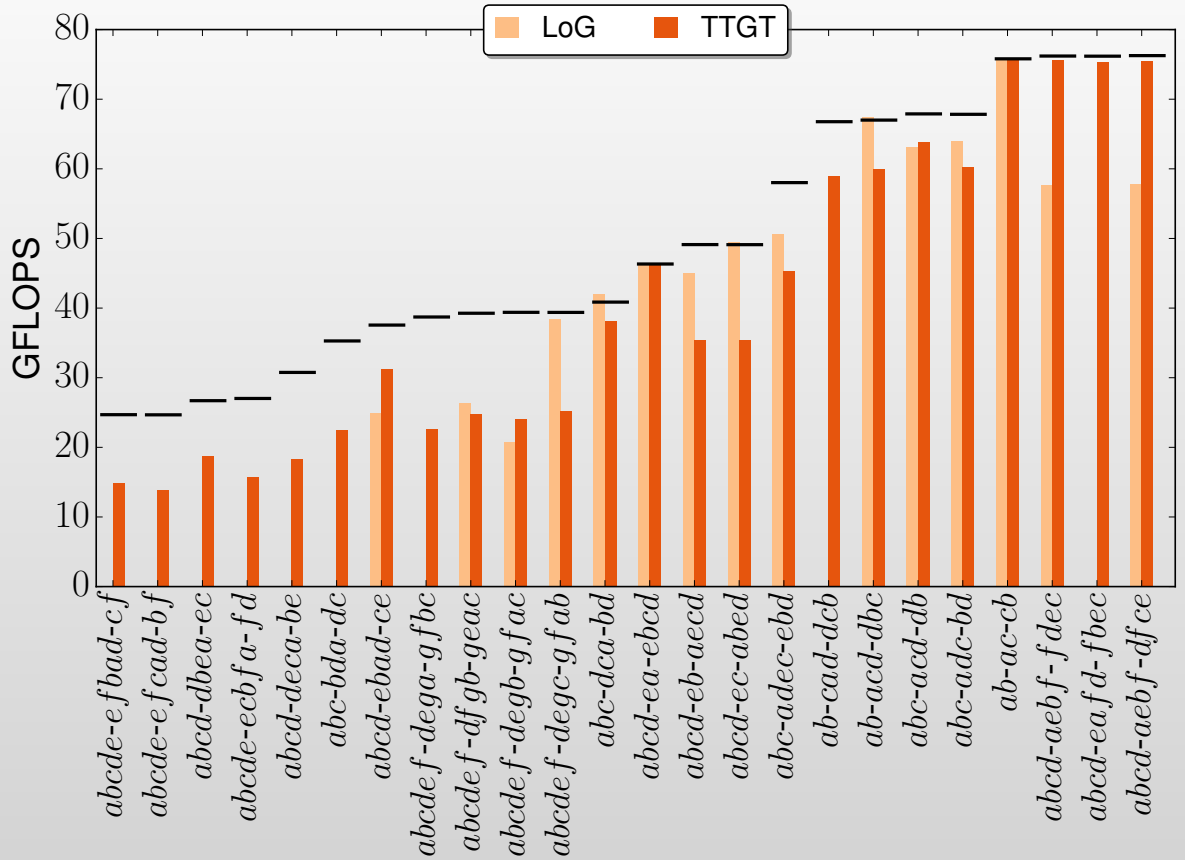

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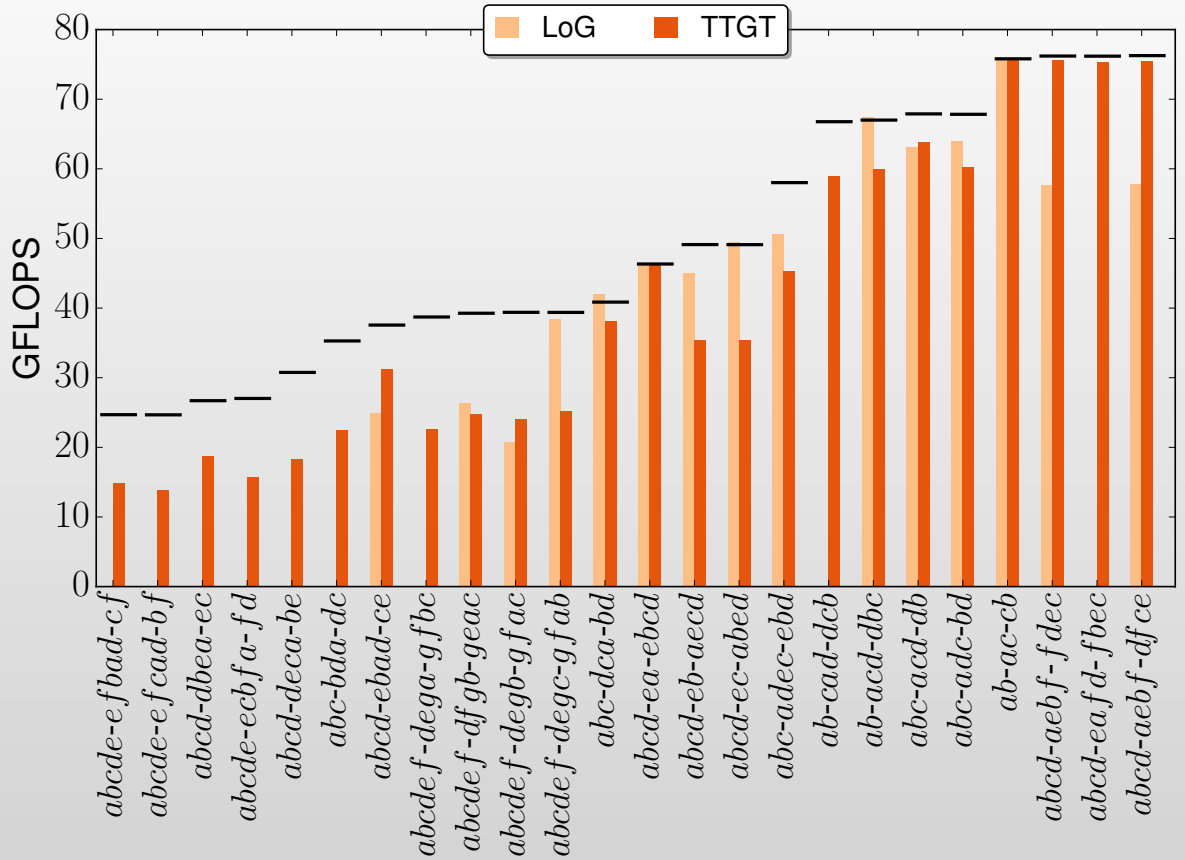
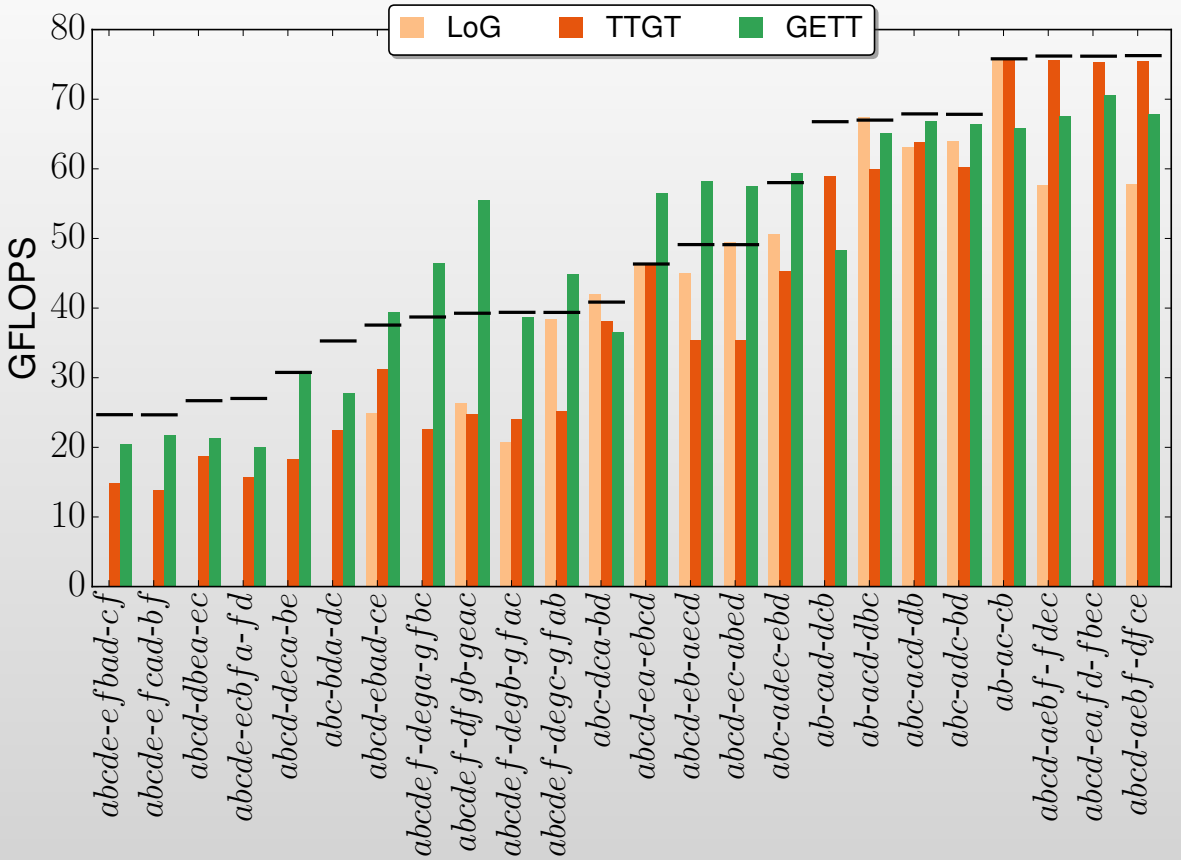


