Communication Avoiding Algorithms in Plasma and Magma

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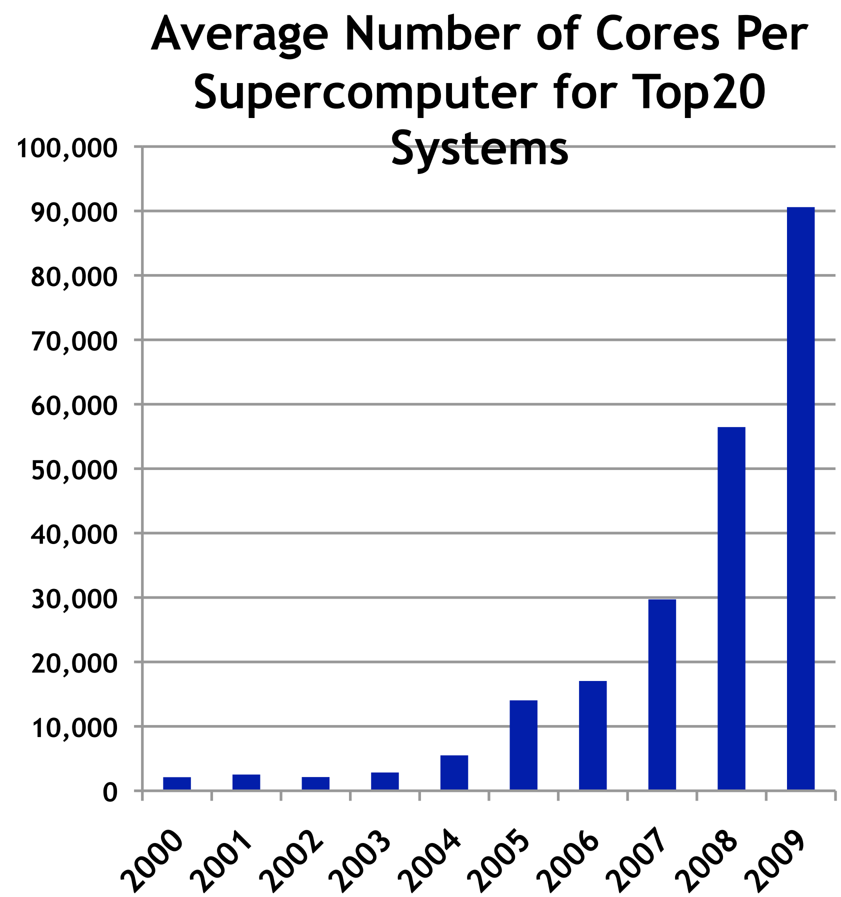
Potential System Architecture

Systems	2009	2018	Difference Today & 2018
System peak	**2 Pflop/s**	**1 Eflop/s**	**O(1000)**
Power	**6 MW**	**~20 MW**	
System memory	0.3 PB	32 - 64 PB **[.03 Bytes/Flop]**	O(100)
Node performance	125 GF	1,2 or 15TF	O(10) – O(100)
Node memory BW	25 GB/s	2 - 4TB/s **[.002 Bytes/Flop]**	O(100)
Node concurrency	12	O(1k) or 10k	O(100) – O(1000)
Total Node Interconnect BW	3.5 GB/s	200-400GB/s (1:4 or 1:8 from memory BW)	O(100)
System size (nodes)	18,700	O(100,000) or O(1M)	O(10) – O(100)
Total concurrency	225,000	O(billion) [O(10) to O(100) for latency hiding]	O(10,000)
Storage	15 PB	500-1000 PB (>10x system memory is min)	O(10) – O(100)
IO	0.2 TB	60 TB/s (how long to drain the machine)	O(100)
MTTI	days	O(1 day)	- O(10)

Factors that Necessitate Redesign of Our Software

- Steepness of the ascent from terascale to petascale to exascale
- Extreme parallelism and hybrid design
  - Preparing for million/billion way parallelism
- Tightening memory/bandwidth bottleneck
  - Limits on power/clock speed implication on multicore
  - Reducing communication will become much more intense
  - Memory per core changes, byte-to-flop ratio will change
- Necessary Fault Tolerance
  - MTTF will drop
  - Checkpoint/restart has limitations



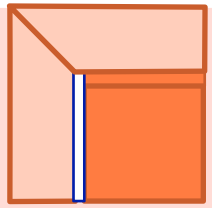
Software infrastructure does not exist today

Major Changes to Software

- Must rethink the design of our software
  - Another disruptive technology
    - Similar to what happened with cluster computing and message passing
  - Rethink and rewrite the applications, algorithms, and software
- Numerical libraries for example will change
  - For example, both LAPACK and ScaLAPACK will undergo major changes to accommodate this

A New Generation of Software:

Parallel Linear Algebra Software for Multicore Architectures (PLASMA)

Software/Algorithms follow hardware evolution in time		
LINPACK (70's) (Vector operations)		Rely on - Level-1 BLAS operations

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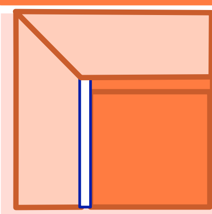
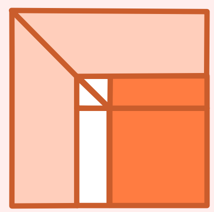
LINPACK (70's) (Vector operations)		Rely on - Level-1 BLAS operations
LAPACK (80's) (Blocking, cache friendly)		Rely on - Level-3 BLAS operations

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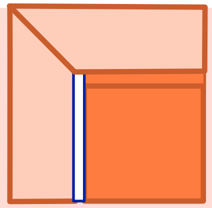
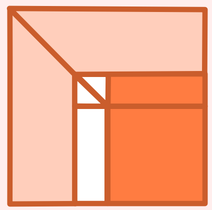
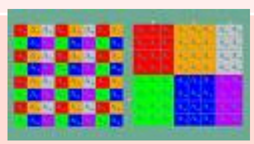
LINPACK (70's) (Vector operations)		Rely on - Level-1 BLAS operations
LAPACK (80's) (Blocking, cache friendly)		Rely on - Level-3 BLAS operations
ScaLAPACK (90's) (Distributed Memory)		Rely on - PBLAS Mess Passing

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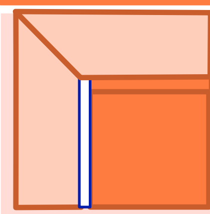
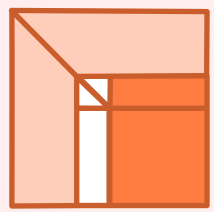
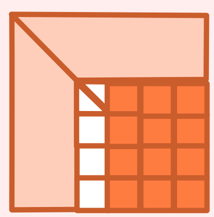
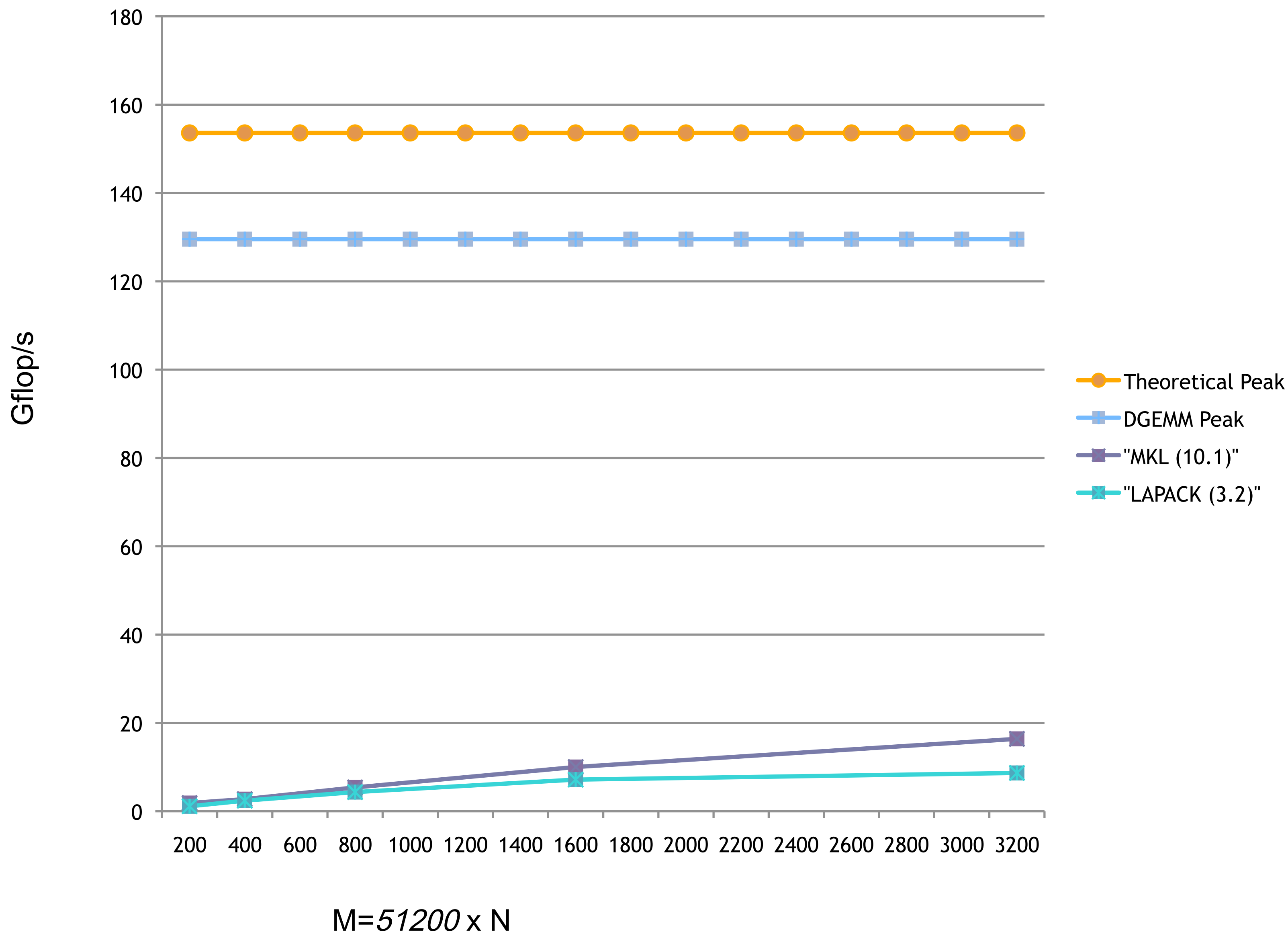
LINPACK (70's) (Vector operations)		Rely on - Level-1 BLAS operations
LAPACK (80's) (Blocking, cache friendly)		Rely on - Level-3 BLAS operations
ScaLAPACK (90's) (Distributed Memory)		Rely on - PBLAS Mess Passing
PLASMA (00's) New Algorithms (many-core friendly)		Rely on - a DAG/scheduler - block data layout - some extra kernels

Those new algorithms
- have a very low granularity, they scale very well (multicore, petascale computing, …)
- removes a lots of dependencies among the tasks, (multicore, distributed computing)
- avoid latency (distributed computing, out-of-core)
- rely on fast kernels

