Algorithm-Based Fault Tolerance for Fail-Stop Failures

Zizhong Chen and Jack Dongarra

Abstract

Fail-stop failures in distributed environments are often tolerated by checkpointing or message logging. In this paper, we show that fail-stop process failures in ScaLAPACK matrix matrix multiplication kernel can be tolerated without checkpointing or message logging. It has been proved in the previous algorithm-based fault tolerance research that, for matrix-matrix multiplication, the checksum relationship in the input checksum matrices is preserved *at the end of the computation* no matter which algorithm is chosen. From this checksum relationship in the final computation results, processor miscalculations can be detected, located, and corrected at the end of the computation. However, whether this checksum relationship in the input checksum matrices can be maintained *in the middle of the computation* or not remains open. In this paper, we first demonstrate that, for many matrix matrix multiplcation algorithms, the checksum relationship in the input checksum matrices is *not* maintained in the middle of the computation. We then prove that, however, for the outer product version matrix matrix multiplcation algorithm, the checksum relationship in the input checksum matrices can be maintained in the middle of the computation. Based on this checksum relationship maintained in the middle of the computation, we demonstrate that fail-stop process failures in ScaLAPACK matrix-matrix multiplcation can be tolerated without checkpointing or message logging. Because no periodical checkpointing is involved, the fault tolerance overhead for this approach is surprisingly low.

Index Terms

Algorithm-based fault tolerance, checkpointing, fail-stop failures, parallel matrix matrix multiplication, ScaLAPACK.

Zizhong Chen is with the Department of Mathematical and Computer Sciences, Colorado School of Mines, Golden, CO 80401-1887. Email: zchen@mines.edu.

Jack Dongarra is with the Department of Electrical Engineering and Computer Science, the University of Tennessee, Knoxville, TN 37996-3450. Email: dongarra@cs.utk.edu.

1 INTRODUCTION

As the number of processors in today's high performance computers continues to grow, the mean-time-to-failure (MTTF) of these systems are becoming significantly shorter than the execution time of many current high performance computing applications. Even making generous assumptions on the reliability of a single processor or link, it is clear that as the processor count in high end systems grows into the tens of thousands, the MTTF can drop from a few years to a few days, or less. For example, with 131,000 processors in the system, the current IBM Blue Gene L experienced failures every 48 hours during initial deployment [28]. In recent years, cluster of commodity off-the-shelf systems becomes more and more popular. While the commodity off-the-shelf cluster systems have excellent price-performance ratios, there is a growing concern with the fault tolerance issues in such systems due to the low reliability of the off-the-shelf components used in these systems. The recently emerging computational grid environments [14] with dynamic computing resources have further exacerbated the problem. However, driven by the desire of scientists for ever higher levels of detail and accuracy in their simulations, many computational science applications are now being designed to run for days or even months. To avoid restarting computations from beginning after failures, the next generation high performance computing applications need to be able to continue computations despite of failures.

Although there are many types of failures in today's parallel and distributed systems, in this paper, we focus on tolerating *fail-stop* process failures where the failed process stops working and all data associated with the failed process are lost. This type of failures is common in today's large computing systems such as high-end clusters with thousands of nodes and computational grids with dynamic computing resources. In order to tolerate such fail-stop failures, it often requires a global consistent state of the application be available or can be reconstructed when the failue occurs. Today's long running scientific applications typically tolerate such failures by checkpoint/restart in which all process states of an application are saved into stable storage periodically. The advantage of this approach is that it is able to tolerate the failure of the whole system. However, in this approach, if one process fails, usually all surviving processes are aborted and the whole application is restarted from the last checkpoint. The major source of overhead in all stable-storage-based checkpoint systems is the time it takes to write checkpoints into stable storage [23]. In order to tolerate partial failures with reduced overhead, diskless checkpointing [23] has been proposed by Plank et. al. By eliminating stable storage from checkpointing and replacing it with memory and processor redundancy, diskless checkpointing removes the main source of overhead in checkpointing [23]. Diskless checkpointing has been shown to achieve a decent performance to tolerate single process failure in [20]. For applications which modify a small amount of memory between checkpoints, it is shown in [9] that, even to tolerate multiple simultaneous process failures, the overhead introduced by diskless checkpointing is still negligible. However, the matrix-matrix multiplication operation considered in this paper often modifys a large mount of memory between checkpoints. Diskless checkpointing for such applications often produces a large size checkpoint. Therefore, even diskless checkpointing still introduces a considerable overhead [20], [22].