- TTGT: good for compute-bound TCs
- TTGT: bad for bandwidth-bound TCs

Performance — Haswell (single core)



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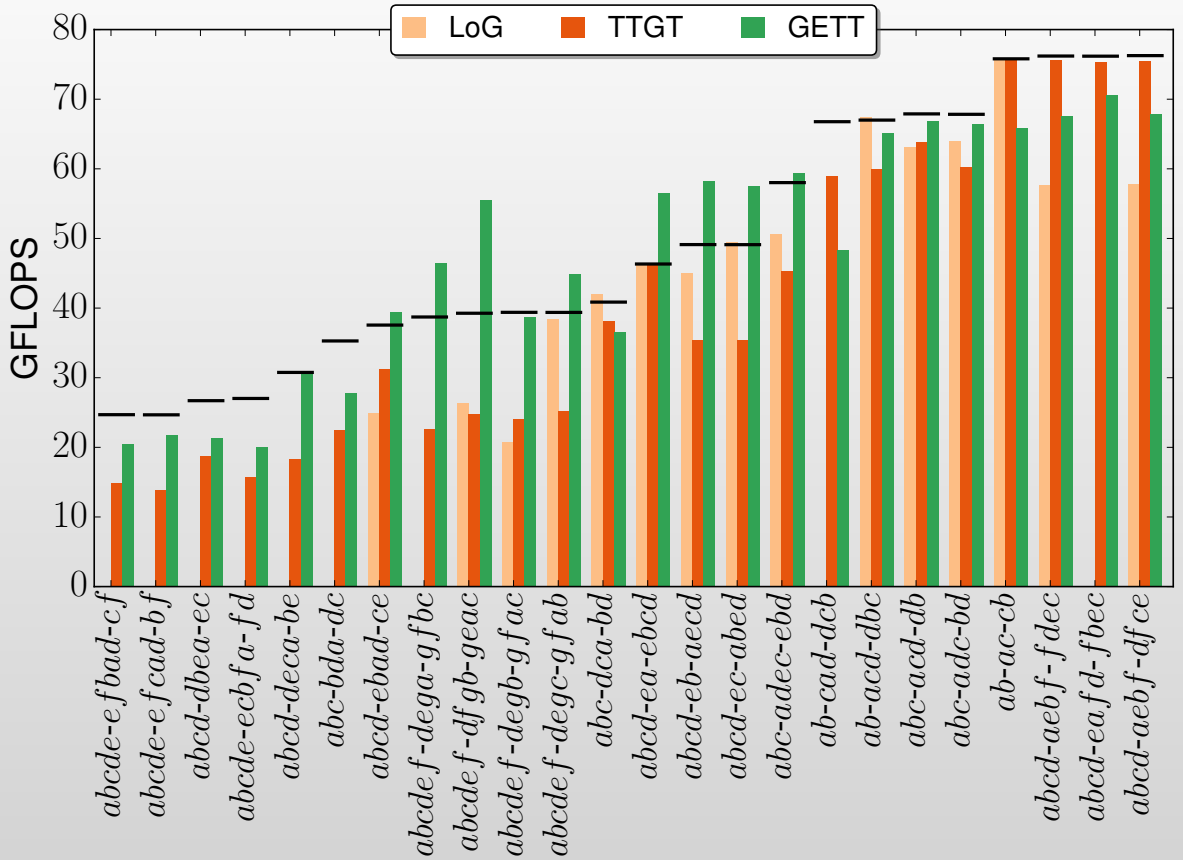

- GETT: excels for bandwidth-bound TCs

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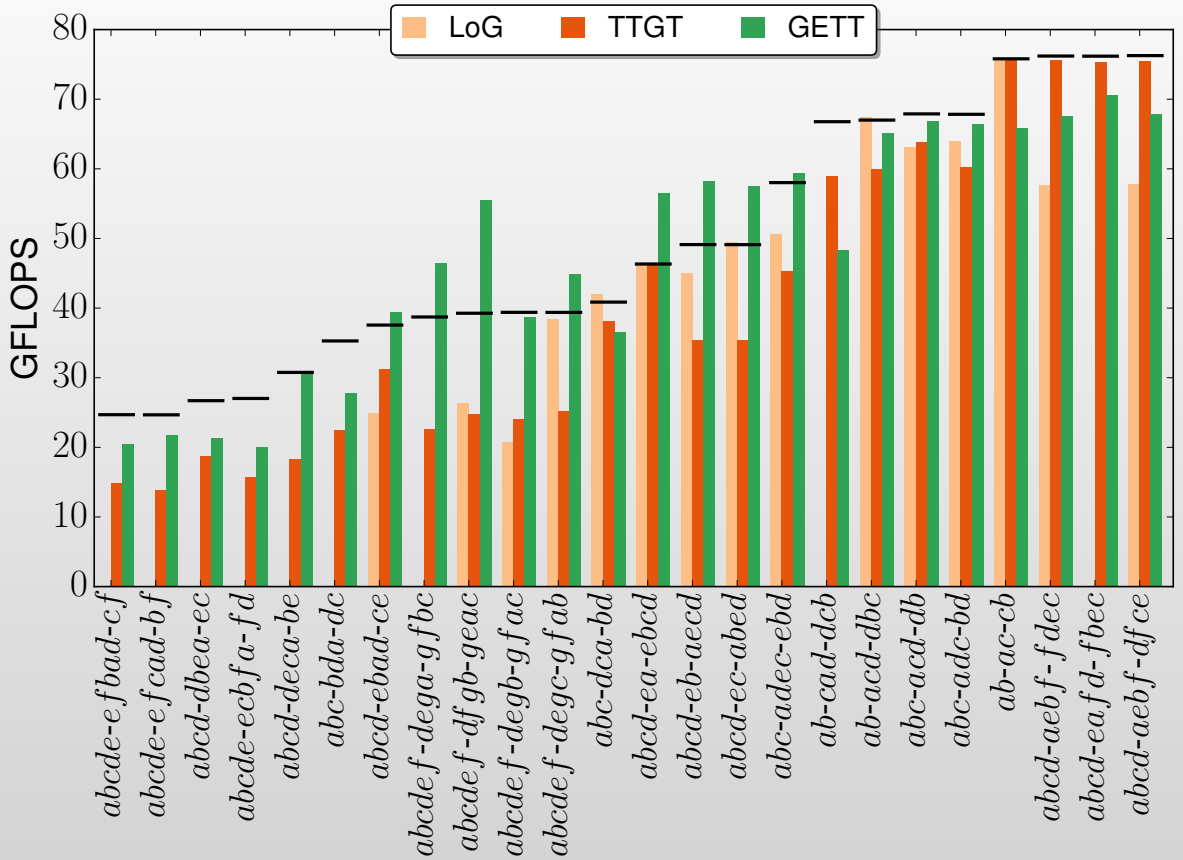
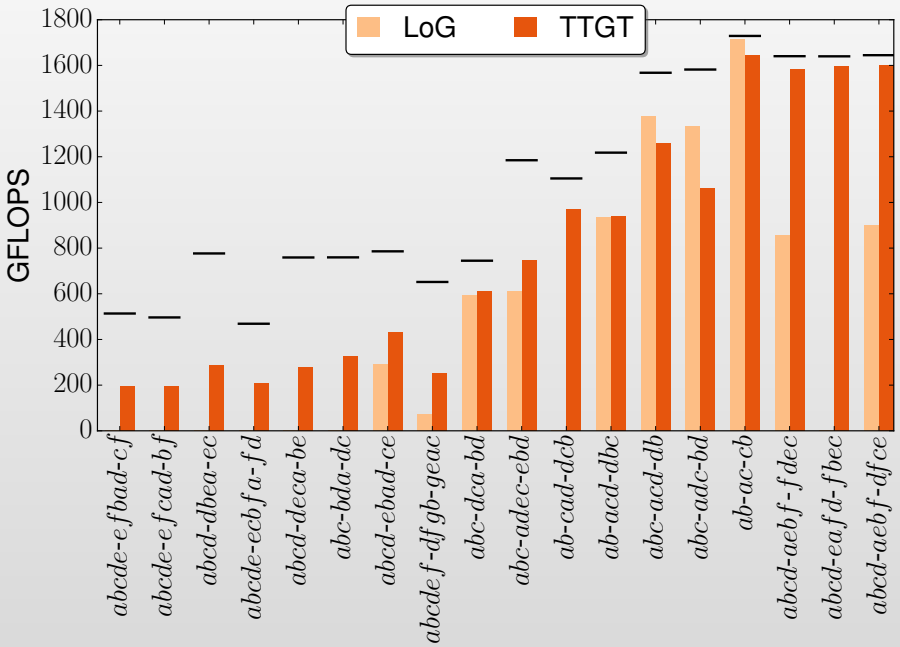