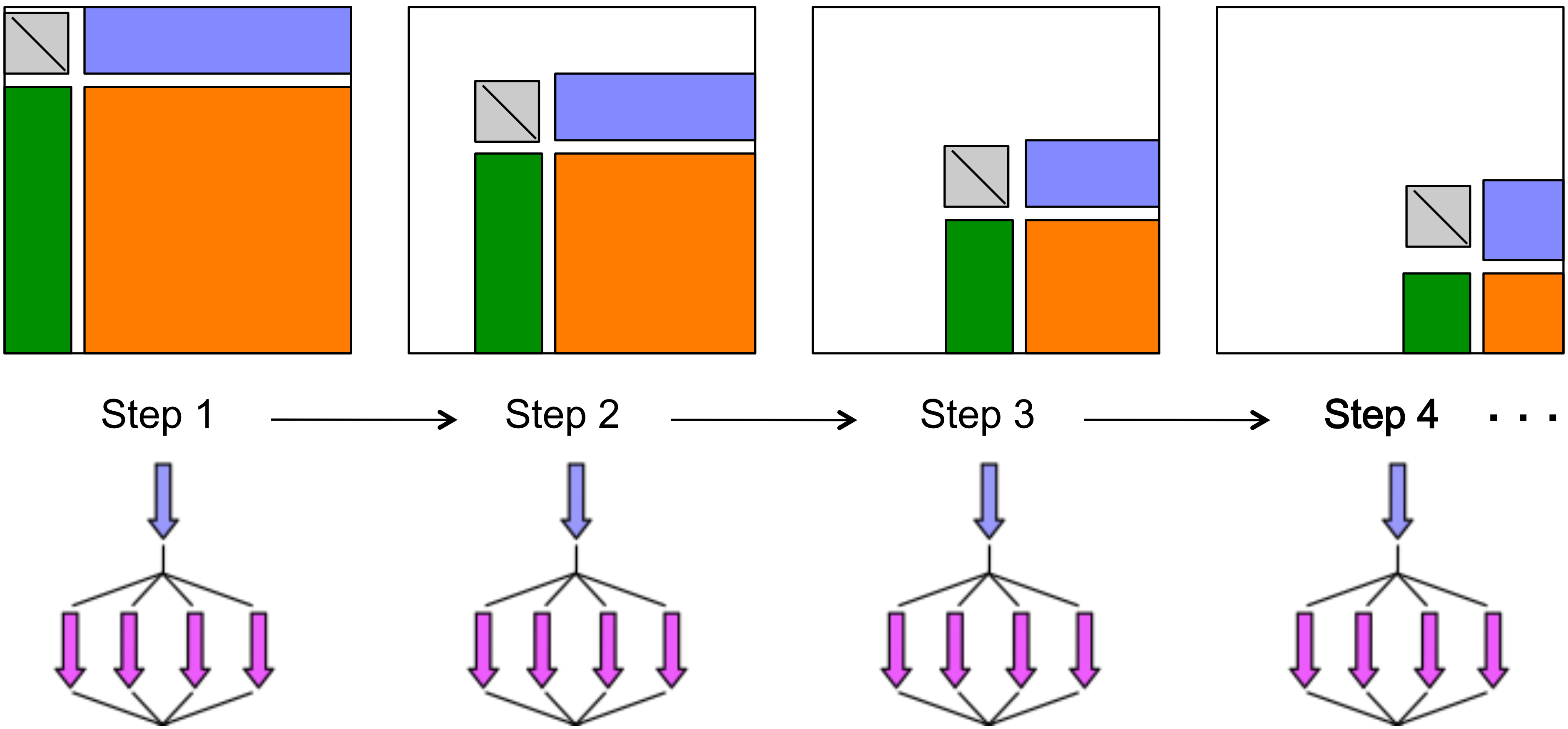
Those new algorithms need new kernels and rely on efficient scheduling algorithms.

QR Factorization Intel 16 cores

Tall Skinny Matrices

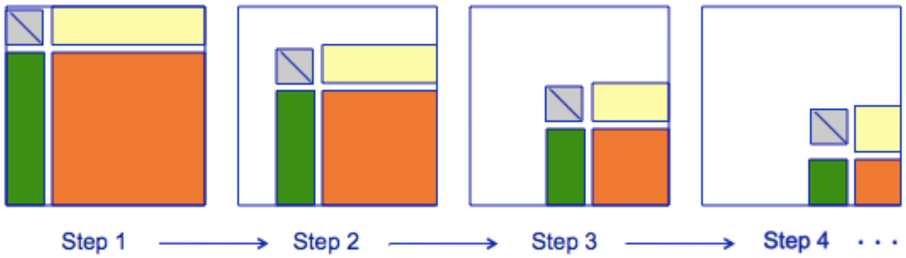


LAPACK QR

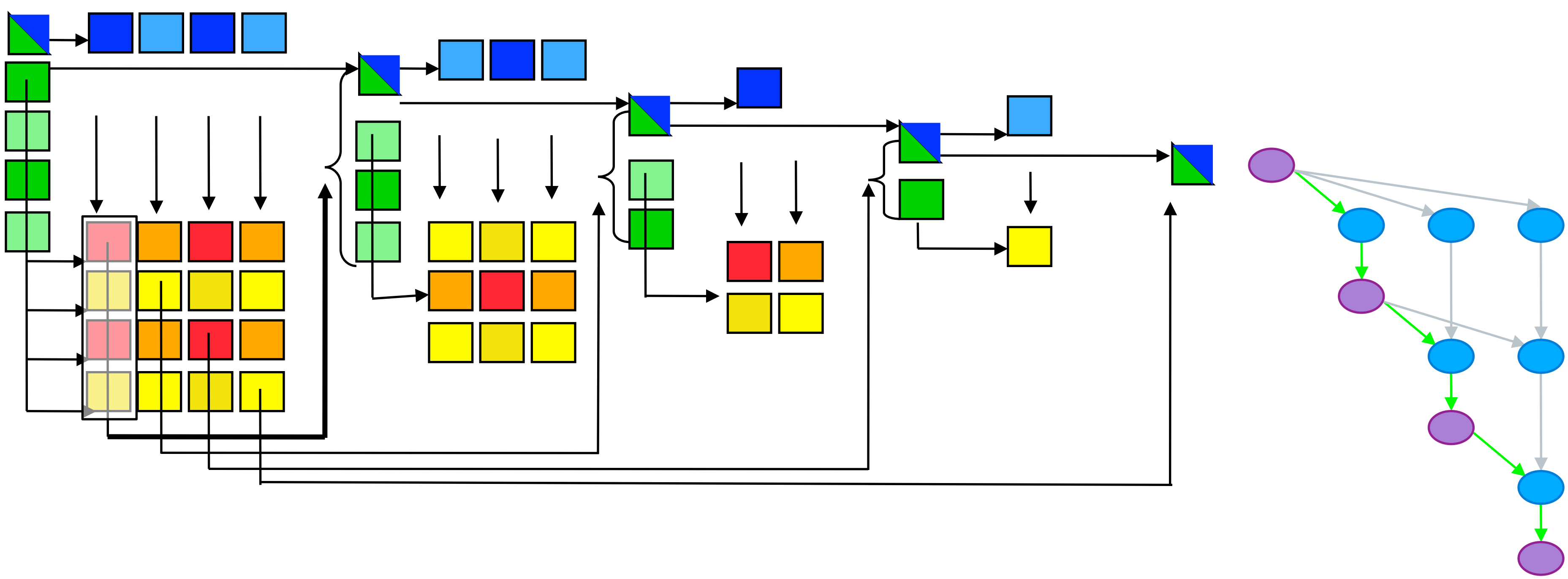


- **Fork-join, bulk synchronous processing**

Parallel Tasks in QR



- Break into smaller tasks and remove dependencies



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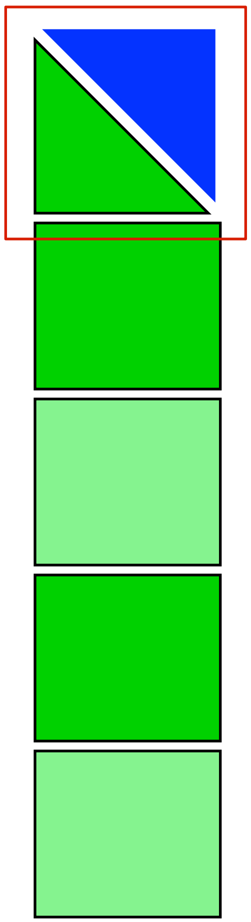
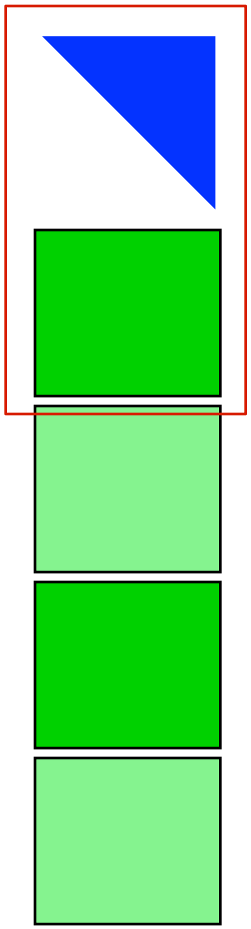


Step 1: QR of block 1,1

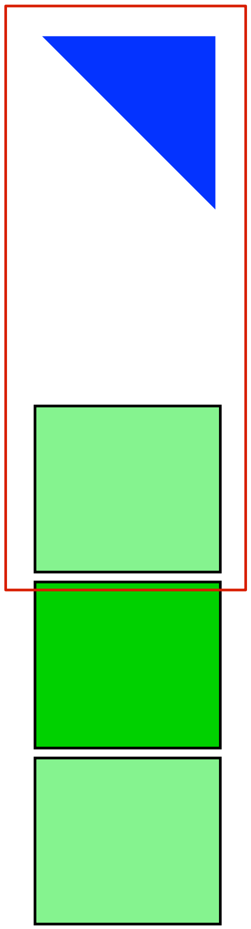
Parallel Tasks in QR



Step 1: QR of block 1,1

Step 2: Use R to zero $A_{1,2}$

Parallel Tasks in QR



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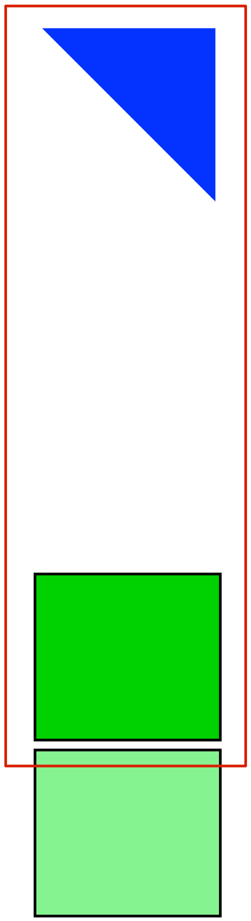

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Step3: Use R to zero $A_{1,3}$

