BENCHMARK
performance
scientific computation

Performance of various computers using standard linear equations software in a Fortran environment

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COMMENTARY

Few published studies enable comparisons to be made between the performances of computers covering the whole spectrum of today's technology. Jack Dongarra's article provides exactly that: execution timings of the same set of programs over a whole range of computers, from the Cray X-MP to the Apple III. Of particular interest is the inclusion of results pertaining to recent machines such as the Fujitsu VP-200, the NAS 9060, the Ridge 32, the ELXSI, etc. Of course, this benchmark, as with any other, has its limitations; one should not accept its results as an absolute measurement of the performance ratios between the machines considered. In particular, the test programs are subroutine from the famous LINPACK library, a set of portable, high-quality primitives; their execution patterns are quite different from what may be found, e.g. in data-intensive applications or text processing systems. Also, one should carefully examine the conditions in which the tests were performed, as given by the author. With these qualifications, however, Dongarra's tables provide a particularly rich source of information . . . and a few surprises.

B. Meyer

The timing information presented here should not be used to judge the overall performance of a computer system. The results reflect only one problem area: solving dense systems of equations using the LINPACK [Dongarra 79] programs in a Fortran environment.

LINPACK programs can be characterized as having a high percentage of floating-point arithmetic operations. The routines involved in this timing study, SGEFA and SGESL, use column-oriented algorithms, i.e. the programs usually reference array elements sequentially down a column, not across a row. Column orientation is important for increasing efficiency because of the way Fortran stores arrays. Most floating-point operations in LINPACK take place in a set of subprograms, the Basic Linear Algebra Subprograms (BLAS) [Lawson 79] that are called repeatedly throughout the calculation. The BLAS reference one-dimensional arrays rather than two-dimensional arrays.

The following two tables report timing results using LINPACK to solve a system of linear equations of order 100. The execution speeds, particularly for vector computers, may not have reached their asymptotic rates. (See the appendix for a different comparison of large scientific computers in Fortran.) It should be noted that, with the exception of replacing the BLAS with assembly language code, no further changes were made to the software. In particular, no attempt was made to use special hardware features on certain machines, or to exploit vector capabilities or multiple processors. (The compilers on some machines may, of course, generate optimized code that itself accesses special features.)

One further point: the information in the tables was compiled over a period of time. Subsequent systems software and hardware changes may alter the timings to some extent.

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TABLE 1

Solving a system of linear equations with LINPACK[a] in full precision[b]