- GETT: excels for bandwidth-bound TCs
- GETT: good for compute-bound TCs

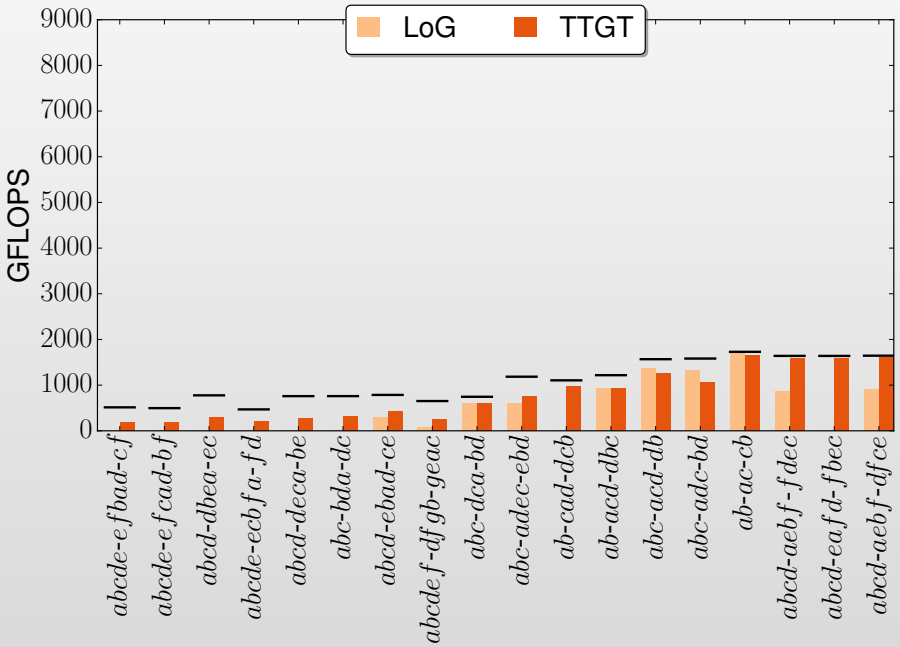
Performance — Multi-threaded



(a) 2×Intel Xeon E5-2680 v3

- Performance gap increases for bandwidth-bound TCs

Performance — Multi-threaded



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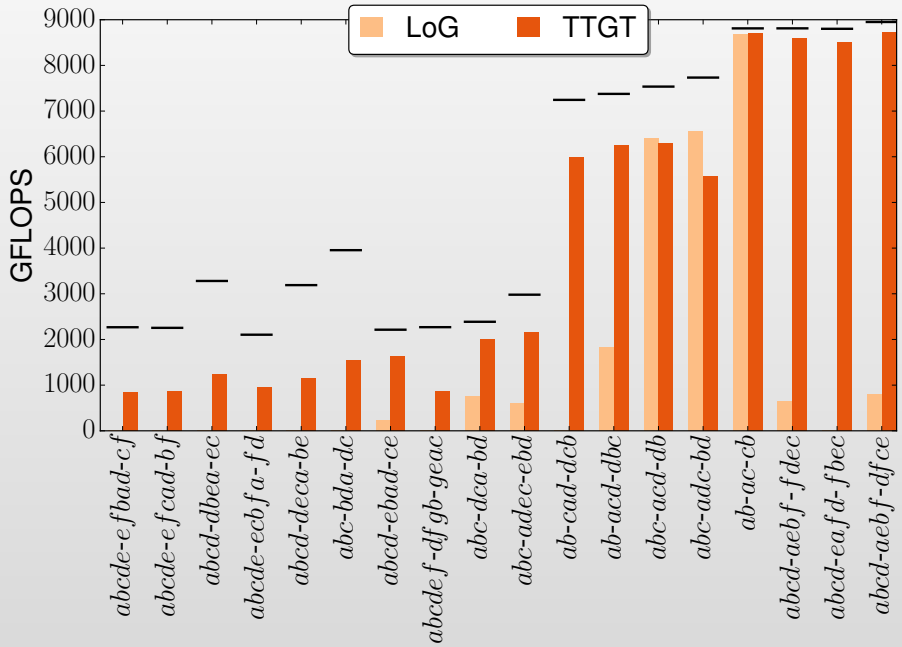

(b) NVIDIA Tesla P100

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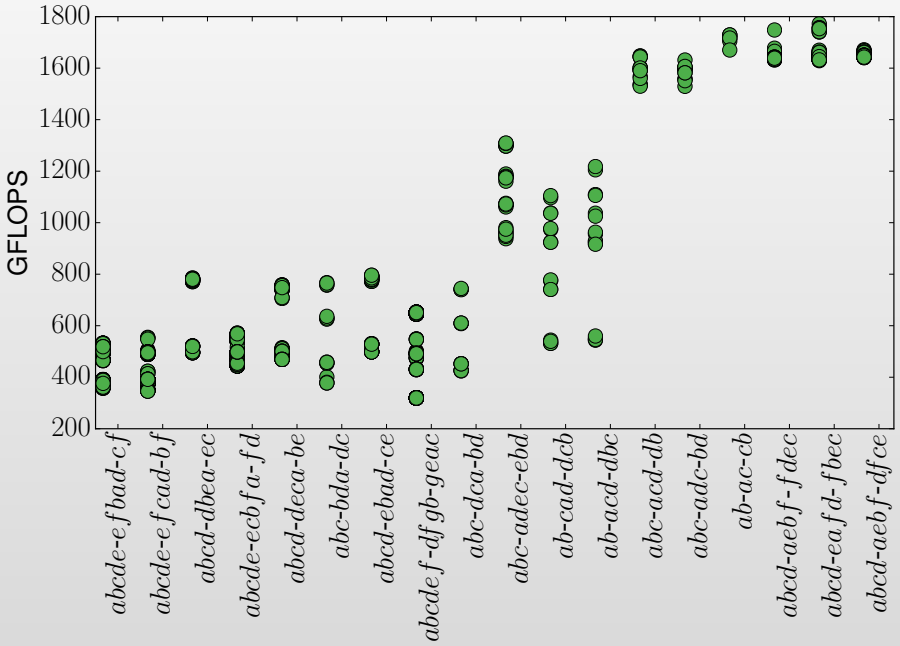
GEMM Performance — Multi-threaded



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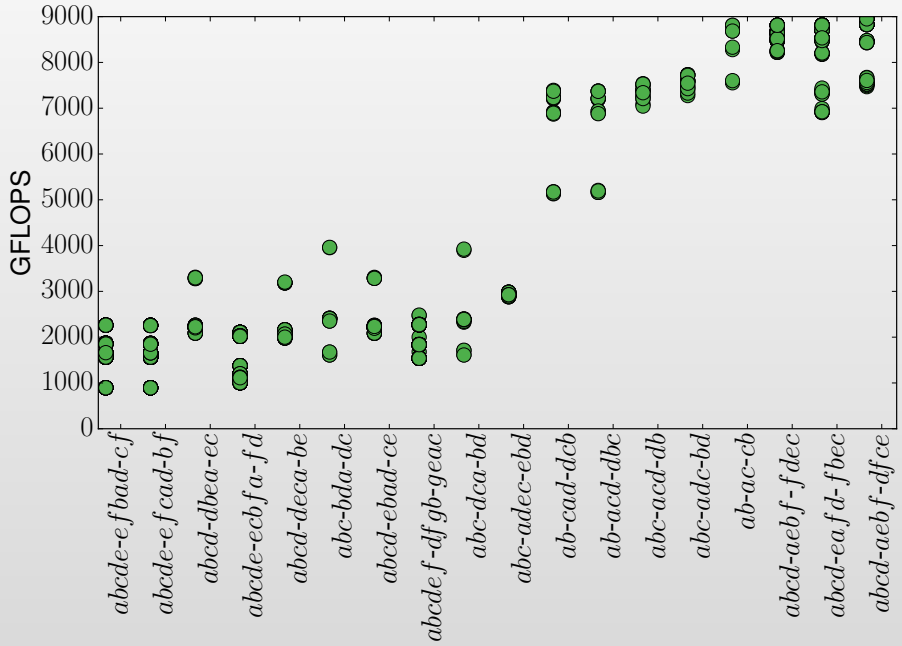

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[6] Elmar Peise et al. "On the Performance Prediction of BLAS-based Tensor Contractions"

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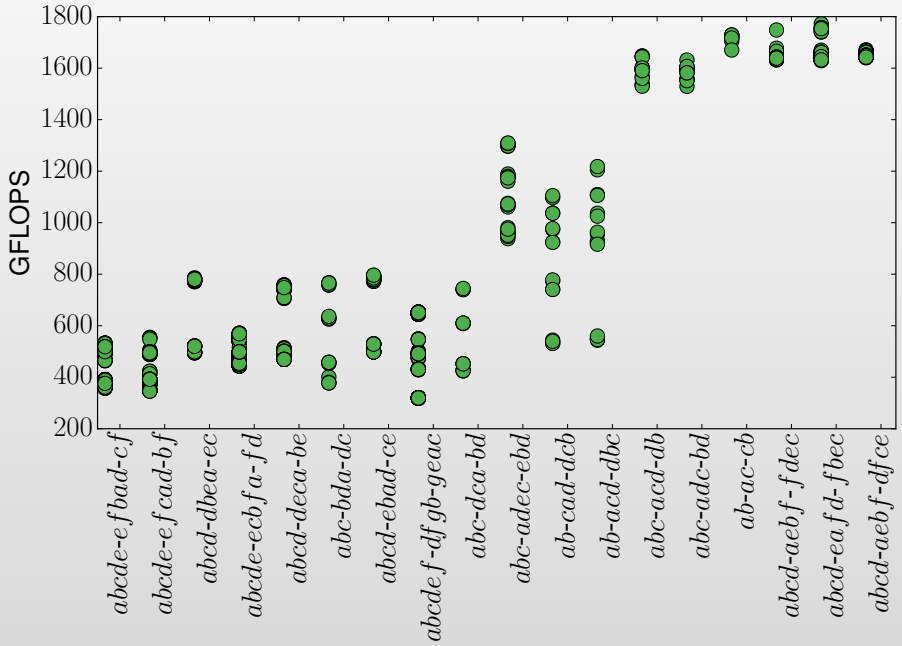