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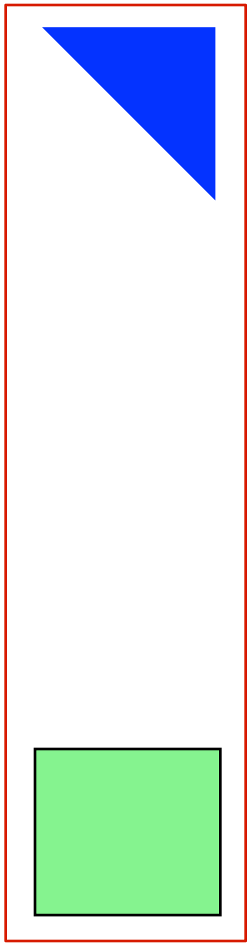
Parallel Tasks in QR



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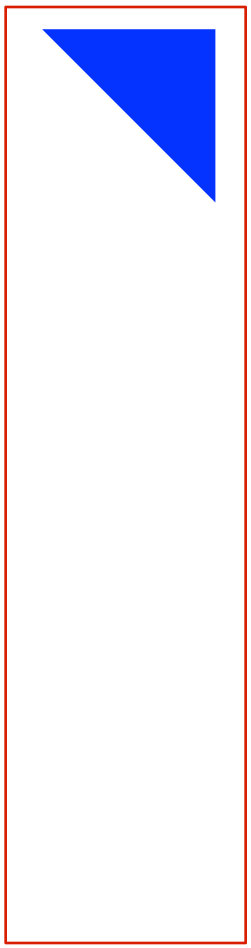

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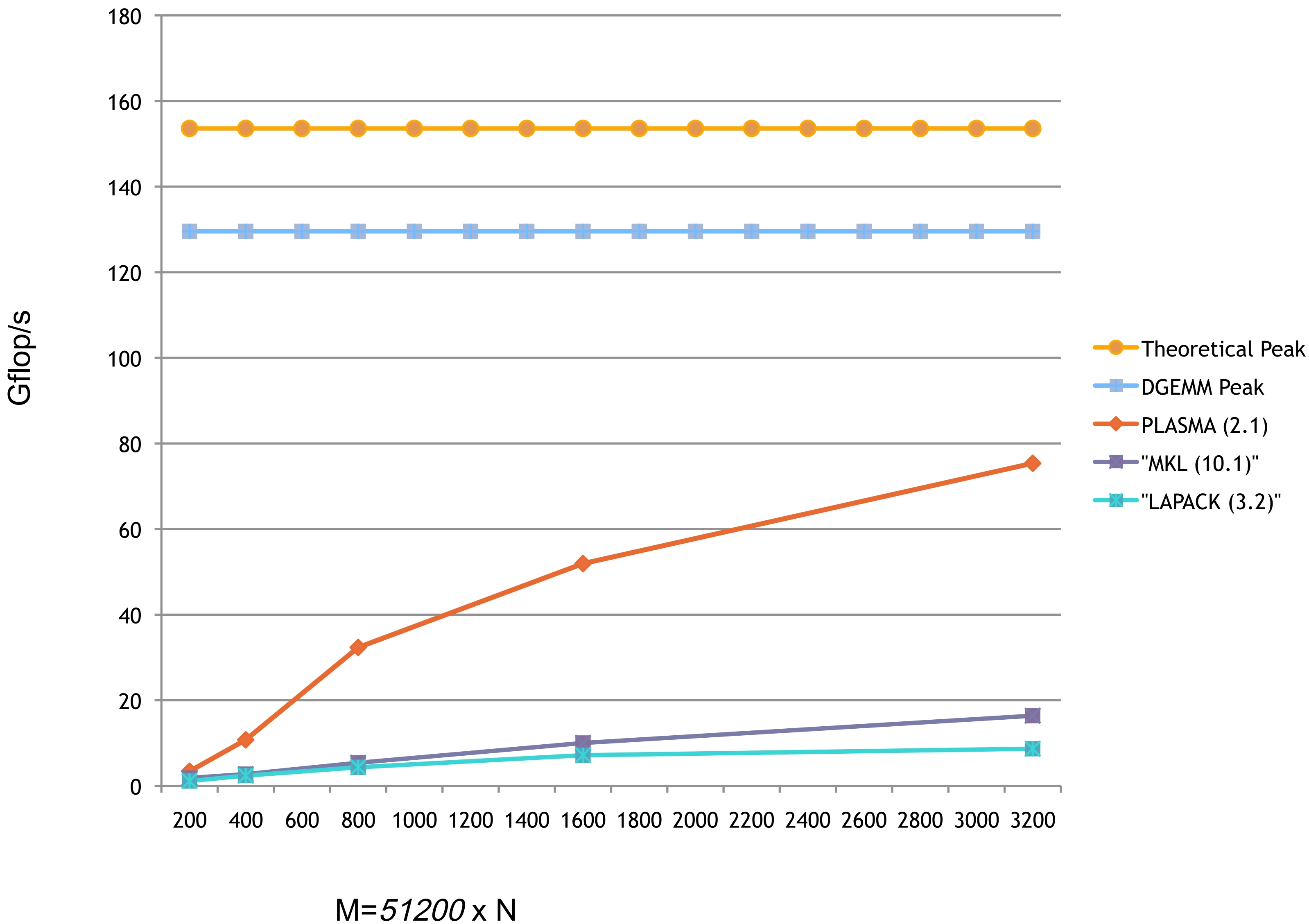
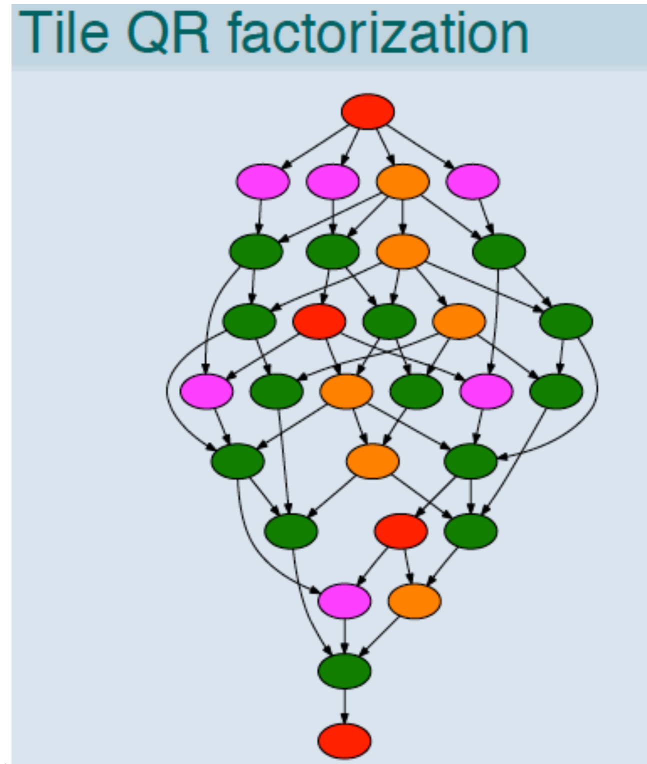
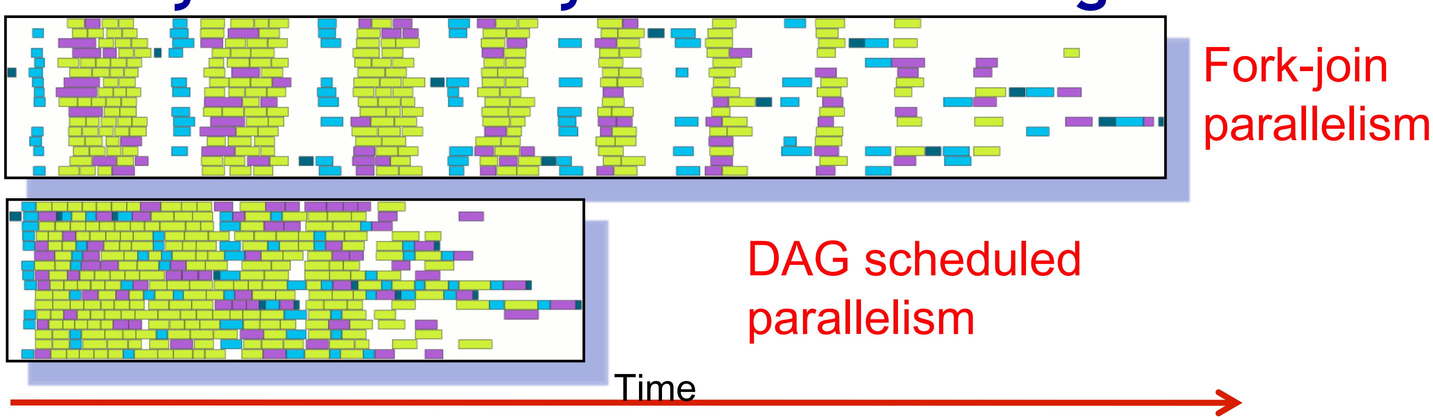


PLASMA: Parallel Linear Algebra s/w for Multicore Architectures

- **Objectives**
  - high utilization of each core
  - scaling to large number of cores
  - shared or distributed memory
- **Methodology**
  - DAG scheduling
  - explicit parallelism
  - implicit communication
  - Fine granularity / block data layout
- **Arbitrary DAG with dynamic scheduling**





Communication Avoiding Algorithms

- Goal: Algorithms that communicate as little as possible
- Jim Demmel and company have been working on algorithms that obtain a provable minimum communication.
- Direct methods (BLAS, LU, QR, SVD, other decompositions)
  - Communication lower bounds for *all* these problems
  - Algorithms that attain them (*all* dense linear algebra, some sparse)
    - Mostly not in LAPACK or ScaLAPACK (yet)
- Iterative methods – Krylov subspace methods for Ax=b, Ax=λx
  - Communication lower bounds, and algorithms that attain them (depending on sparsity structure)
    - Not in any libraries (yet)
- For QR Factorization they can show:

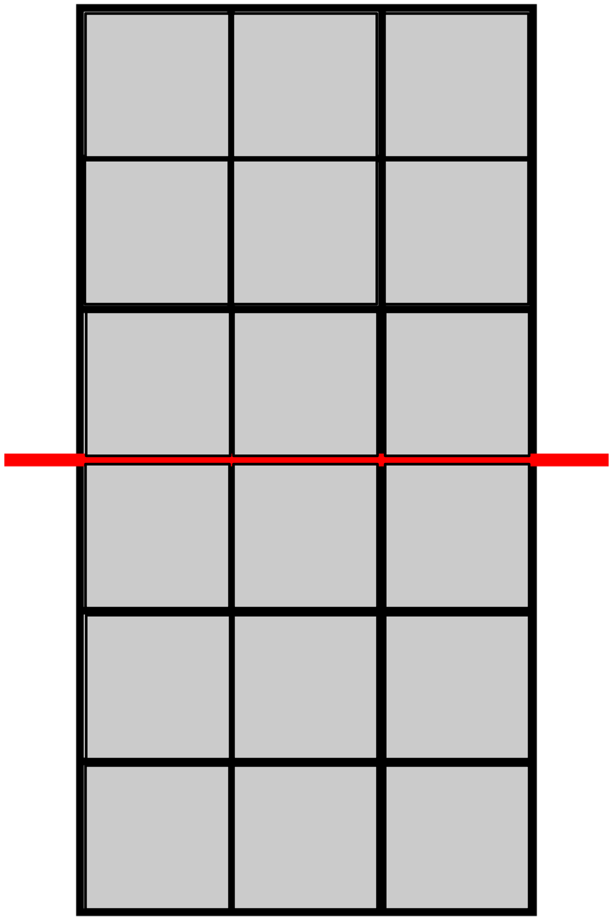
	Lower bound
# flops	$\Theta(mn^2)$
# words	$\Theta(\frac{mn^2}{\sqrt{W}})$
# messages	$\Theta(\frac{mn^2}{W^{3/2}})$