It has been proved in previous research [19] that, for some matrix operations, the checksum relationship in input checksum matrices is preserved *in the final computation results at the end of the operation*. Based on this checksum relationship in the final computation results, Huang and Abraham have developed the famous algorithm-based fault tolerance (ABFT) [19] technique to detect, locate, and correct certain processor miscalculations in matrix computations with low overhead. The algorithm-based fault tolerance proposed in [19] was later extended by many researches [1], [2], [3], [5], [21].

However, previous ABFT researches have mostly focused on detecting, locating, and correcting miscalculations or data corruption where failed processors are often assumed to be able to continue their work but produce incorrect calculations or corrupted data. The error detection are often performed at the end of the computation by checking whether the final computation results satisfy the checksum relationship or not.

In order to be able to recover from a fail-stop process failure in the middle of the computation, a global consistent state of the application is often required when a process failure occurs. Checkpointing and message logging are typical approaches to maintain or construct such global consistent state in a distributed environment. But if there exists a checksum relationship between application data on different processes, such checksum relationship can actually be treated as a global consistent state. However, it is still an open problem that whether the checksum relationship in input checksum matrices in ABFT can be maintained during computation or not. Therefore, whether ABFT can be extended to tolerate fail-stop process failures in a distributed environment or not remains open.

In this paper, we extend the ABFT idea to recover applications from fail-stop failures in the middle of the computation by maintaining a checksum relationship during the whole computation. We show that fail-stop process failures in ScaLAPACK [4] matrix-matrix multiplication kernel can be tolerated without checkpointing or message logging. We first demonstrate that, for many matrix matrix multiplication algorithms, the checksum relationship in input checksum matrices does not preserve during computation. We then prove that, however, for the outer product version matrix matrix multiplcation algorithm, it is possible to maintain the checksum relationship in input checksum matrices during computation. Based on this checksum relationship maintained during computation, we demonstrate that it is possible to tolerate fail-stop process failures (which are typically tolerated by checkpoting or message logging) in the outer product version distributed matrix matrix multiplcation without checkpointing or message logging. Because no periodical checkpoint or rollback-recovery is involved in this approach, process failures can often be tolerated with a surprisingly low overhead. We show the practicality of this technique by applying it to the ScaLAPACK matrix-matrix multiplication kernel which is one of the most important kernels for the widely used ScaLAPACK library to achieve high performance and scalability.

The rest of this paper is organized as follows. Section 2 explores properties of matrix matrix multiplication with input checksum matrices. Section 3 presents the basic idea of algorithm-based checkpoint-free fault tolerance. In Section 4, we demonstrate how to tolerate fail-stop process failures in ScaLAPACK matrix-matrix multiplcation without checkpointing or message logging. In Section 5, we evaluate the performance overhead of the proposed fault tolerance approach. Section 6 compares algorithm-based checkpoint-free fault tolerance with existing works and discusses the limitations of this technique. Section 7 concludes the paper and discusses future work.

2 MATRIX MATRIX MULTIPLICATION WITH CHECKSUM MATRICES

In this section, we explore the properties of different matrix matrix multiplication algorithms when the input matrices are checksum matrices defined in [19].

It has been proved in [19] that the checksum relationship of the input checksum matrices is preserved in the final computation results at the end of computation no mater which algorithm is used in the operation. However, whether this checksum relationship in input checksum matrices can be maintained during computation or not remains open.

In this section, we demonstrate that, for many algorithms to perform matrix matrix multiplication, the checksum relationship in the input checksum matrices does not preserve during computation. We prove that, however, for the outer product version matrix matrix multiplcation algorithm, it is possible to maintain the checksum relationship in the input checksum matrices during computation.