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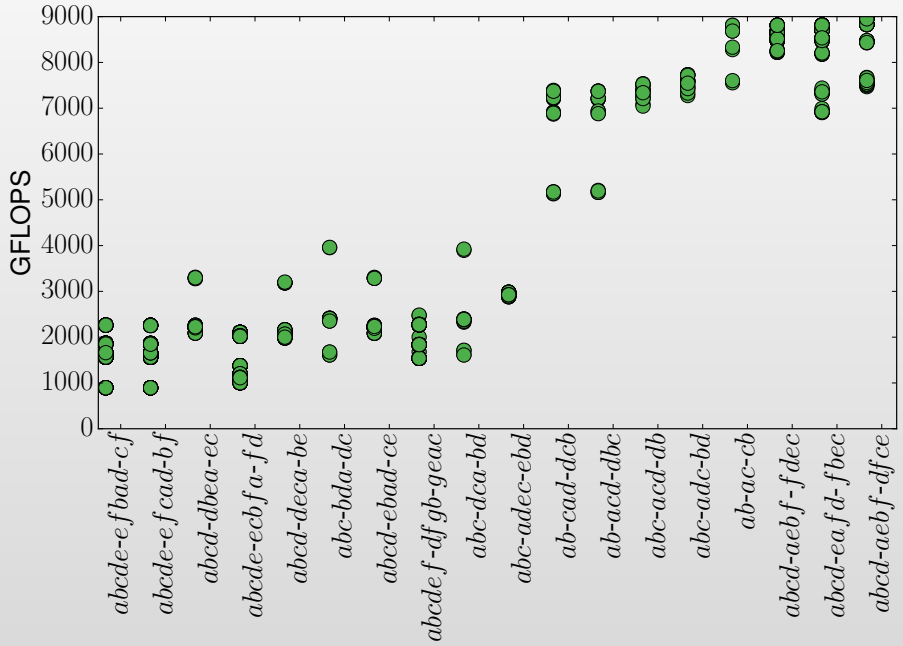

(b) NVIDIA Tesla P100

- Performance for equally-sized GEMMs varies greatly
  - For different settings: `opA`, `opB`, interchanged $\mathcal{A}$ and $\mathcal{B}$

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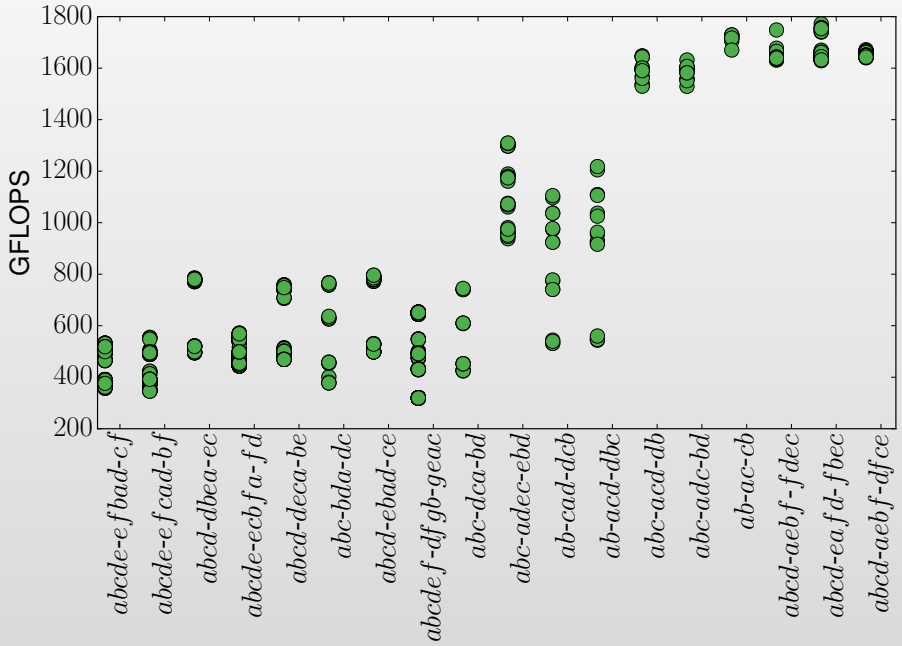

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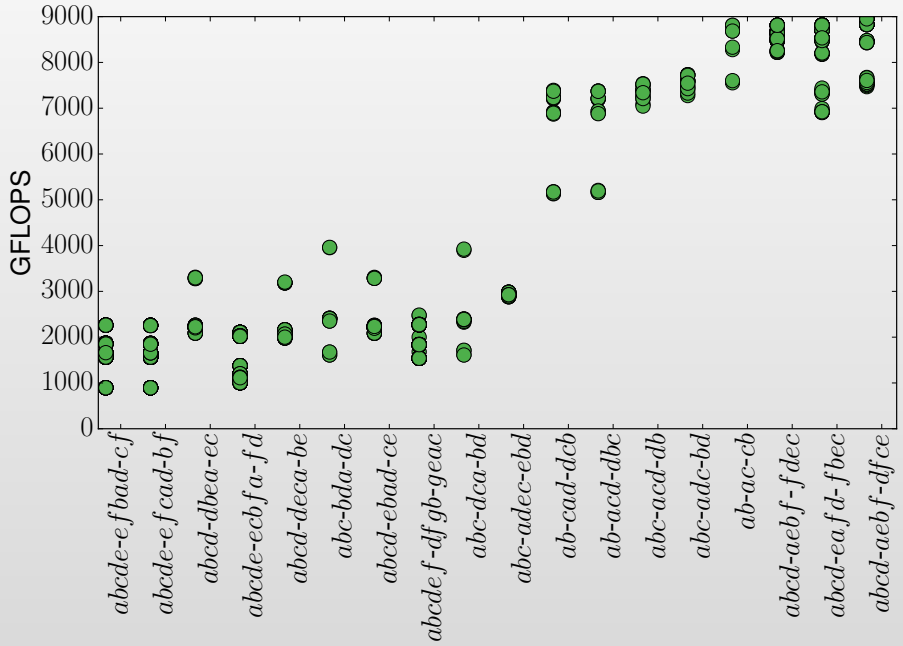

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- Performance for equally-sized GEMMs varies greatly
  - For different settings: `opA`, `opB`, interchanged $\mathcal{A}$ and $\mathcal{B}$
- Performance Model for TTGT and LoG:
  - Account for varying GEMM perf[6]

[6] Elmar Peise et al. "On the Performance Prediction of BLAS-based Tensor Contractions"

Conclusion

- A survey of different approaches to TCs has been presented
- GETT exhibits high performance across a wide range of TCs
- TCCG is available at `https://github.com/HPAC/tccg`

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Future Work

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- Parallelize GETT

Conclusion and Future Work

RWTH AACHEN UNIVERSITY

Conclusion

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- GETT exhibits high performance across a wide range of TCs
- TCCG is available at `https://github.com/HPAC/tccg`

Future Work

- Implement TC library based on GETT
- Parallelize GETT

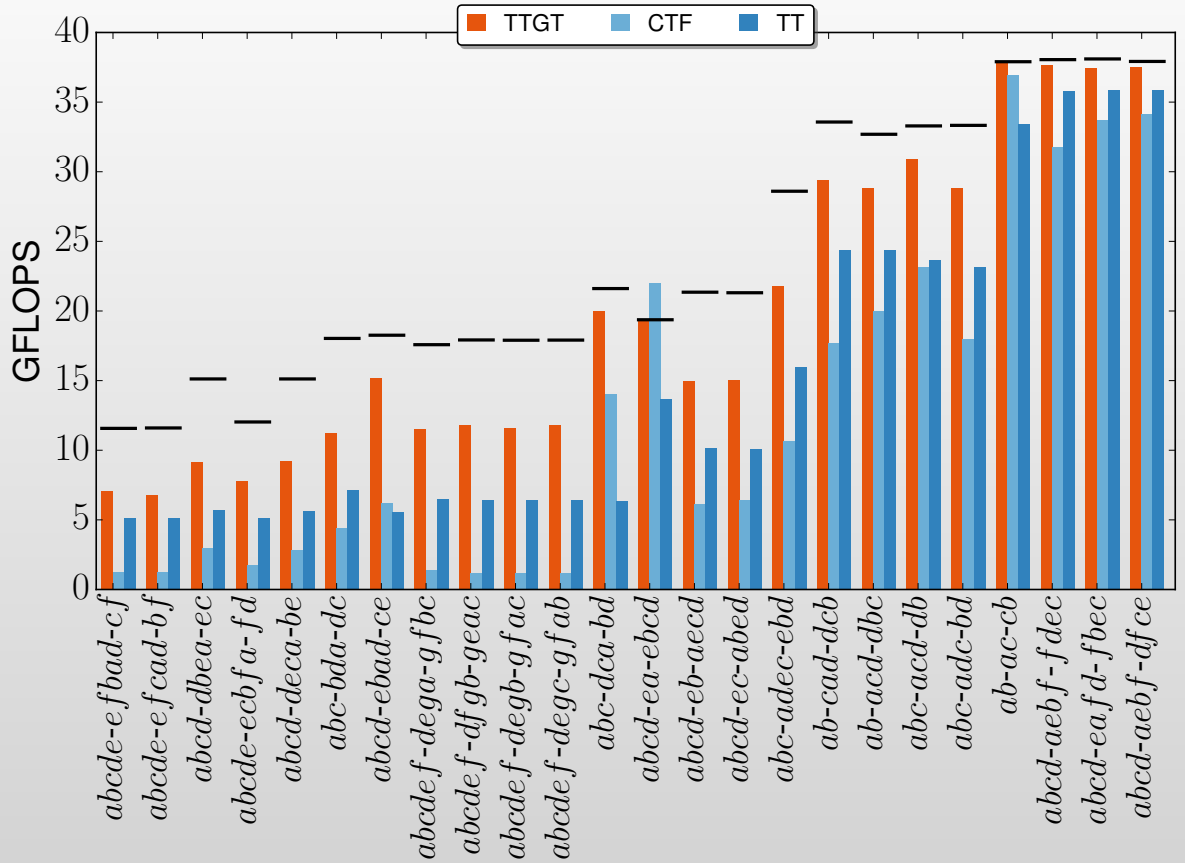
Thank you for your attention.