Communication Reducing QR Factorization

TS matrix
- MT=6 and NT=3
- split into 2 domains

3 overlapped steps

- panel factorization
- updating the trailing submatrix
- merge the domains



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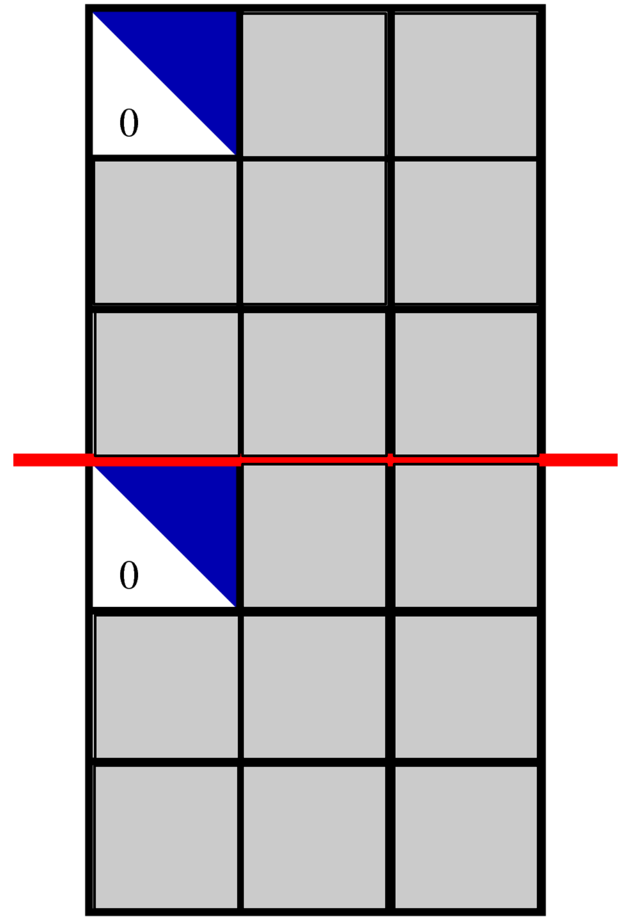

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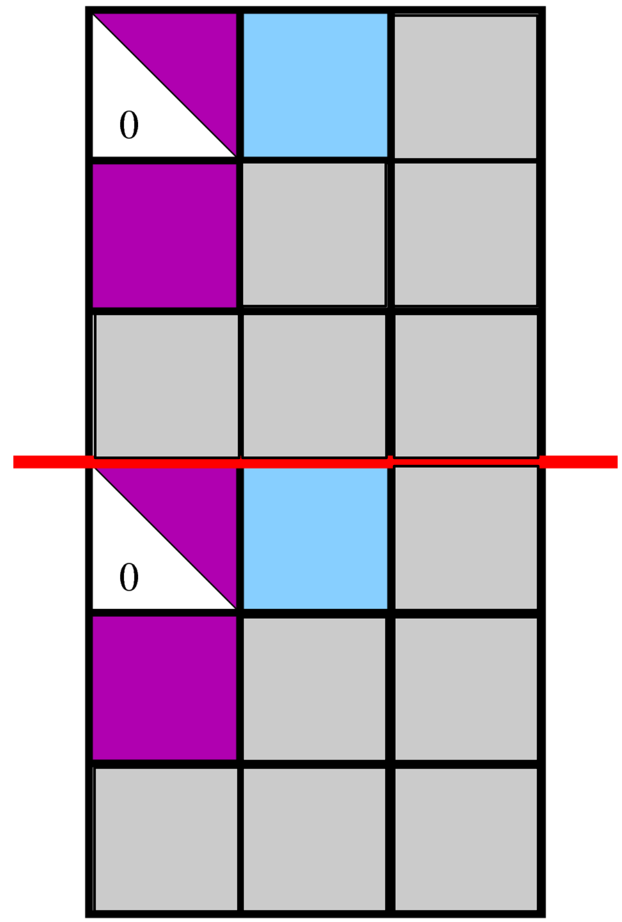

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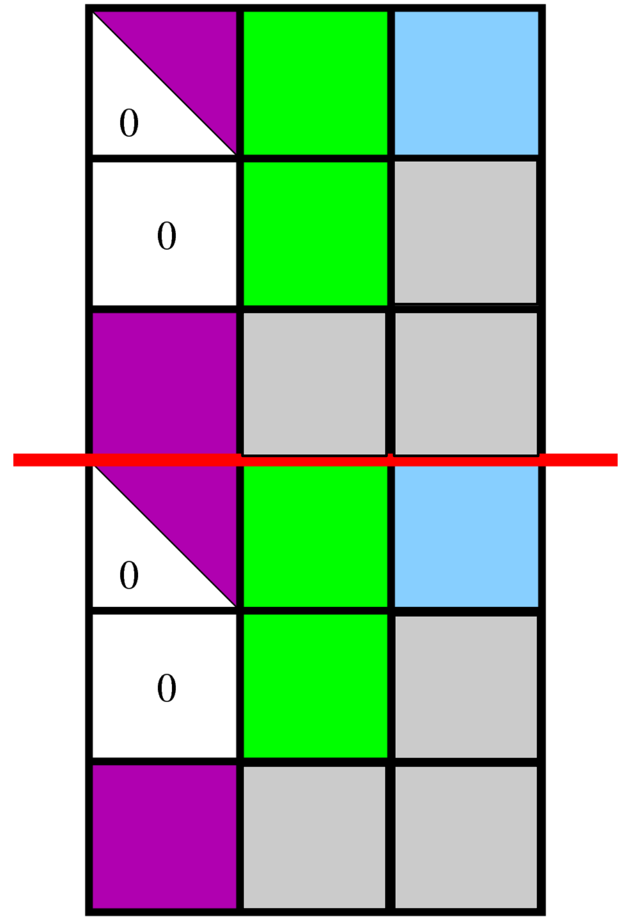

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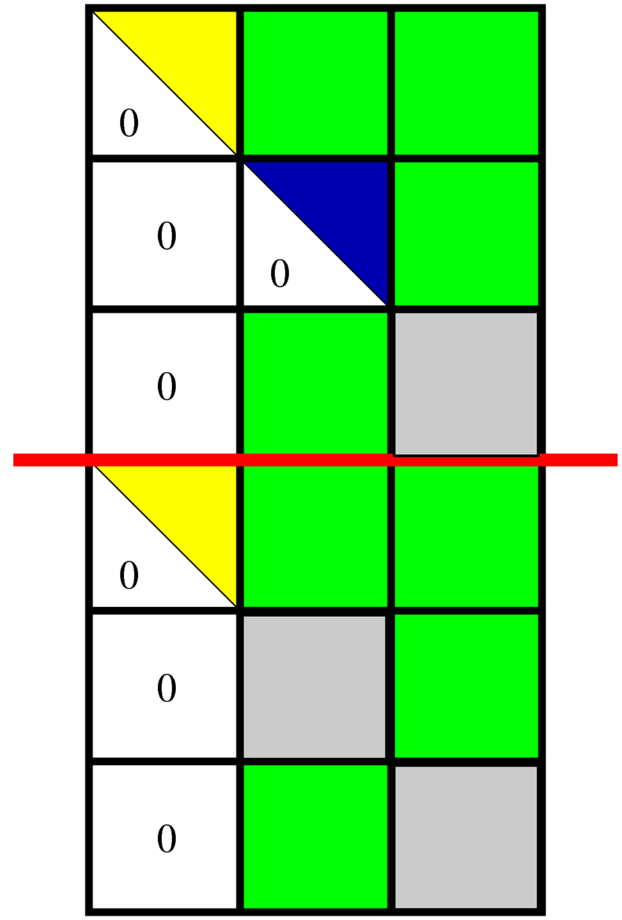

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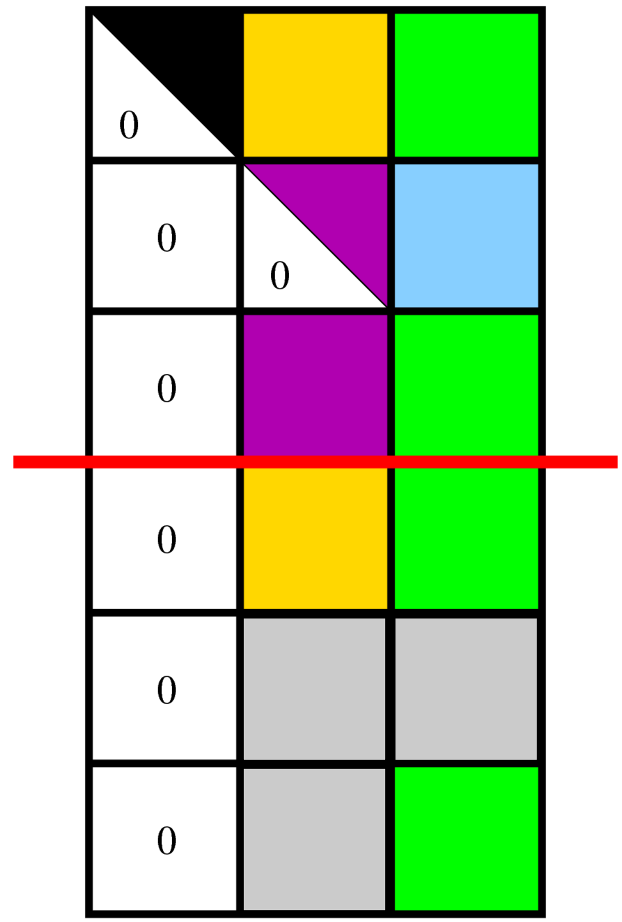

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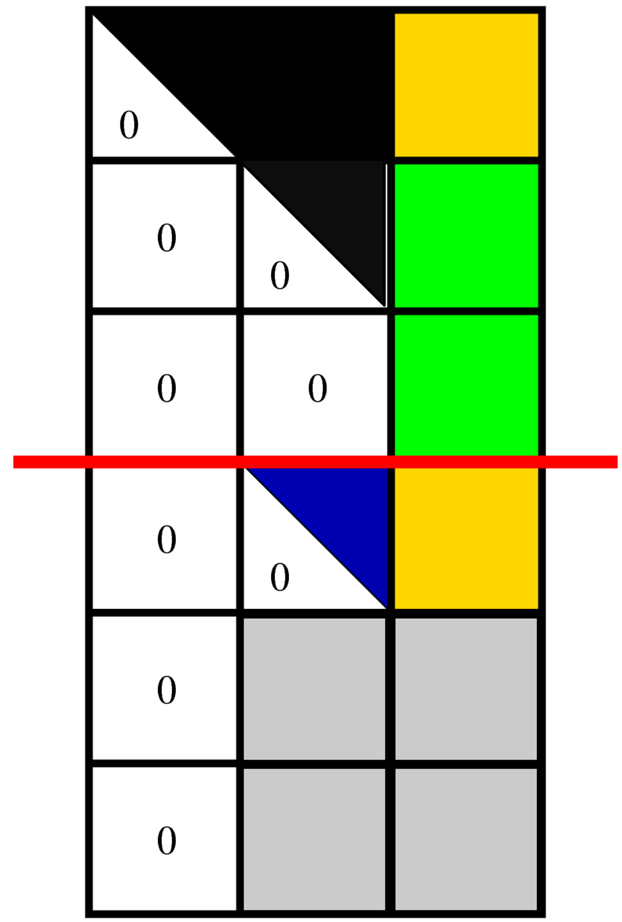