Computer	Compiler[c]	Ratio[d]	MFLOPS[e]	Time (s)	Unit[f] (μs)
Cray X-MP	CFT (Coded BLAS)	0.36	33	0.021	0.061
CDC Cyber 205	FTN (Coded BLAS)	0.48	25	0.027	0.079
Cray X-MP	CFT	0.57	21	0.032	0.093
Cray1-S	CFT (Coded BLAS)	0.68	18	0.038	0.11
Fujitsu VP-200	Fortran 77	0.69	18	0.039	0.11
Cray1-S	CFT	1.0	12	0.056	0.16
CDC Cyber 205	FTN	1.5	8.4	0.082	0.24
NAS 9060 w/VPF	VS opt=2 (Coded BLAS)	1.8	6.8	0.101	0.28
NAS 9060	VS opt=2	2.3	5.3	0.130	0.29
CDC Cyber 875	FTN opt=3	2.5	5.0	0.138	0.40
CDC 7600	FTN (Coded BLAS)	2.6	4.6	0.148	0.43
CDC Cyber 176	FTN 5.1 opt=2	2.6	4.6	0.148	0.43
Amdahl 5860 HSFPF	H enhanced opt=3	3.1	3.9	0.176	0.51
Amdahl 5860 HSFPF	VS opt=3	3.2	3.8	0.181	0.53
CDC 7600	FTN	3.8	3.3	0.210	0.61
FPS-164	D opt=3 (Coded BLAS)	4.7	2.6	0.264	0.77
IBM 370/195	H enhanced opt=3	4.9	2.5	0.275	0.80
IBM 3081 K	H enhanced opt=3	5.7	2.1	0.321	0.94
CDC Cyber 175	FTN 5 opt=2	5.8	2.1	0.322	0.94
IBM 3081 K	VS opt=3	6.2	2.0	0.346	1.01
CDC 7600	Local	6.4	2.0	0.359	1.05
CDC Cyber 175	FTN 5 opt=1	6.8	1.8	0.381	1.11
IBM 3033	H enhanced opt=3	7.0	1.8	0.390	1.14
IBM 3033	VS opt=3	7.1	1.7	0.396	1.15
IBM 3081 D	VS opt=2	7.4	1.7	0.415	1.21
Amdahl 470 V/8	H enhanced opt=3	7.7	1.6	0.429	1.25
Amdahl 470 V/8	VS opt=3	8.2	1.5	0.458	1.33
FPS-164	D, opt=3	9.5	1.3	0.529	1.54
CDC 7600	Chat, no opt	9.9	1.2	0.554	1.61
IBM 370/168	H Ext Fast Mult	10	1.2	0.579	1.69
Amdahl 470 V/6	H opt=2	11	1.1	0.631	1.84
IBM 370/165	H Ext Fast Mult	16	0.77	0.890	2.59
ELXSI	Embos, F77 (Coded BLAS)	22	0.56	1.23	3.57
CDC 6600	FTN 4.6 opt=2	26	0.48	1.44	4.19
CDC Cyber 170-835	FTN 5 opt=2	26	0.47	1.45	4.22
CDC Cyber 170-835	FTN 5 opt=1	28	0.44	1.57	4.58
ELXSI	EMBOS, F77	28	0.43	1.60	4.66
Univac 1100/81	ASCII opt=ZEO	32	0.38	1.80	5.24
CDC 6600	Run	34	0.36	1.93	5.62
Data General MV/10000	f77 opt level 2	40	0.30	2.26	6.58
Harris 800	Fortran 77	53	0.23	2.99	8.70
IBM 370/158	H opt=3	53	0.23	2.99	8.71
Vax 11/785 FPA	VMS (Coded BLAS)	54	0.23	3.01	8.77
IBM 370/158	VS opt=3	56	0.22	3.15	9.17
Itel AS/5 mod 3	H	63	0.19	3.54	10.3
Norsk Data ND-500	Fortran-500-E	63	0.19	3.54	10.3
CDC Cyber 170-825	FTN 5 opt=2	65	0.19	3.63	10.6
IBM 4341 MG10	VS opt=3	66	0.19	3.70	10.8
Vax 11/785 FPA	VMS	68	0.18	3.79	11.0
CDC Cyber 170-825	FTN 5 opt=1	68	0.18	3.81	11.1
Vax 11/780 FPA	VMS (Coded BLAS)	76	0.16	4.25	12.4
ICL 2988	f77 opt=2	85	0.14	4.78	13.9
Vax 11/750 FPA	VMS (Coded BLAS)	88	0.14	4.92	14.3
Vax 11/780 FPA	VMS	98	0.13	5.48	16.0
Ridge 32	Fort 77	100	0.12	5.61	16.3
CDC 6500	Fun	102	0.12	5.69	16.6
Denelcor HEP	f77	107	0.11	5.98	17.4
Vax 11/780 FPA	Unix xf77	107	0.11	5.98	17.4
Vax 11/750 FPA	VMS	119	0.10	6.66	19.4
Prime 850	Primos	130	0.095	7.26	21.1
Univac 1100/62	ASCII opt=ZEO	132	0.093	7.38	21.5
Data General MV/8000	f77 opt level 2	157	0.078	8.80	25.6
Vax 11/750	VMS	216	0.057	12.1	35.3
HP 9000 Series 500	Fortran 1.7	285	0.043	16.0	46.6
Vax 11/730 FPA	VMS (Coded BLAS)	286	0.043	16.0	46.6
Vax 11/725 FPA	VMS (Coded BLAS)	286	0.043	16.0	46.6
Apollo	4.1 PEB (Coded BLAS)	323	0.038	18.1	52.7
IBM 4331	H opt=3	326	0.038	18.3	53.2
Vax 11/730 FPA	VMS	348	0.036	19.5	56.9