AICES

Performance

- Systems:
  - Intel *Xeon E5-2680 v3* CPU (Haswell)
  - NVIDIA *Tesla P100* GPU (Pascal)
- Compilers:
  - *icpc 16.0.1 20151021*
  - *nvcc v8.0.44*
- Benchmark
  - Collection of 48 TCs
  - Compiled from four publications
  - Each TC is at least 200 MiB
- Correctness checked against naive loop-based implementation

Performance - DP



- TTGT faster than CTF everywhere.
- TTGT good in compute-bound regime
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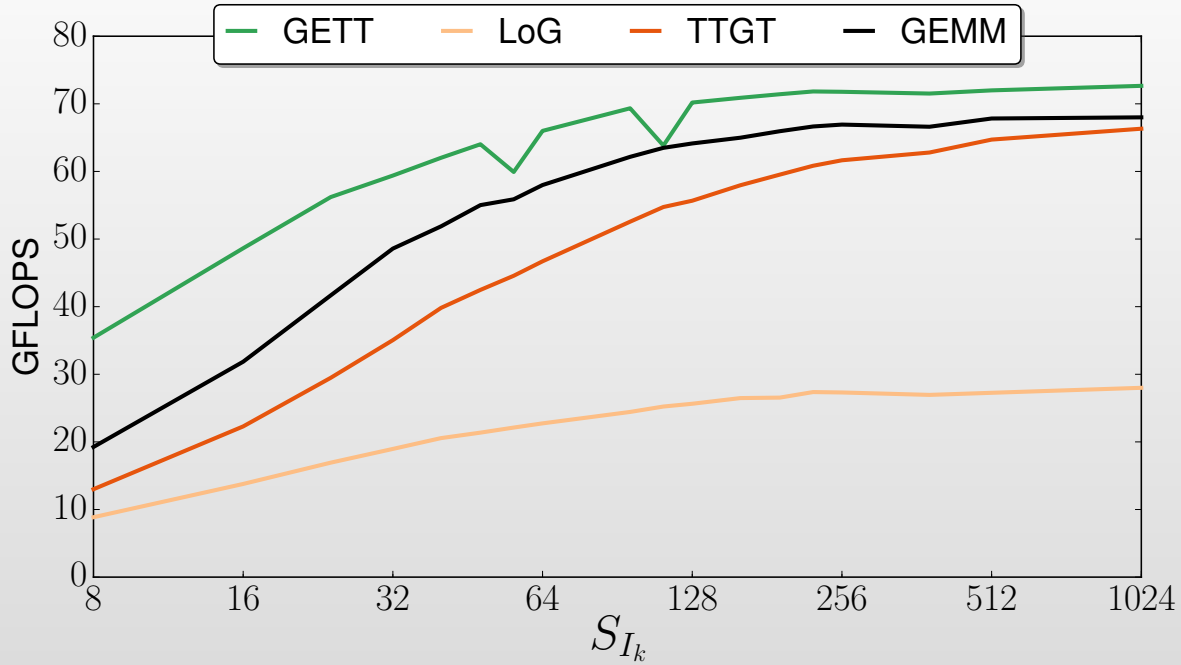
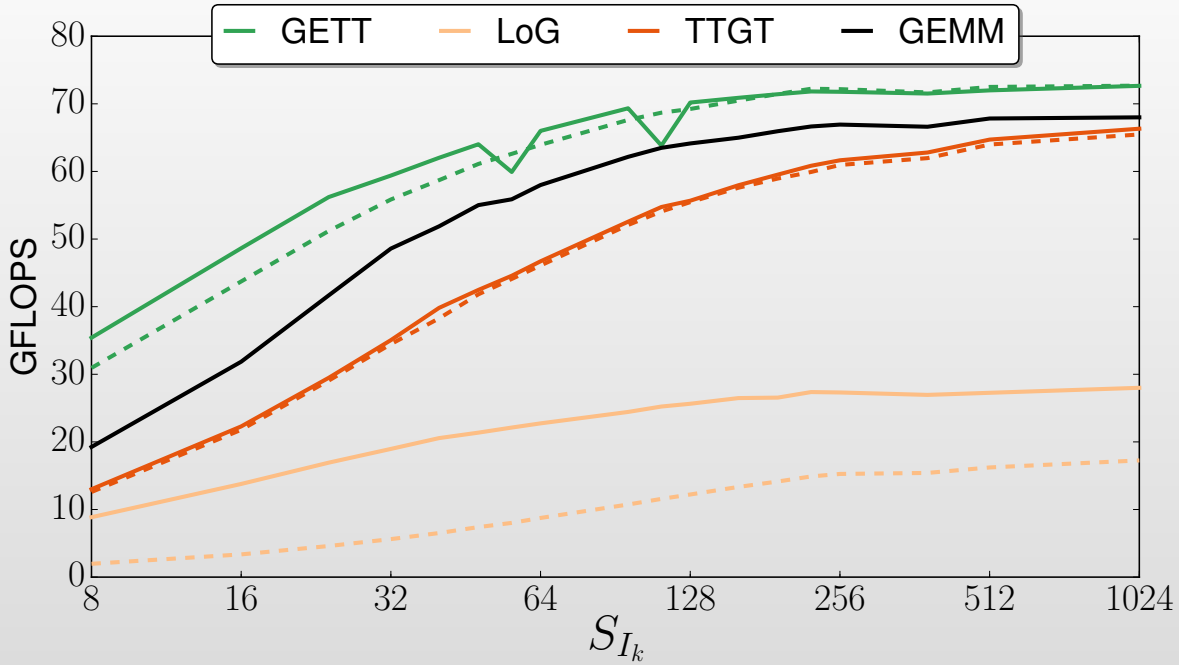
Performance: $m_1 n_1 m_2$-$m_1 k_1 m_2$-$n_1 k_1$



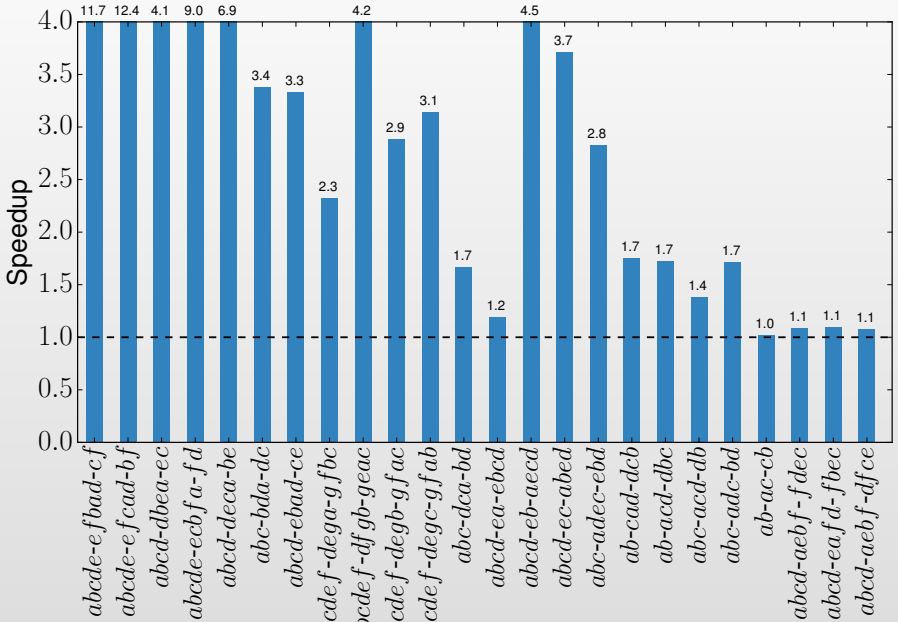
- GETT especially good in bandwidth-bound regime
  - GETT still attains up to 91.3% of peak floating-point performance
- TTGT poor in bandwidth-bound regime

Performance: $m_1 n_1 m_2 n_2$-$m_1 k_1 m_2$-$n_1 k_1 n_2$

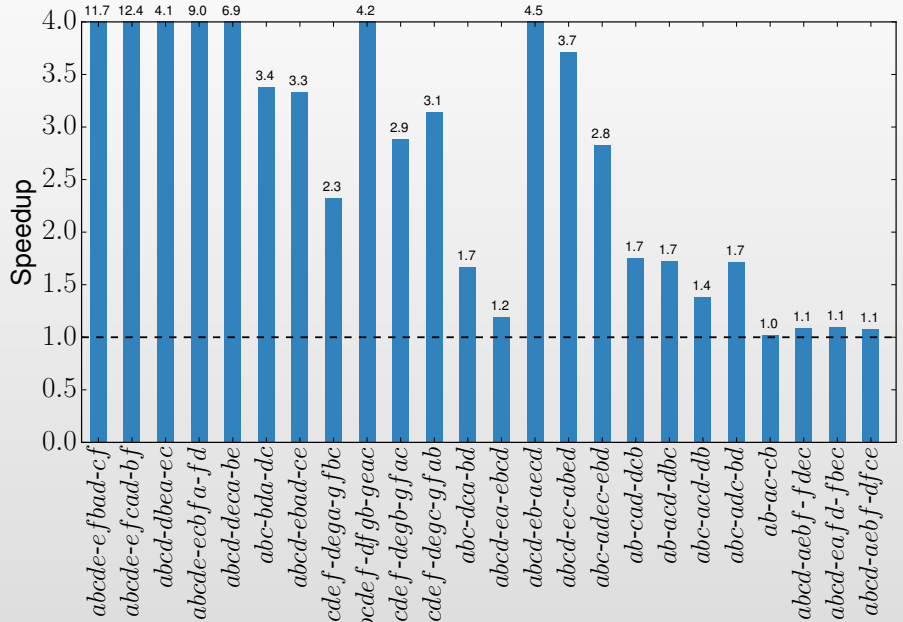


- GETT especially good in bandwidth-bound regime
  - GETT still attains up to 91.3% of peak floating-point performance
- TTGT poor in bandwidth-bound regime
- LoG performance can become arbitrarily bad
- GETT and TTGT barely affected by higher dimensions