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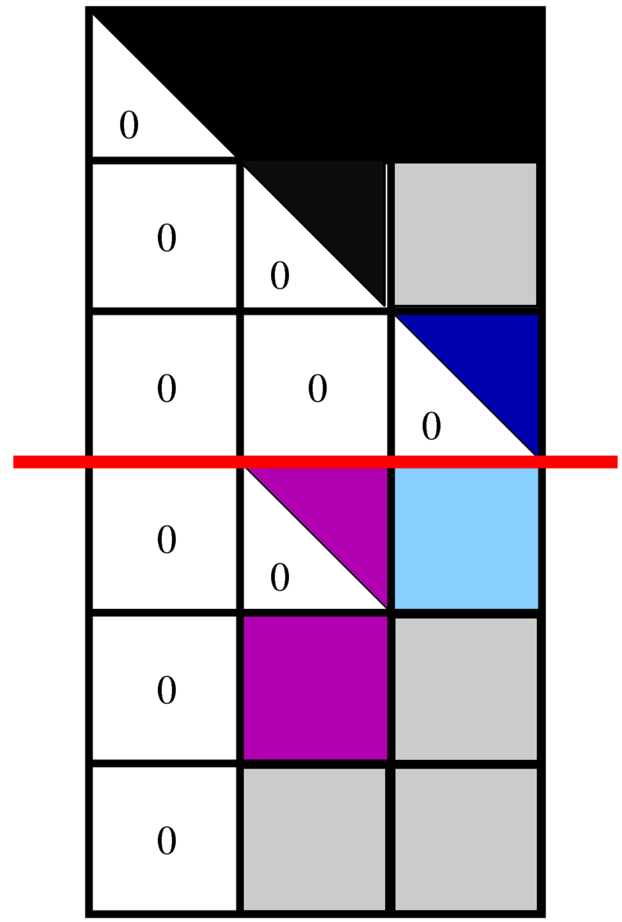

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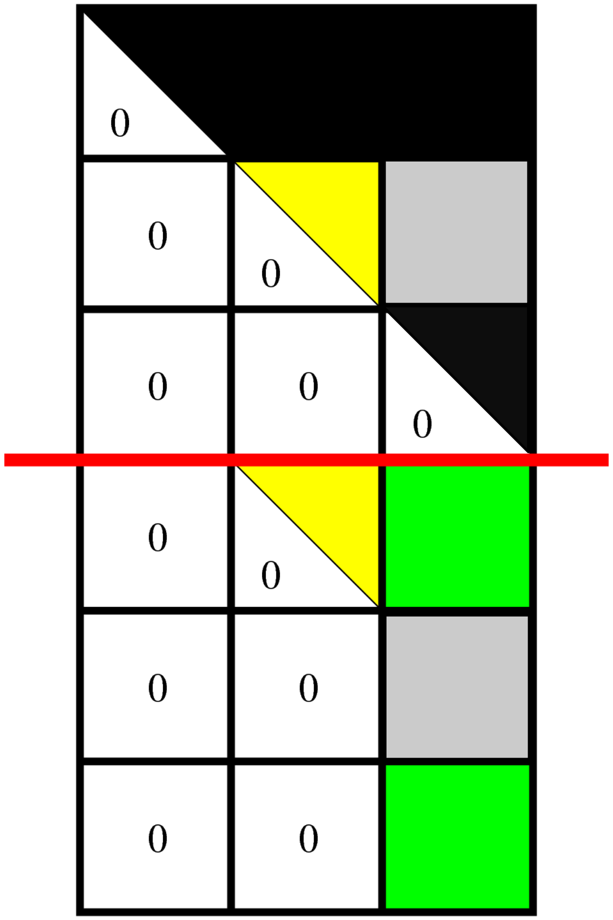

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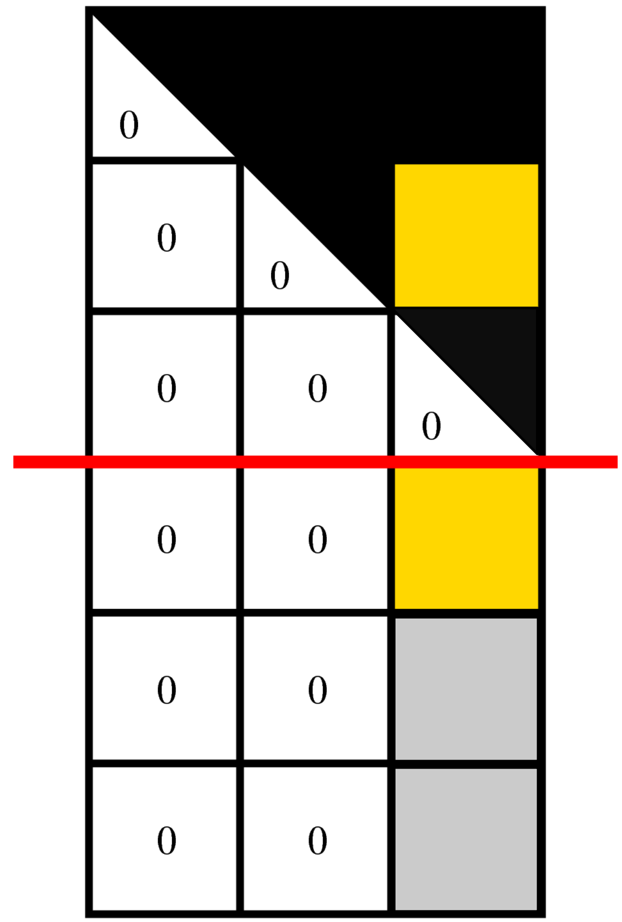

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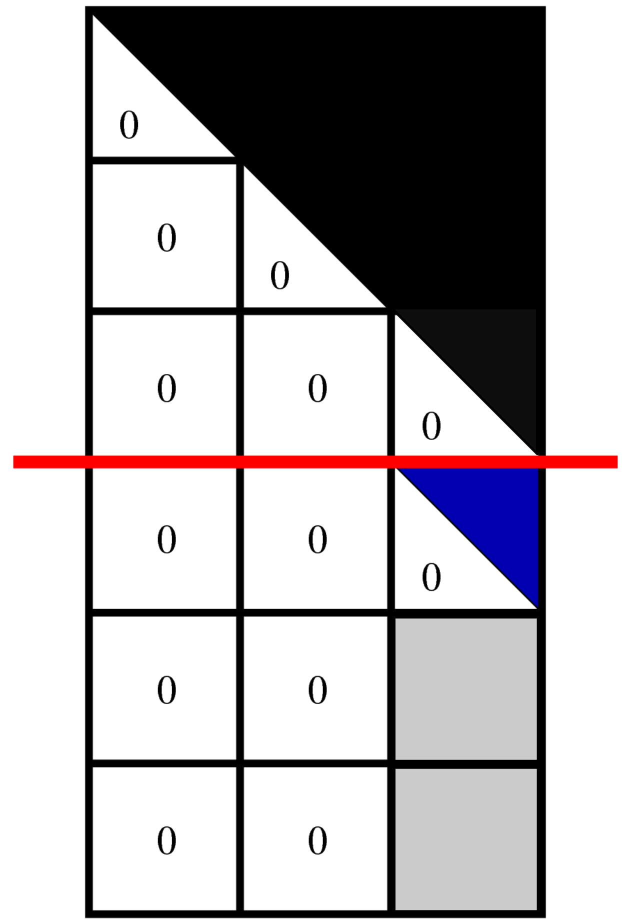

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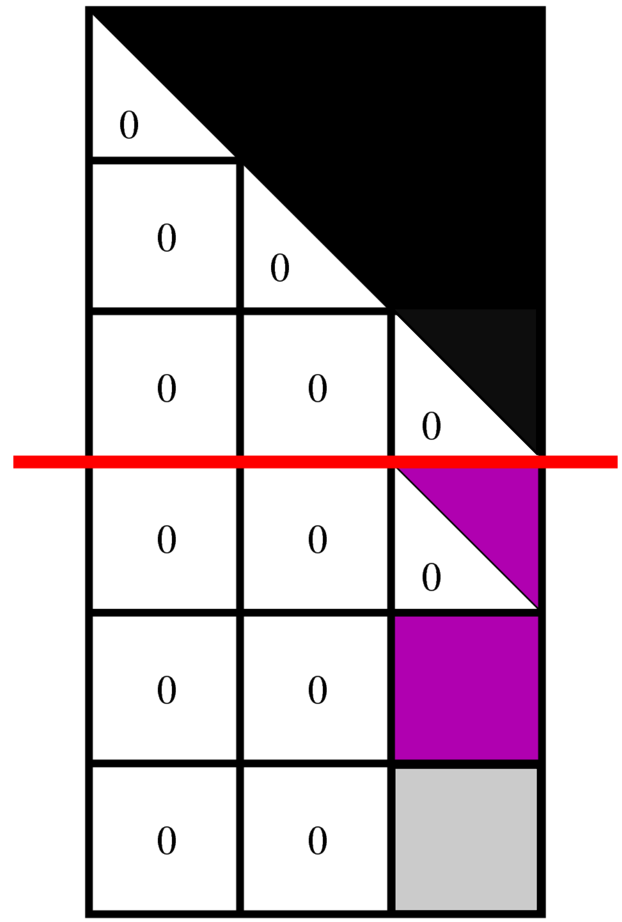

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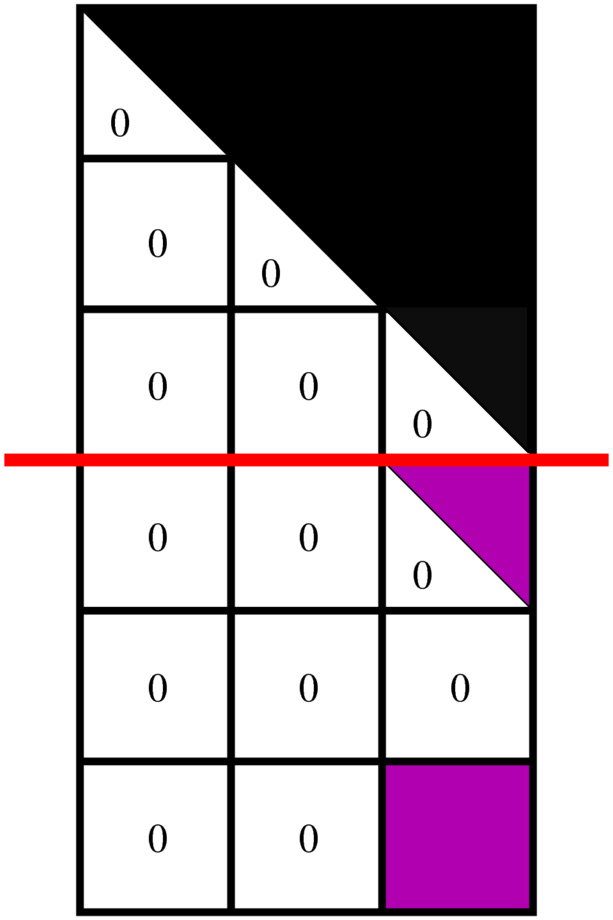