2.1 Maintaining Checksum at the End of Computation

Assume $I_{m\times m}$ is the identity matrix of dimmension m, $E_{m\times n}$ is the m-by-n matrix with all elements being 1. Let $H_m^c = [I_{m\times m}, E_{m\times 1}]^T$, $H_n^r = [I_{n\times n}, E_{n\times 1}]$. It's trival to verify $H_m^c = H_n^{rT}$ if $m = n$. For any m-by-n matrix A, the *column checksum* matrix A^c of A is defined by $A^c = H_m^c * A$, the *row checksum* matrix A^r of A is defined by $A^r = A * H_n^r$, and the *full checksum* matrix A^f of A is defined by $A^f = H_m^c * A * H_n^r$.

Theorem 1: Assume A is an m-by-k matrix, B is a k-by-n matrix, and C is an m-by-n matrix. If $A * B = C$, then $A^c * B^r = C^f$.

Proof:

$$\begin{aligned} A^c * B^r &= (H_m^c * A) * (B * H_n^r) \\ &= H_m^c * (A * B) * H_n^r \\ &= H_m^c * C * H_n^r \\ &= C^f. \end{aligned}$$

□

Theorem 1 was first proved by Huang and Abraham in [19]. We prove it here again to show that the proof of *Theorem 1* is independent of the algorithms used for the matrix

matrix multiplication operation. Therefore no mater which algorithm is used to perform the matrix matrix multiplication, the checksum relationship of the input matrices will always be preserved in the final computation results at the end of the computation.

Based on this checksum relationship in the final computation result, the low-overhead ABFT technique has been developed in [19] to detect, locate, and correct certain processor miscalculations in matrix computations.

2.2 Is the Checksum Maintained During Computation?

Algorithm-based fault tolerance usually detects, locates, and corrects errors at the end of the computation. But in today's highperformance computing environments such as PVM [27] and MPI [26], after a fail-stop process failure occurs in the middle of the computation, it is often required to recover from the failure first before the continuation of the rest of the computation.

In order to be able to recover from fail-stop failures occured in the middle of the computation, a global consistent state of an application is often required in the middle of the computation. The checksum relationship, if exists, can actually be treated as a global consistent state. However, from *Theorem 1*, it is still uncertain whether the checksum relationship is preserved in the middle of the computation or not.

In what follows, we demonstrate, for both Cannon's algorithm and Fox's algorithm for matrix matrix multiplication, this checksum relationship in the input checksum matrices is generally not preserved in the middle of the computation.

Assume A is an $(n-1)$-by-n matrix, B is an n-by-$(n-1)$ matrix. Then $A^c = (a_{ij})_{n\times n}$, $B^r = (b_{ij})_{n\times n}$, and $C^f = A^c * B^r$ are all n-by-n matrices. For convenience of description, but without loss of generality, assume there are n^2 processors with each processor stores one element from A^c, B^r,and C^f respectively. The n^2 processors are organized into a n-by-n processor grid.

Consider using the Cannon's algorithm [6] in Fig. 1 to perform $A^c * B^r$ in parallel on an n-by-n processor grid. We can prove the following *Theorem 2*.

Theorem 2: If the Cannon's algorithm in Fig. 1 is used to perform $A^c * B^r$, then there exist matrices A and B such that, at the end of each step s, where $s = 0, 1, 2, \cdots, n-2$, the partial sum matrix $C = (c_{ij})$ in Fig. 1 is not a full checksum matrix.

```
/* Calculate C = A^c * B^r by cannon's algorithm. */
initialize C = 0;
for i = 0 to n - 1
    left-circular-shift row i of A^c by i
        so that a_{i,j} is overwritten by a_{i, (j+i) mod n};
end
for i = 0 to n - 1
    up-circular-shift column i of B^r by i
        so that b_{i,j} is overwritten by b_{(i+j) mod n, j};
end
for s = 0 to n - 1
    every processor (i,j) performs c_ij = c_ij + a_ij * b_ij
        locally in parallel;
    left-circular-shift each row of A^c by 1;
    up-circular-shift each column of B^r by 1;
    /* Here is the end of the s^th step. */
end
```

Fig. 1. Matrix-matrix multiplication by cannon's algorithm with checksum input matrices

Proof: This can be proved by giving a simple example.