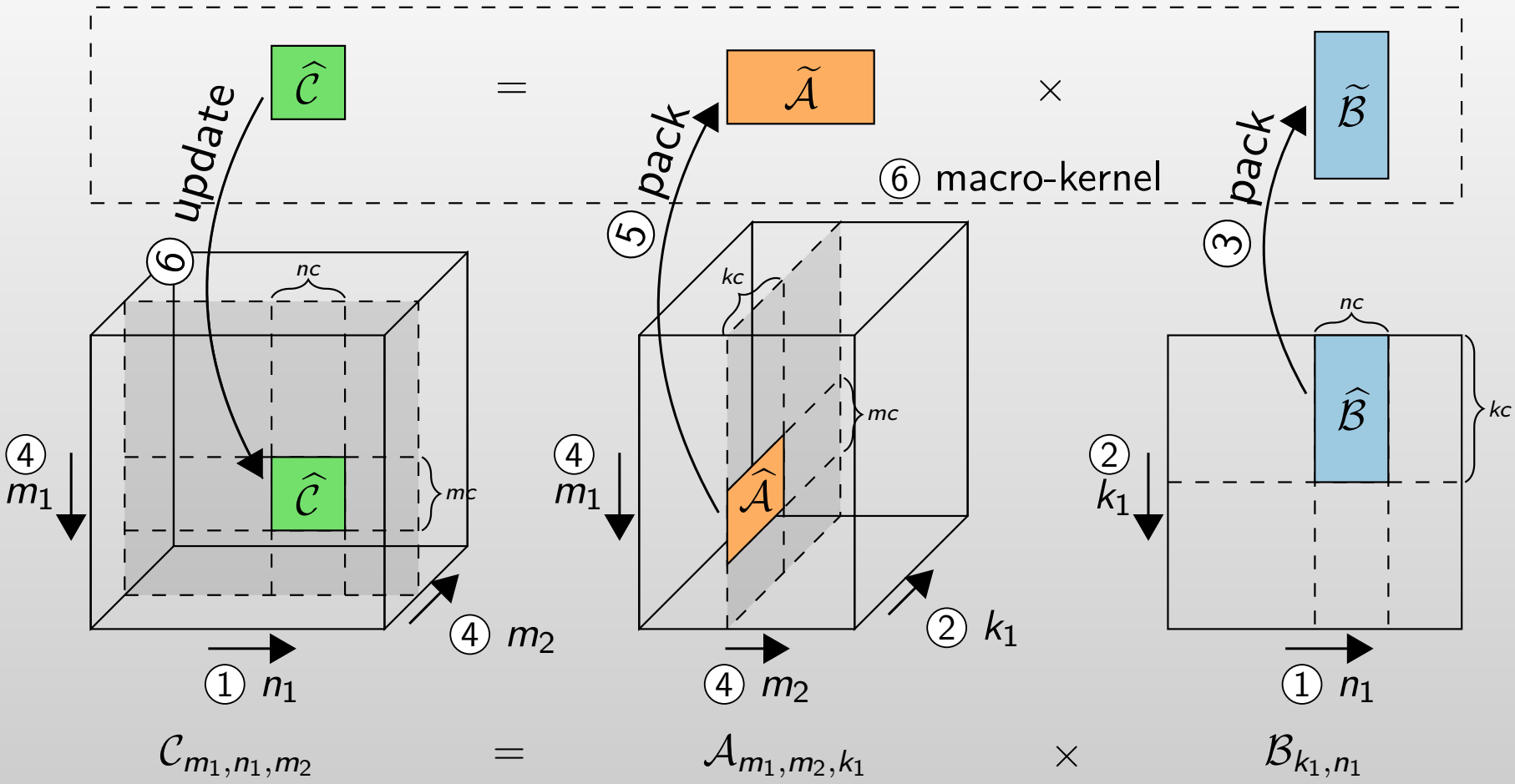
Speedup



(a) Single-Precision.

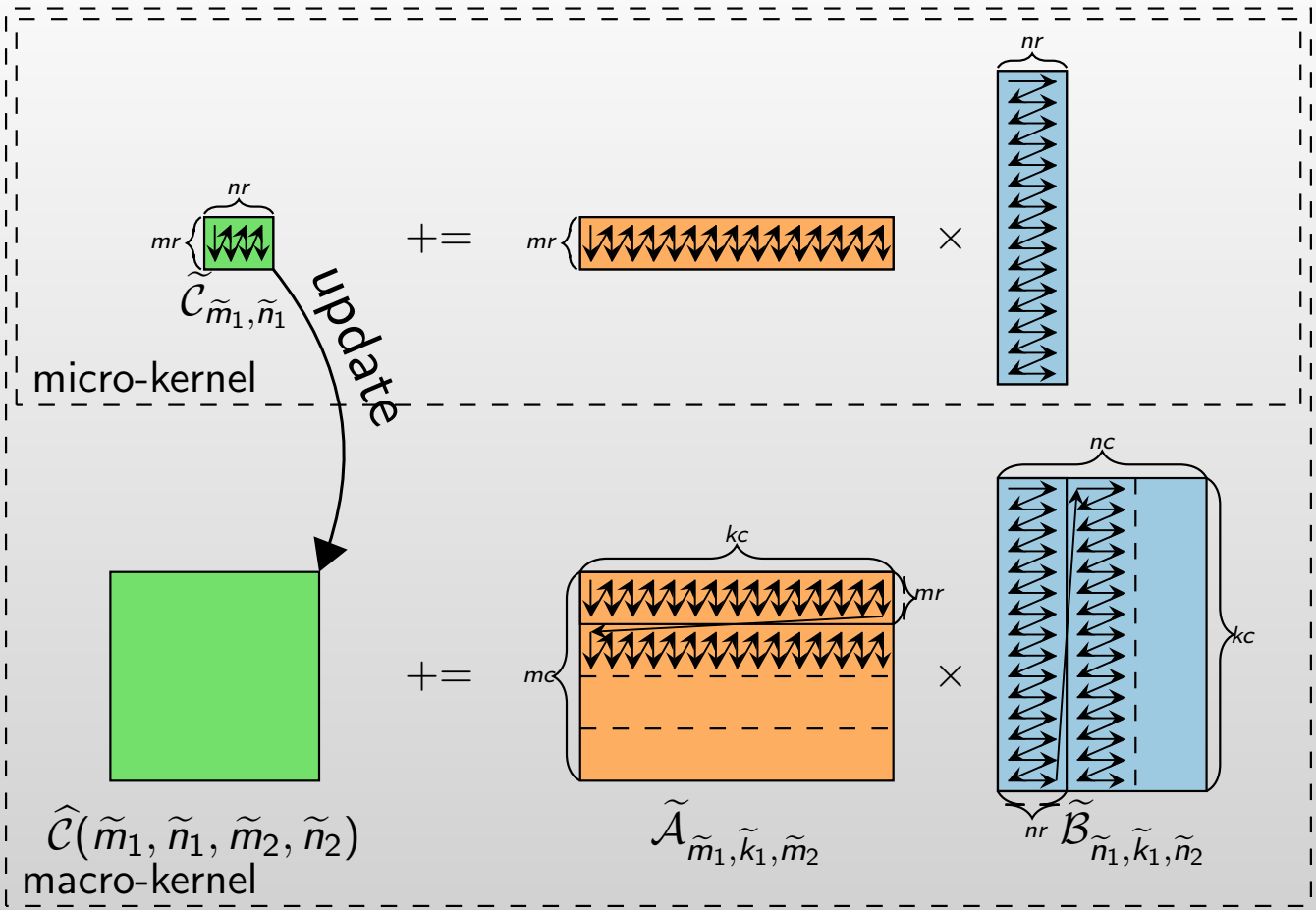


(b) Double-Precision.



$$\mathcal{C}_{m_1,n_1,m_2} = \mathcal{A}_{m_1,m_2,k_1} \times \mathcal{B}_{k_1,n_1}$$