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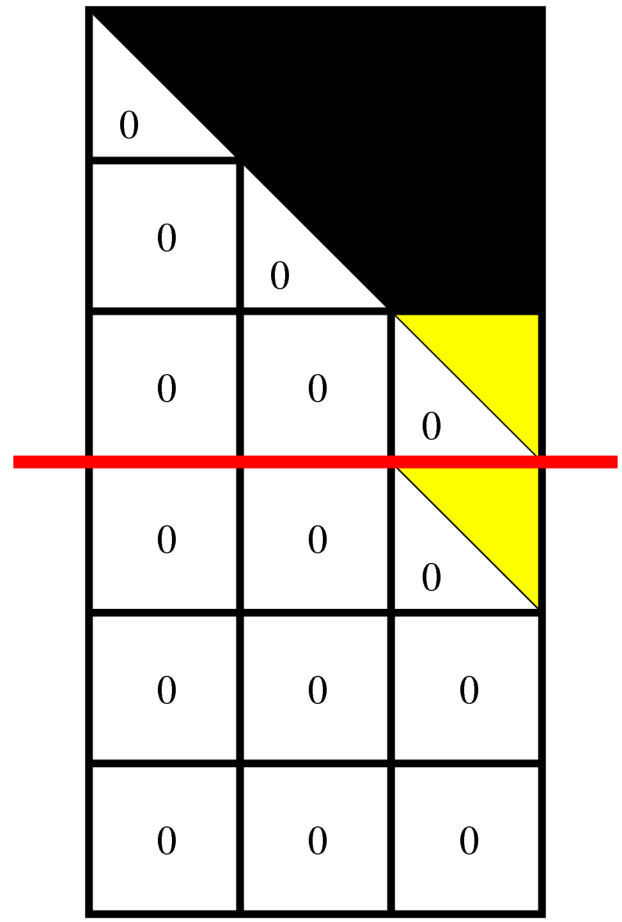

Communication Reducing QR Factorization

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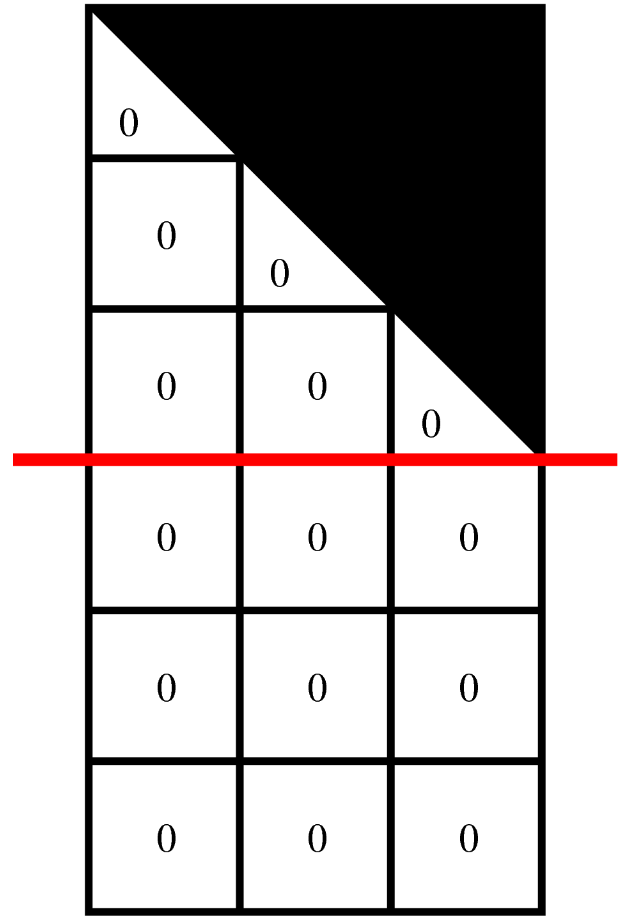
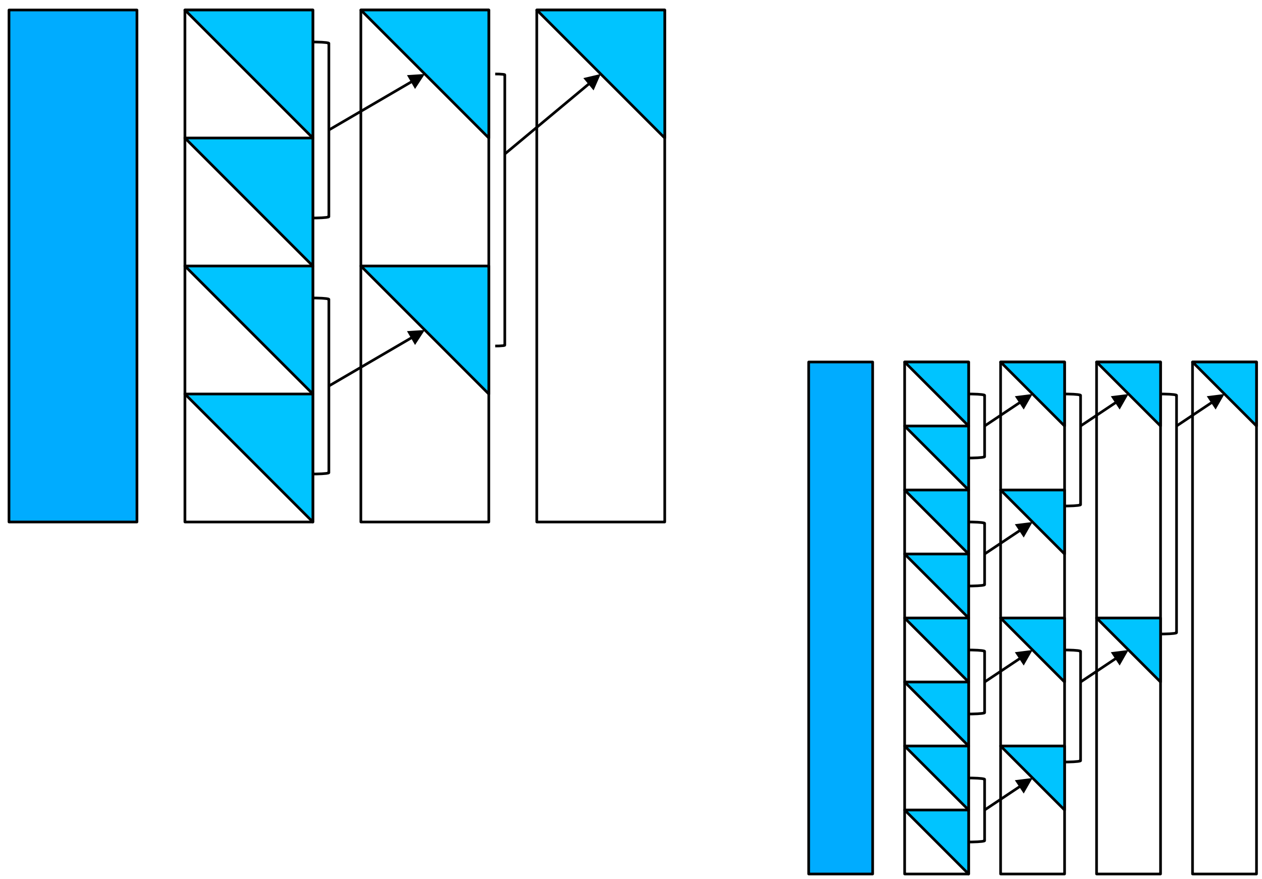
- merge the domains
- Final R computed



Example with 4 and 8 Domains



A. Pothen and P. Raghavan. Distributed orthogonal factorization. In *The 3rd Conference on Hypercube Concurrent Computers and Applications, volume II, Applications,* pages 1610–1620, Pasadena, CA, Jan. 1988. ACM. Penn. State.