TABLE 1 *cont*

Computer	Compiler[c]	Ratio[d]	MFLOPS[e]	Time (s)	Unit[f] (μs)
Vax 11/725 FPA	VMS	348	0.036	19.5	56.9
Prime 2250	Fortran 77	365	0.034	20.5	59.6
IBM PC-XT/370	H opt=3	391	0.031	21.9	63.7
Masscomp MC500 w/FP	Unix, f77 opt	452	0.027	25.3	73.7
Sun 2 + SKY board	Unix, f77 opt	557	0.022	31.2	90.1
Apollo	4.1 PEB	559	0.022	31.3	91.2
Canaan	VS	588	0.021	33.0	96.0
Chas. River Data 6835 + SKY	SVS Fortran 77	700	0.018	39.2	114
Cadtrak DS1/8087	Intel Fortran 77	1143	0.011	64.0	186
Chas. River Data 6835	SVS Fortran 77	1401	0.0088	78.5	229
HP 9000 Series 200	HP-UX	1982	0.0062	111	323
Sun 2	Unix, f77 opt	1991	0.0062	112	325
Masscomp MC500	Unix, f77 opt	2588	0.0047	145	422
Sun	Unix, f77 no opt	2661	0.0046	149	434

[a] LINPACK routines *SGEFA* and *SGESL* were used for single precision, and routines *DGEFA* and *DGESL* were used for double precision. These routines perform standard *LU* decomposition with partial pivoting and backsubstitution.
[b] *Full precision* implies the use of (approximately) 64 bit arithmetic, e.g. CDC single precision or IBM double precision. *Half precision* implies the use of (approximately) 32 bit arithmetic, e.g. IBM single precision.
[c] *Compiler* refers to the compiler used and (Coded BLAS) refers to the use of assembly langauge coding of the BLAS.
[d] *Ratio* is the number of times faster or slower a particular machine configuration is when compared to the Cray-1S using a Fortran coding for the BLAS.
[e] *MFLOPS* is a rate of execution, the number of million floating-point operations completed per second. For solving a system of n equations, approximately $(2/3n^3) + 2n^2$ operations are performed (we count both additions and multiplications).
[f] *Unit* is the time in microseconds required to execute the statement $y_i = y_i + tx_i$. This involves one floating-point multiplication, one floating-point addition, and a few one-dimensional indexing operations and storage references. The actual statement occurs in SAXPY, which is called roughly n^2 times by SGEFA and n times by SGESL with vectors of varying lengths. The statement is executed approximately $(n^3/3) + n^2$ times. Thus for $n = 100$:

$$Unit = 10^6\ Time/((100^3/3) + 100^2)$$

TABLE 2
Solving a system of linear equations with LINPACK[a] in half precision[b]

Computer	Compiler[c]	Ratio[d]	MFLOPS[e]	Time (s)	Unit[f] (μs)
NAS 9060 w/VPF	VS opt=2 (Coded BLAS)	1.5	8.4	0.082	0.24
Amdahl 5860 HSFPF	H enhanced opt=3	2.2	5.5	0.125	0.36
NAS 9060	VS opt=2	2.4	5.2	0.133	0.38
Amdahl 5860 HSFPF	VS opt=3	2.4	5.1	0.135	0.39
Amdahl 470 V/8	H enhanced opt=3	4.4	2.8	0.246	0.71
Amdahl 470 V/8	VS opt=3	4.5	2.7	0.254	0.74
IBM 3081 K	H enhanced opt=3	5.1	2.4	0.283	0.82
IBM 3081 K	VS opt=3	5.6	2.2	0.311	0.91
IBM 3033	VS Fortran	6.3	1.9	0.353	1.03
IBM 3081 D	VS opt=2	6.7	1.8	0.376	1.10
ELXSI	Embos, F77 (Coded BLAS)	17	0.71	0.967	2.82
Vax 11/785 FPA	VMS (Coded BLAS)	23	0.53	1.30	3.79
ELXSI	Emobs, F77	23	0.51	1.35	3.92
Univac 1100/81	ASCII opt=ZEO	24	0.52	1.32	3.85
Data General MV/10000	f77 opt level 2	31	0.39	1.75	5.09
Vax 11/785 FPA	VMS	36	0.34	2.01	5.85
Vax 11/780 FPA	VMS (Coded BLAS)	37	0.33	2.08	6.07
Ridge 32	Fort 77 (Coded BLAS)	39	0.31	2.19	6.38
IBM 370/158	H opt=3	42	0.29	2.35	6.86
Norsk Data ND-500	Fortran-500-E	43	0.27	2.58	7.51
Dec KL-20	F20	46	0.27	2.59	7.53
IBM 370/158	VS opt=3	46	0.26	2.60	7.58
Univac 1100/62	ASCII opt=ZEO	49	0.25	2.77	8.09
ICL 2988	f77 opt=2	50	0.25	2.79	8.13
Harris 800	Fortran 77	53	0.23	2.99	8.70
Vax 11/750 FPA	VMS (Coded BLAS)	56	0.22	3.14	9.16
IBM 4341 MG10	VS opt=3	57	0.22	3.18	9.25
Vax 11/780 FPA	VMS	59	0.21	3.28	9.57
Vax 11/780 FPA	Unix xf77	61	0.20	3.41	9.93
Honeywell 6080	Y	62	0.20	3.46	10.1
Ridge 32	Fort 77	62	0.20	3.48	10.1
Data General MV/8000	f77 opt level 2	69	0.18	3.84	11.2
Vax 11/780	VMS	74	0.17	4.13	12.0
Vax 11/750 FPA	VMS	86	0.14	4.80	14.0