GETT: Macro- /Micro-Kernel



- Blocking for L3, L2, L1 cache as well as registers

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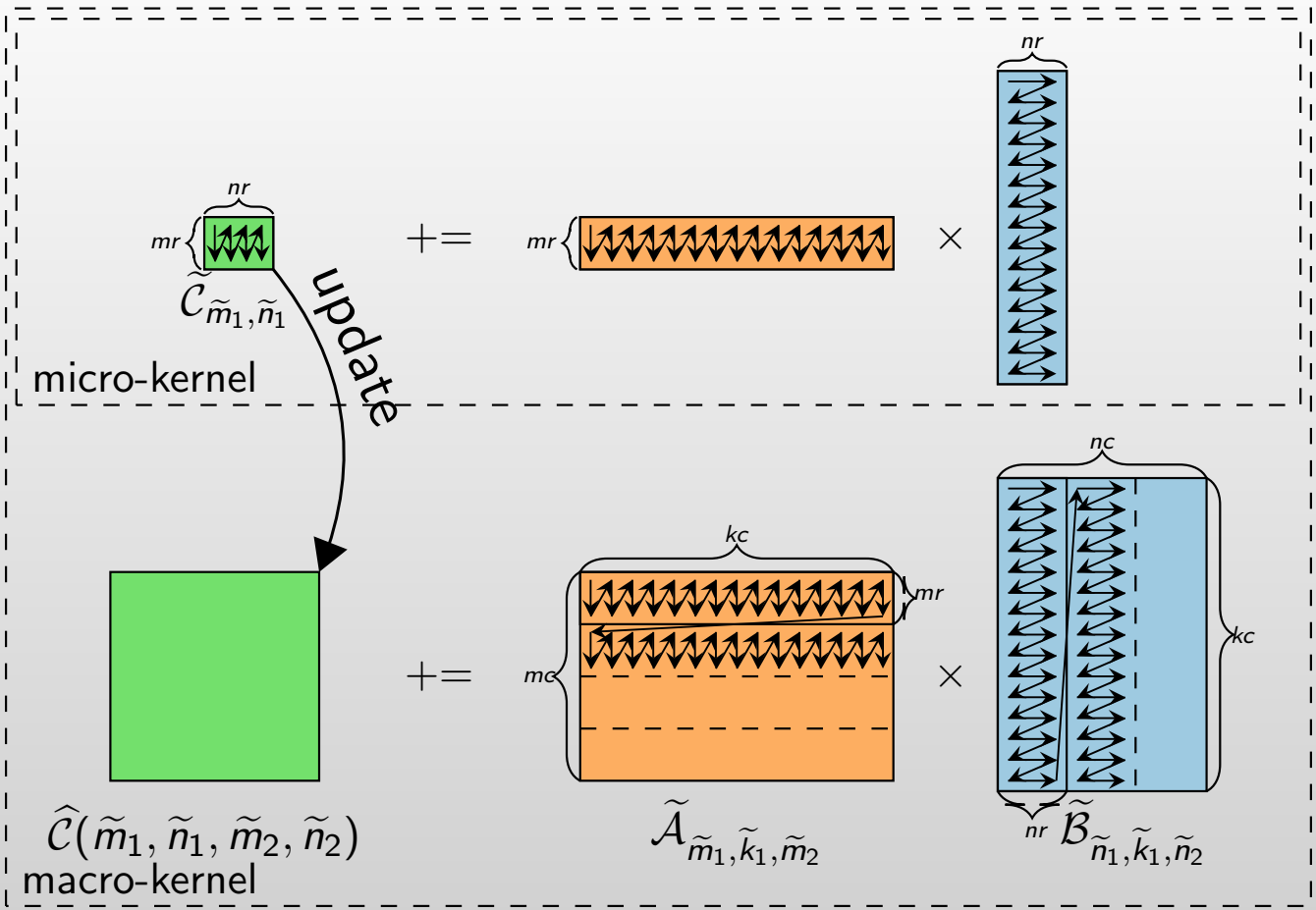
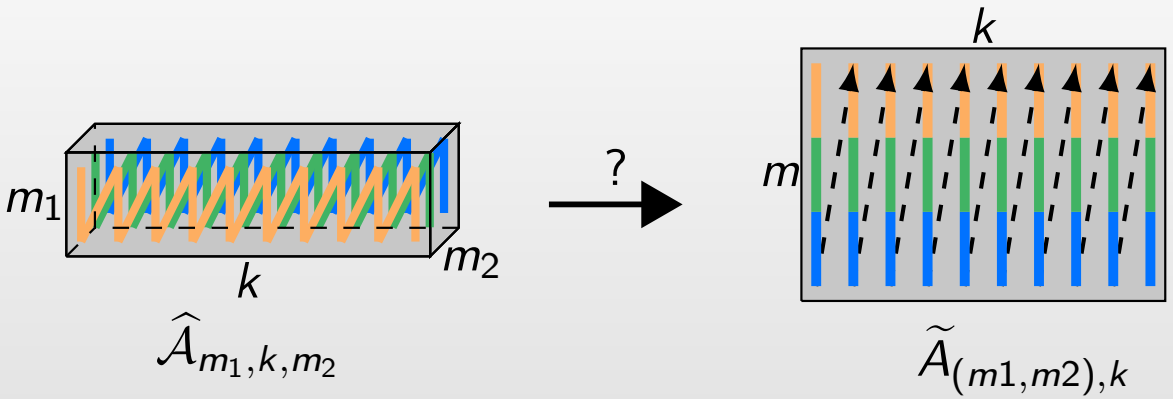


- Blocking for L3, L2, L1 cache as well as registers
- Written in AVX2 intrinsics

Packing via Tensor Transpositions



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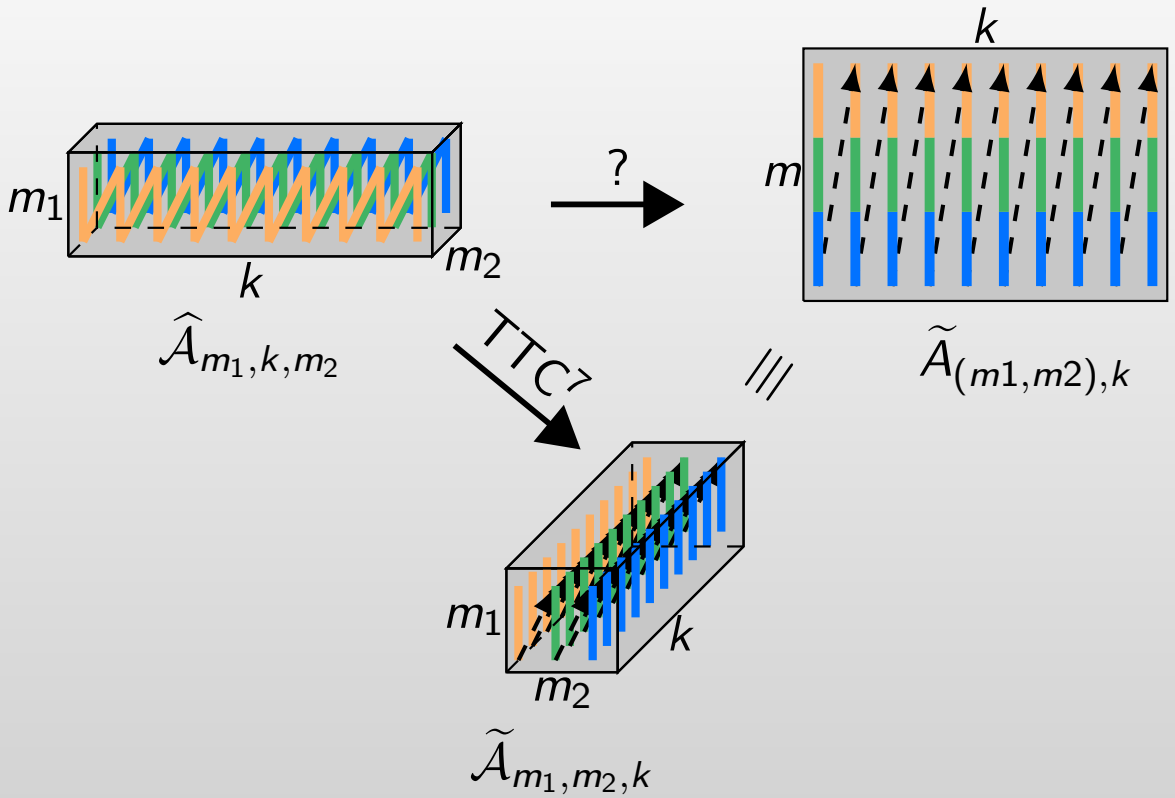
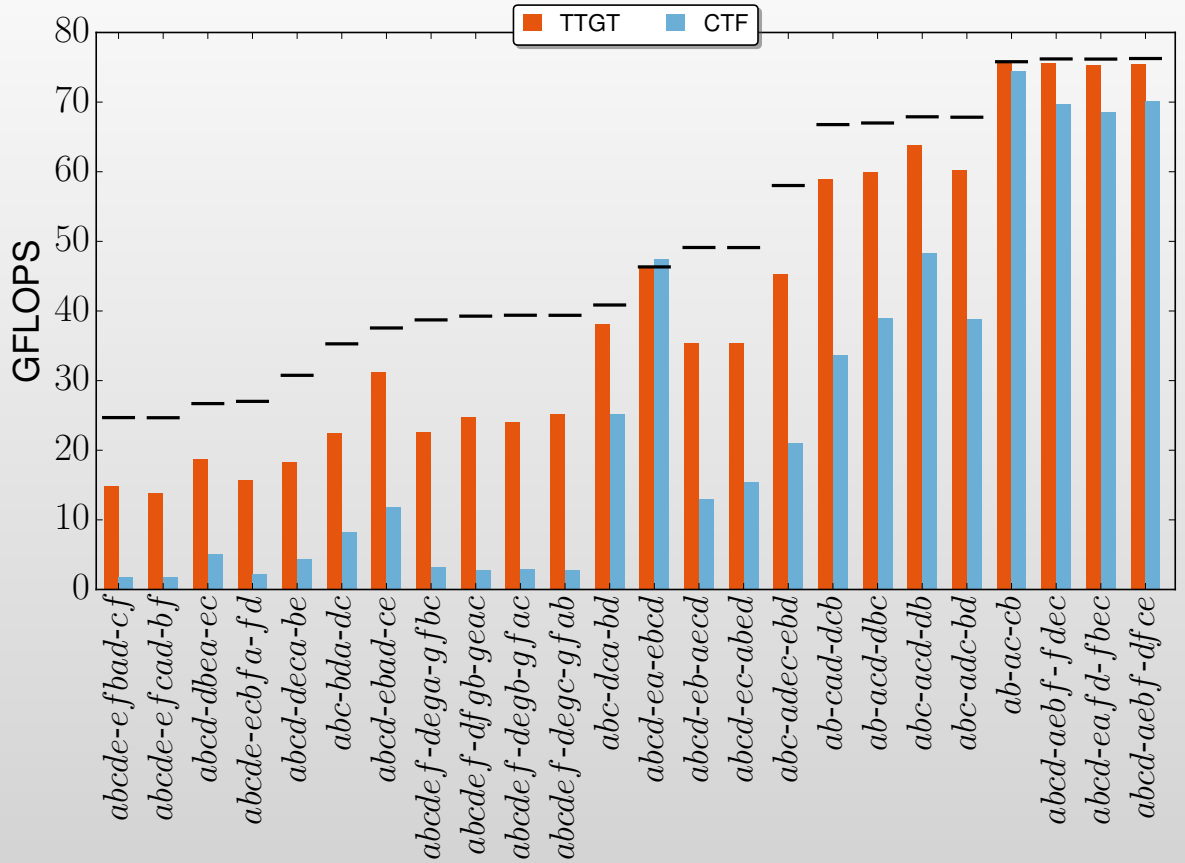