Let

$$A = \begin{pmatrix} 1 & 2 \end{pmatrix},$$

$$B = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$$

Then

$$A^c = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix},$$

$$B^r = \begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}.$$

In this example, $n = 2$, thus it is enough to just check one case: $s = 0$.

When the Cannon's algorithm in Fig. 1 is used to perform $A^c * B^r$, at the end of $s = 0^{th}$ step

$$C = \begin{pmatrix} 3 & 8 \\ 8 & 3 \end{pmatrix},$$

which is not a full checksum matrix. □

Actually, when the Cannon's algorithm in Fig. 1 is used to perform $A^c * B^r$ in parallel for matrix A and B in *Theorem 2*, it can be proved that at the end of the s^{th} step

$$c_{ij} = \sum_{k=0}^{s} a_{i,\ (i+j+k)\ mod\ n} \ * \ b_{(i+j+k)\ mod\ n,\ j}$$

It can be verified that $C = (c_{ij})_{n \times n}$ is not a full checksum matrix unless $s = n - 1$ which is the end of the computation. Therefore the checksum relationship in the matrix C is generally not preserved during computation in the cannon's algorithm for matrix-multiplication.

Each step of Cannon's algorithm updates the partial sum matrix C by adding a rank one matrix T where each entry of A^c and each entry of B^r contribute to some different entry of the matrix T. The rank one matrix T is a outer product between a column vector of entries from different columns of A^c and a row vector of entries from different rows of B^r. From the irreducibility of outer products, the rank one matrix T cannot algebraically equals to a full checksum matrix. Therefore, the partial sum matrix C cannot algebraically equals to a full checksum matrix.

How about if Fox's algorithm [16] in Fig. 2 is used to perform $A^c * B^r$?

Theorem 3: If the Fox's algorithm in Fig. 2 is used to perform $A^c * B^r$, then there exist matrices A and B such that, at the end of each step s, where $s = 0, 1, 2, \cdots, n-2$, the partial sum matrix $C = (c_{ij})$ in Fig. 2 is not a full checksum matrix.

Proof: The same example matrices A and B in the proof of *Theorem 2* can be used to prove *Theorem 3*. Because $n = 2$, it is also enough to just check only one case: $s = 0$.

When the Fox's algorithm in Fig. 2 is used to perform $A^c * B^r$, at the end of 0^{th} step

$$C = \begin{pmatrix} 3 & 3 \\ 8 & 8 \end{pmatrix},$$

which is not a full checksum matrix. □

```
/* Calculate A^c * B^r by fox's algorithm. */
initialize C = (c_ij) = 0;
for s = 0 to n - 1
    for i = 0 to n - 1 in parallel
        processor (i, (i + s) mod n) broadcast local
        t = a_{i, (i+s) mod n} to other processors in row i;
    for i, j = 0 to n - 1 in parallel
        every processor (i, j)
        performs c_ij = c_ij + t * b_ij locally;
    up-circular-shift each column of B^r by 1;
    /* Here is the end of the s^th step. */
end
```

Fig. 2. Matrix matrix multiplication by Fox's algorithm with input checksum matrices

When the Fox's algorithm in Fig. 2 is used to perform $A^c * B^r$ in parallel, it can actually be proved that at the end of the s^{th} step

$$c_{ij} = \sum_{k=0}^{s} a_{i,\ (i+k)\ mod\ n} \ * \ b_{(i+k)\ mod\ n,\ j}$$

It can be verified that $C = (c_{ij})_{n \times n}$ is not a full checksum matrix either unless $s = n - 1$ which is the end of the computation. Therefore the checksum relationship in the matrix C is generally not preserved during computation either in the Fox's algorithm for matrix-multiplication.