Table 2 *cont*

Computer	Compiler[c]	Ratio[d]	MFLOPs[e]	Time (s)	Unit[f] (μs)
Prime 850	Primos	97	0.13	5.41	15.8
HP 9000 Series 500	Fortran 1.7	125	0.098	7.00	20.4
Vax 11/750	VMS	137	0.089	7.69	22.4
IBM 4331	H opt=3	140	0.088	7.84	22.8
Apollo	4.1 PEB (Coded BLAS)	177	0.069	9.92	28.9
Vax 11/730 FPA	VMS (Coded BLAS)	205	0.060	11.5	33.4
Vax 11/725 FPA	VMS (Coded BLAS)	205	0.060	11.5	33.4
Masscomp MC500 w/FP	Unix, f77 opt	227	0.054	12.7	37.1
Burroughs 6700	H	234	0.052	13.1	38.2
Prime 2250	Fortran 77	258	0.048	14.5	42.1
Vax 11/730 FPA	VMS	259	0.047	14.5	42.2
Vax 11/725 FPA	VMS	259	0.047	14.5	42.2
Chas. River Data 6835 + SKY	SVS Fortran 77	284	0.043	15.9	46.3
IBM PC-XT/370	H opt=3	303	0.040	17.0	49.5
Dec KA-10	F40	305	0.040	17.1	49.8
Canaan	VS	306	0.040	17.1	49.9
Sun 2 + SKY board	Unix, f77 opt	314	0.039	17.6	51.1
Apollo	4.1 PEB	334	0.037	18.7	54.5
Chas. River Data 6835	SVS Fortran 77	770	0.016	43.1	126
Cadtrak DS1/8087	Intel Fortran 77	893	0.013	50.0	146
Sun 2	Unix, f77 opt	966	0.013	54.1	158
Masscomp MC500	Unix, f77 opt	1015	0.012	56.8	166
IBM PC/8087	Microsoft 3.1	1071	0.011	60.0	175
HP 9000 Series 200	HP-UX	1196	0.010	67.0	195
Sun	Unix, f77 no opt	1298	0.0094	72.7	212
IBM PC	Microsoft 3.1	21875	0.00056	1225	3568
Apple III	Pascal	50232	0.00024	2813	8193

[a], [b], [c], [d], [e] and [f] are the same as in Table 1.

Anyone interested in adding to or updating these tables is encouraged to contact the author. Please send suggestions and interesting results to:

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Appendix: Performance of large scientific computers in a Fortran environment

The LINPACK routines used to generate the timings in the previous tables do not reflect the true performance of 'advanced scientific computers'. A different implementation of the solution of linear equations, presented in a report by Dongarra and Eisenstat [Dongarra 83], better describes the performance on such machines. That algorithm is based on matrix–vector operations rather than just vector operations. This produces a program that has a high level of modularity or larger granularity, having the potential for better performance across a wide range of machines, especially on high-performance computers. The number of floating-point operations required and the roundoff errors produced by both algorithms are exactly the same, only the way in which the matrix elements are accessed is different. As before, a Fortran program was run and the time to complete the solution of equations for a matrix of order 300 is reported.

Note that these numbers are for a problem of order 300 and all runs are for full precision.