- Preserve stride-1 index
  - $\Rightarrow$ Efficient packing routines

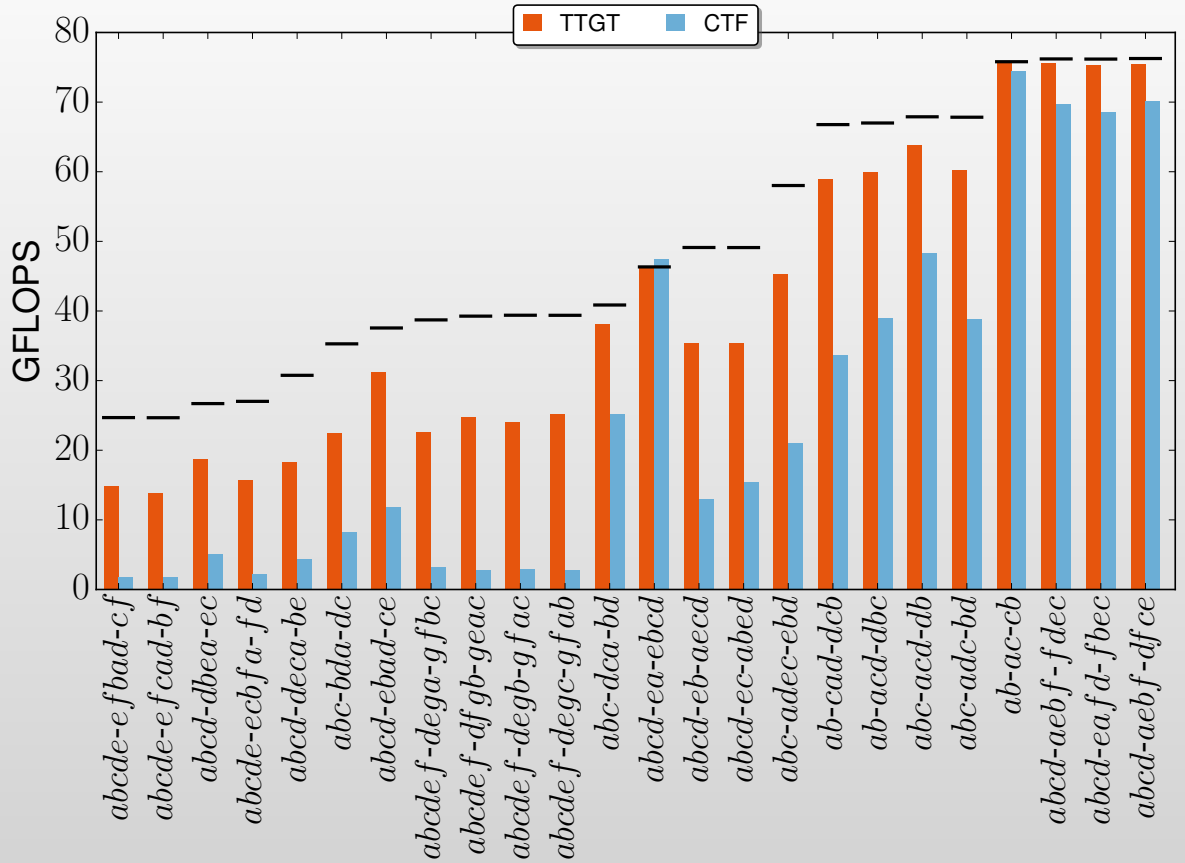
[7] Paul Springer et al. "TTC: A high-performance Compiler for Tensor Transpositions"

GETT: Summary

- Blocking for caches
- Blocking for registers
- Explicitly vectorized
- Use TTC to generate high-performance packing routines
  - Exploits full cache line (avoids non-stride-one memory accesses)
- Explore large search-space:
  - Different GEMM-variants (e.g., panel-matrix, matrix-panel)
  - Different permutations
  - Different values for mc, nc and kc
- Prune the search space via a performance model


TTGT
CTF
GFLOPS
0
10
20
30
40
50
60
70
80
abcde-efbad-cf
abcde-efcad-bf
abcd-dbea-ec
abcde-ecbfa-fd
abcd-deca-be
abc-bda-dc
abcd-ebad-ce
abcdef-dega-gfbc
abcdef-dfgb-geac
abcdef-degb-gfac
abcdef-degc-gfab
abc-dca-bd
abcd-ea-ebcd
abcd-eb-aecd
abcd-ec-abed
abc-adec-ebd
ab-cad-dcb
ab-acd-dbc
abc-acd-db
abc-adc-bd
ab-ac-cb
abcd-aebf-fdec
abcd-eafd-fbec
abcd-aebf-dfce

Performance



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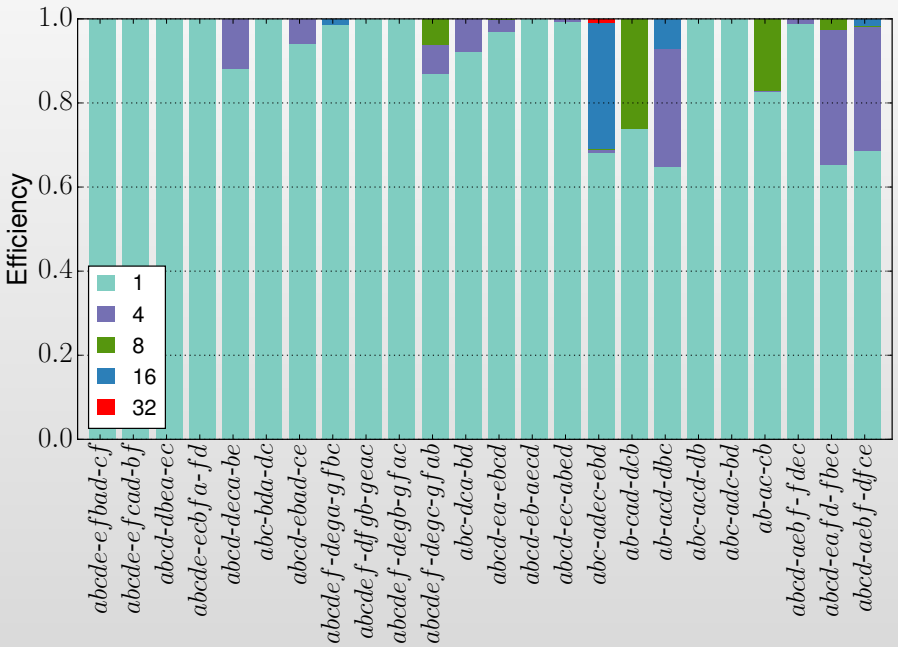
GETT Performance Model



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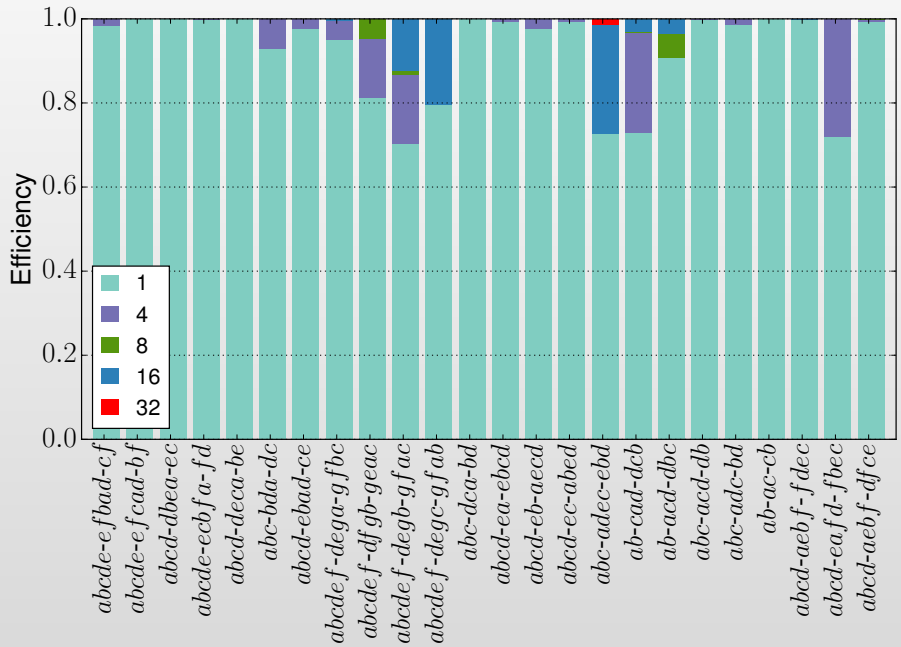

(b) Double-Precision.

Figure: Limit the GETT candidates to 1, 4, 8, 16 or 32, respectively.

- Average performance without search: 90.7% / 92.3%
- Average performance of the four best candidates: 98.3% / 97.2%