Execution Trace

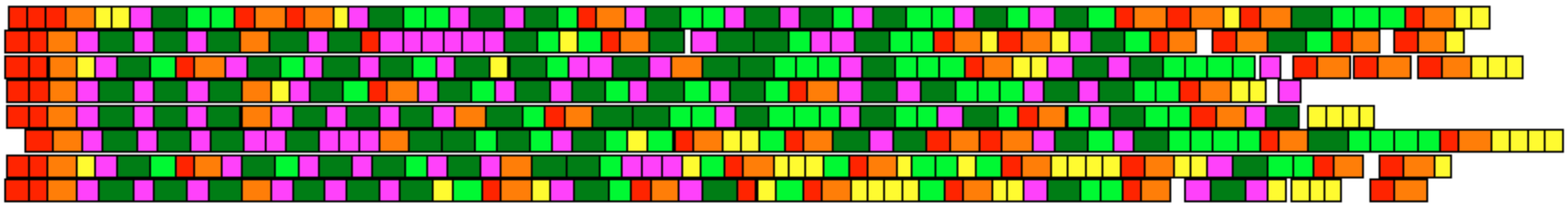


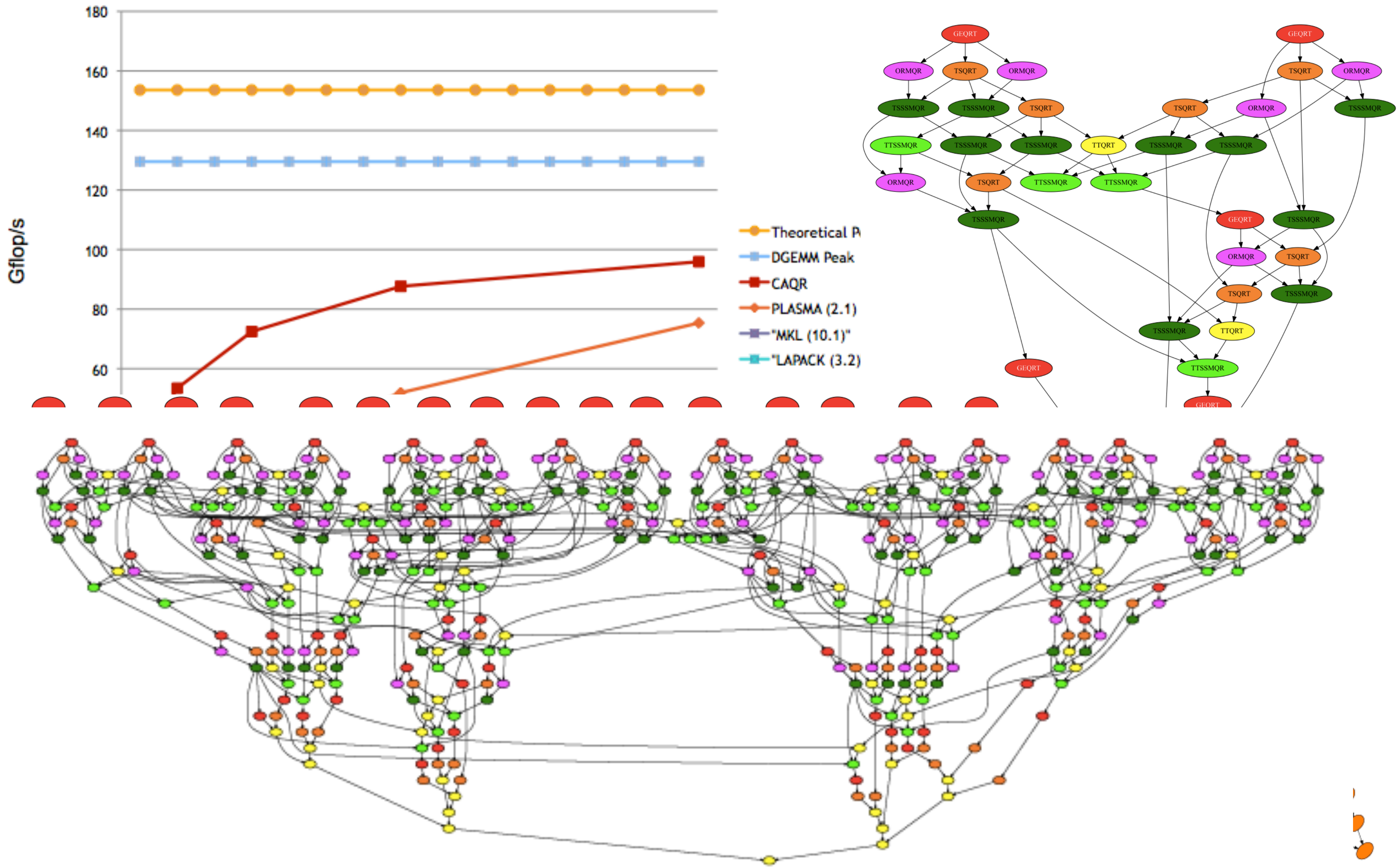
Fig. 11. Parallel execution traces of SP-16 with MT=32 and NT=4 on 8 cores.

TABLE III
IMPROVEMENT OF SP-CAQR AGAINST OTHER LIBRARIES (PERFORMANCE RATIO).

Matrix sizes	PLASMA	MKL	ScaLAPACK	LAPACK
51200 – 200	9.54	8.77	3.38	28.63
51200 – 3200	1.27	4.10	2.88	11.05

16 core run

Communication Reducing QR Factorization



(a) One domain: SP-1 (or PLASMA-like Tile QR factorization).

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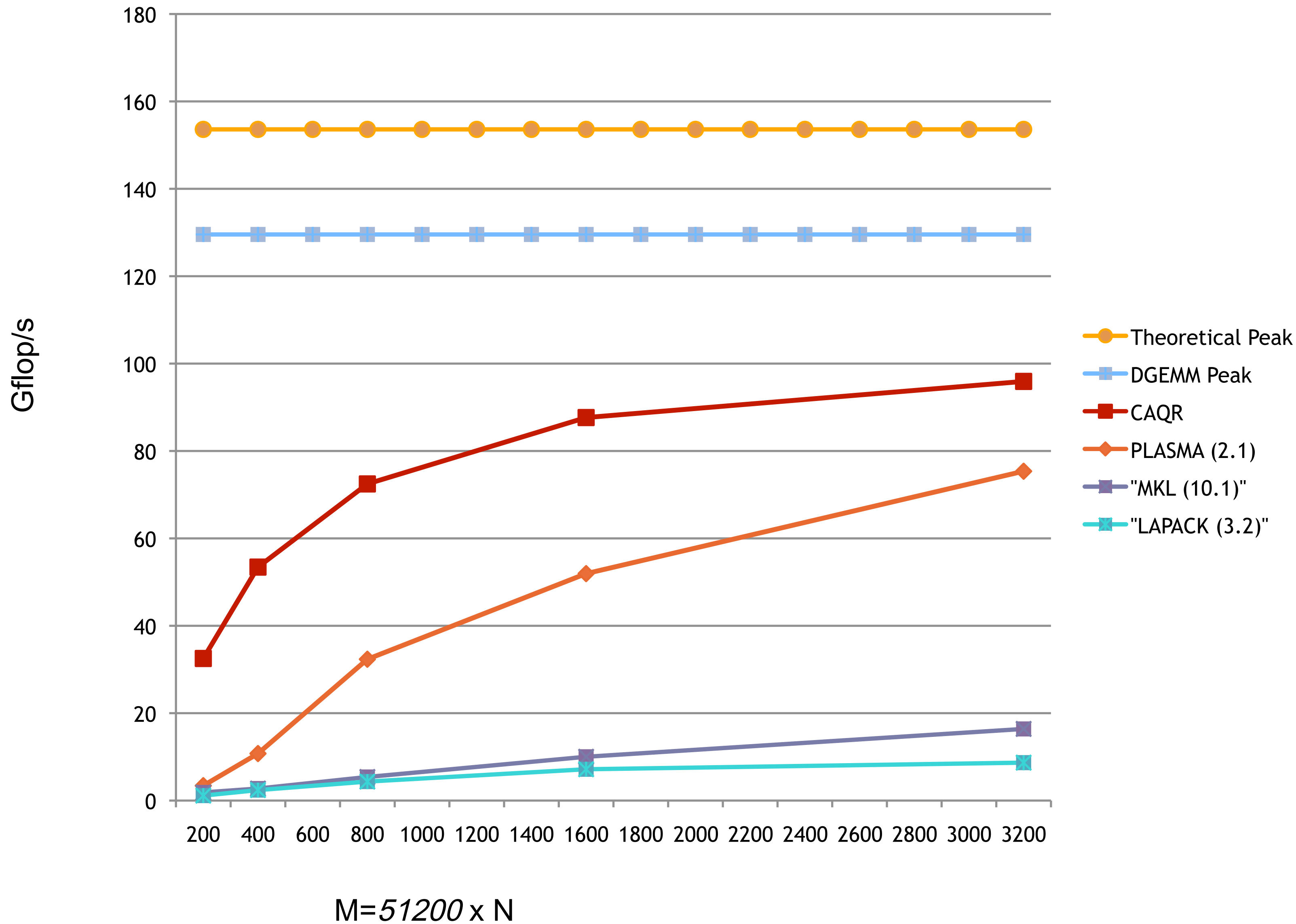
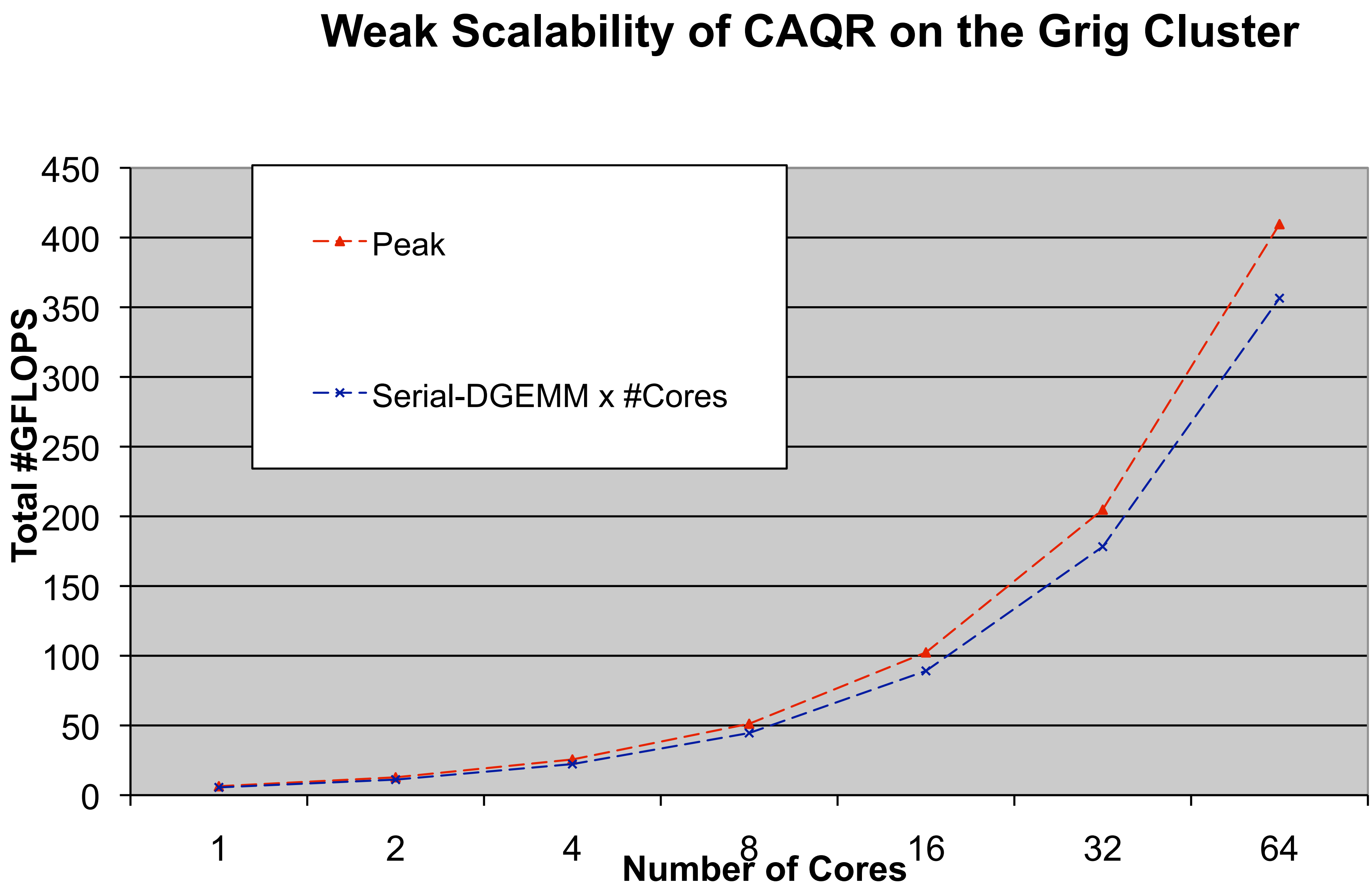


Cluster Experiment

- grig.sinrg.cs.utk.edu
- 61 nodes
  - Two CPUs per node
  - Intel Xeon 3.20GHz
  - Peak performance 6.4 GFLOPS
  - Myrinet interconnection (MX 1.0.0)
- Goto BLAS 1.26
  - DGEMM performance 5.57 GFLOPS (87%)
- MPICH-MX
- gcc 64 bits

Weak Scalability (8 columns of tiles)