The data presented in the following table was compiled over a period of time. Subsequent software and hardware changes to a computer system may alter the timing to some extent.

TABLE 3

Solving a system of linear equations using the vector unrolling technique

Computer	Compiler[a]	MFLOPS[b]	Time (s)	Unit[c] (μs)
Cray X-MP[d]	CFT (Coded ISAMAX)	240	0.076	0.0083
Fujitsu VP-200	Fortran 77 (Comp directive)	220	0.083	0.0091
Fujitsu VP-200	Fortran 77	183	0.099	0.011
Cray X-MP[d]	CFT	161	0.113	0.012
Cray X-MP[e]	CFT (Coded ISAMAX)	134	0.136	0.015
Cray X-MP[e]	CFT	106	0.172	0.019
Cray 1-M[f]	CFT (Coded ISAMAX)	83	0.215	0.024
Cray 1-S[f]	CFT (Coded ISAMAX)	76	0.236	0.026
Cray 1-M	CFT	69	0.259	0.029
Cray 1-S	CFT	66	0.273	0.030
NAS 9060 w/VPF	VS opt=2 (Coded BLAS)	9.7	1.9	0.204
NAS 9060	VS opt=2	6.9	2.6	0.285
IBM 370/195	VS opt=2	4.4	4.1	0.455
FPS 164	D, opt=3 (Coded ISAMAX)	4.1	4.4	0.488
FPS 164	D, opt=3	4.0	4.5	0.500
IBM 3033	VS opt=2	2.5	7.1	0.800
Vax 11/780 FPA	Unix xf77	0.11	177	19.5

[a] *Compiler* refers to the compiler used, (Coded ISAMAX) refers to the use assembly language coding of the BLAS ISAMAX, and *Comp Directive* refers to the use of compiler directives in the matrix–vector routines.

[b] *MFLOPS* is a rate of execution, the number of million floating-point operations completed per second. For solving a system of n equations, approximately $(2/3n^3) + 2n^2$ operations are performed (we count both additions and multiplications).

[c] *Unit* is the time in microseconds required to execute the statement $y_i = y_i + tx_i$. This involves one floating-point multiplication, one floating-point addition, and a few one-dimensional indexing operations and storage references (additions and multiplications).

[d] These timings are for two processors.

[e] These timings are for one processor.

The major difference between the Cray 1-M and Cray 1-S is the memory speed, the Cray 1-M having a slower memory. The timings show the Cray 1-M to be faster than the Cray 1-S. After much discussion and examination of the generated assembly language code it was determined that, in fact, the Cray 1-M was faster for this program. The code generated by the compiler causes the Cray 1-S to miss a chain-slot. On the Cray 1-M, because of a slower memory, the chain-slot is not missed, and hence the faster execution time.

Abbreviations used in TSI

Technology and Science of Informatics uses standard abbreviations for organizations and institutions in France. A list of the relevant organizations is given below for the benefit of readers who may not be familiar with all the abbreviations.

ADI	Agence de l'Informatique
AFCET	Association Française pour la Cybernétique Economique et Technique
CERI	Centre d'Etudes et Recherches Informatique
CERT	Centre d'Etudes et de Recherches de Toulouse
CII-HB	CII-Honeywell Bull
CNAM	Conservatoire National des Arts et Métiers
CNET	Centre National d'Etudes des Télécommunications
CNRS	Centre National de la Recherche Scientifique
CRIN	Centre de Recherche en Informatique de Nancy
EDF	Electricité de France
ENSEEIHT	Ecole Normale Supérieure d'Electrotechnique, d'Electronique, d'Informatique et d'Hydraulique de Toulouse
ENST	Ecole Normale Supérieure des Télécommunications
IMAG	Institut de Mathématiques Appliquées de Grenoble
INRIA	Institut National de la Recherche en Informatique et en Automatique
IP	Institut de Programmation
IRISA	Institut de Recherche en Informatique et Systèmes Aléatoires
IUT	Institut Universitaire de Technologie
LAAS	Laboratoire d'Automatique et d'Analyse des Systèmes
ONERA/CERT	Office National d'Etudes et de Recherches Aérospatiale CERT
RNUR	Régie Nationale des Usines Renault
STI (EDF-GDF)	Service Technique Informatique EDF (see above) - Gaz de France
UTC	Université de Technologie de Compiègne