It can also be demonstrated that the checksum relationship in the input matrix C is not preserved during computation in many other parallel algorithms for matrix matrix multiplication.

2.3 Maintaining Checksum During Computation

Despite the checksum relationship of the input matrices is preserved in final results at the end of computation no mater which algorithm is used, from last subsection, we konw that the checksum relationship is not necessarily preserved during computation.

However, it is interesting to ask: is there any algorithm that preserves the checksum relationship during computation?

Consider using the outer product version algorithm [18] in Fig. 3 to perform $A^c * B^r$ in parallel. Assume the matrices A^c, B^r, and C have the same data distribution scheme as the matrices in Subsection 2.2.

Theorem 4: If the algorithm in Fig. 3 is used to perform $A^c * B^r$, then the partial sum matrix $C = (c_{ij})$ in Fig. 3 is a full checksum matrix at the end of each step s, where $s = 0, 2, \cdots, n-1$.

Proof: Let $A(:, 1:s)$ be the first s columns of A, $B(1:s, :)$ be the first s rows of B, and C(s) be the partial sum matrix C at the end of the s^{th} step of the outer product version algorithm in Fig. 2. Then

$$\begin{aligned} C(s) &= A^c(:, 1:s) * B^r(1:s, :) \\ &= (H^c_{n-1} * A(:, 1:s)) * (B(1:s, :) * H^r_{n-1}) \\ &= H^c_{n-1} * (A(:, 1:s) * B(1:s, :)) * H^r_{n-1} \end{aligned}$$

which is the full checksum matrix of the matrix $A(:, 1:s) * B(1:s, :)$. □

```
/*Calculate C = A^c * B^r by outer product algorithm.*/
initialize C = 0;
for s = 0 to n - 1
    row broadcast the s^th row of A^c;
    column broadcast the s^th column of B^r;
    every processor (i,j) performs c_ij = c_ij + a_is * b_sj
        locally in parallel;
    /* Here is the end of the s^th step. */
end
```

Fig. 3. Matrix-matrix multiplication by outer product algorithm with checksum input matrices

Theorem 4 implies that a coded global consistent state of the critical application data (i.e. the checksum relationship in A^c, B^r, and C^f) can be maintained in memory at the end of each iteration in the outer product version matrix matrix multiplication if we perform the computation with the checksum input matrices.

However, in a high performance distributed environment, different processes may update their data in local memory asynchronously. Therefore, if a failure happens at a time when some processes have updated their local matrix in memory and other processes are still in the communication stage, then the checksum relationship in the distributed matrix will be damaged and the data on all processes will not form a global consistent state.

But this problem can be solved by simply performing a synchronization before performing local memory update. Therefore, it is possible to maintain a coded global consistent state (i.e. the checksum relationship) of the matrix A^c, B^r and C^f in the distributed memory at any time during computation. Hence, a single fail-stop process failure in the middle of the computation can be recovered from the checksum relationship.

Note that it is also the outer product version algorithm that is often used in today's highperformance computing practice. The outer product version algorithm is more popular due to both its simplicity and and it's efficiency in modern high performance computer architecture. In the widely used parallel numerical linear algebra library ScaLAPACK [4], it is also the outer product version algorithm that is chosen to perform the matrix matrix mulitiplication.

More importantly, it can also be proved that similar checksum relationship exists for the outer product version of many other matrix operations (such as Cholesky and LU factorization).

3 Algorithm-Based Checkpoint-Free Fault Tolerance for Fail-Stop Failures

In this section, we develop some general princples for recovering fail-stop failures in the middle of computation by maintaining checksum relationship in the algorithm instead of checkpointing or message logging.

3.1 Failure Detection and Location

Handling fault-tolerance typically consists of three steps: 1) fault detection, 2) fault location, and 3) fault recovery. Fail-stop process failures can often be detected and located with the aid of the programming environment. For example, many current programming environments such as PVM [27], Globus [15], FT-MPI [12], and Open MPI [17] provide this type of failure detection and location capability. We assume the loss of partial processes in the message passing system does not cause the aborting of the survival processes and it is possible to replace the failed processes in the message passing system and continue the communication after the replacement. FT-MPI [12] is one such programming environments that support all these functionalities.