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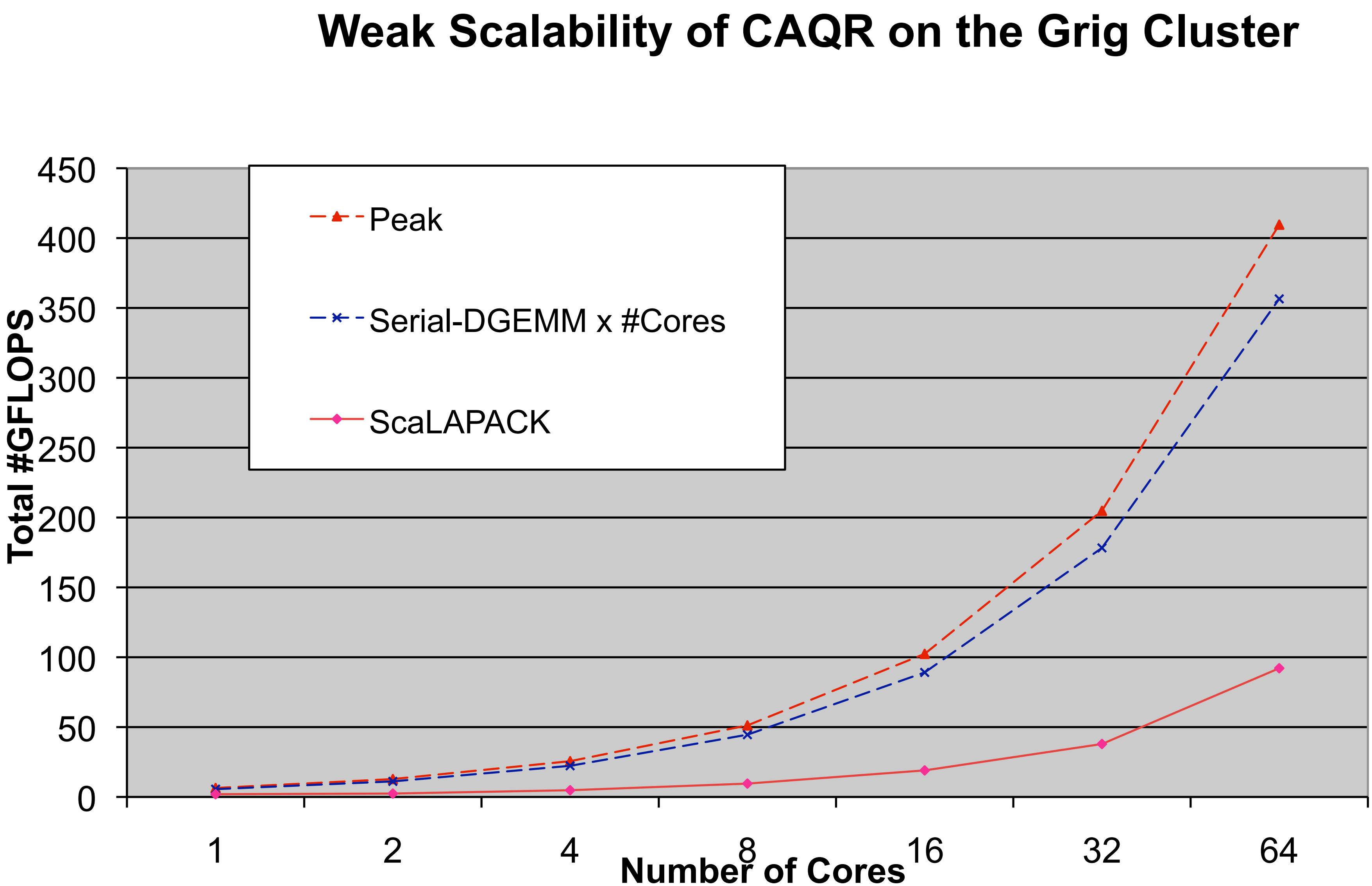

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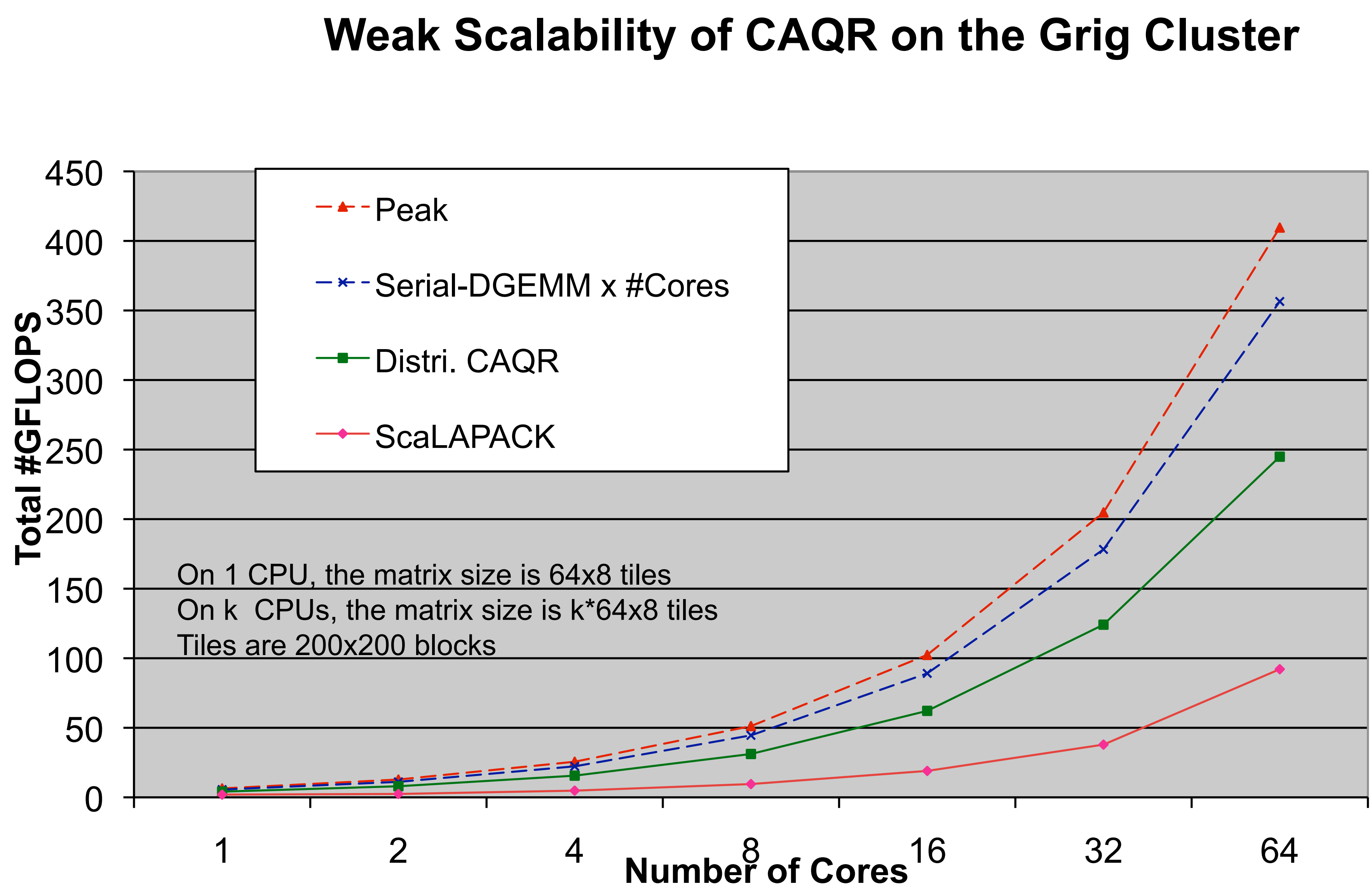

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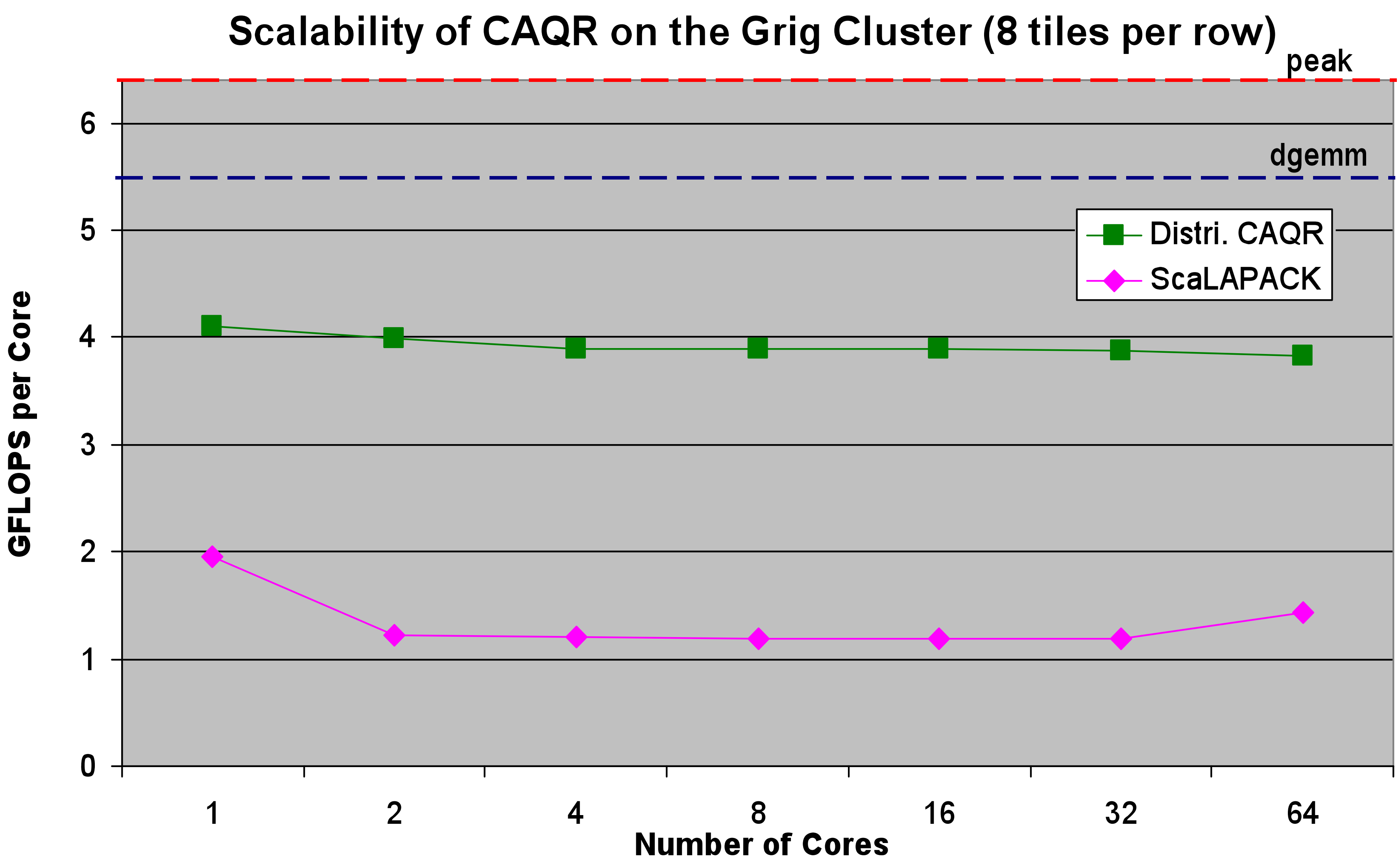

Futures

- Architectural trends are forcing major changes to our algorithms
- Communication avoiding algorithms will be critical for performance.
- PLASMA and MAGMA will make use of CA algorithms.