In this paper, we use FT-MPI to detect and locate failures. FT-MPI is a fault tolerant version of MPI that is able to provide basic system services to support fault survivable applications. FT-MPI implements the complete MPI-1.2 specification, some parts of the MPI-2 document and extends some of the semantics of MPI for allowing the application the possibility to survive process failures. FT-MPI can survive the failure of n-1 processes in a n-process job, and, if requested, can re-spawn the failed processes. However, the application is still responsible for recovering the data structures and the data of the failed processes. Interested readers are refered to [12], [13] for more detail on how to recover FT-MPI programming environment. In the rest of this paper, we will mainly focus on how to recover the lost data in the failed processes.

3.2 Failure Recovery

Consider the simple case where there will be only one process failure. Before the failure actually occurs, we do not know which process will fail, therefore, a scheme to recover only the lost data on the failed process actually need to be able to recover data on *any* process. It seems difficult to be able to recover data on any process without saving all data on all processes somewhere. However, if we assume, at any time during the computation, the data on the i^{th} process P_i satisfies

$$P_1 + P_2 + \cdots + P_{n-1} = P_n, \tag{1}$$

where n is the total number of process used for the computation. Then the lost data on *any* failed process would be able to be recovered from formula (1). Assume the j^{th} process failed, then the lost data P_j can be recovered from

$$P_j = P_n - (P_1 + \cdots + P_{j-1} + P_{j+1} + \cdots + P_{n-1})$$

In this very special case, we are lucy enough to be able to recover the lost data on *any* failed process without checkpoint due to the special *checksum relationship* (1). In practice, this kind of special relationship is by no means natural. However, it is natural to ask: *is it possible to design an application to maintain such a special checksum relationship throughout the computation on purpose?*

Assume the original application is designed to run on n processes. Let P_i denotes the data on the i^{th} computation process. In some algorithms for matrix operations (such as the outer product version algorithm for matrix-matrix multiplication), the special checksum relationship above can actually be designed on purpose as follows

- **step 1:** Add another encoding process into the application. Assume the data on this encoding process is C. For numerical computations, P_i is often an array of floating-point numbers, therefore, at the beginning of the computation, we can create a checksum relationship among the data of all processes by initializing the data C on the encoding process as

$$P_1 + P_2 + \cdots + P_n = C \tag{2}$$

- **step 2:** During the execution of the application, redesign the algorithm to operate both on the data of computation processes and on the data of encoding process in such a way that the checksum relationship (2) is always maintained during computation.

The specially designed checksum relationship (2) actually establishes an equality between the data P_i on computation processes and the encoding data C on the encoding process. If any processor fails then the equality (2) becomes an equation with one unknown. Therefore, the data in the failed processor can be reconstructed through solving this equation.

The above fault tolerance technique can be used to tolerate single fail-stop process failure in parallel matrix-matrix multiplication without checkpointing or message logging. The special checksum relationship between the data on different processes can be designed on purpose by: (1). using the checksum matrices of the original matrices as the input matrices, and (2). choosing the outer product version algorithm to perform the matrix-matrix multiplication. Section 2.3 is the application of the technique to the case where each element of the matrix is on a different process. In the next section, we will apply this technique to the case where matrices are distributed onto processes according to the two-dimensional block cyclic distribution.

4 Incorporating Fault Tolerance into the ScaLAPACK Matrix-Matrix Multiplication

In this section, we apply the algorithm-based checkpoint-free technique developed in Section 3 to the ScaLAPACK matrix-matrix multiplication kernel which is one of the most important kernels for the widely used ScaLAPACK library to achieve high performance and scalability.

Actually, it is also possible to incorporate fault tolerance into many other ScaLAPACK routines through this approach. However, in this section, we will restrict our presentation to the matrix-matrix multiplication kernel. For the simplicity of presentation, in this section, we only discuss the case where there is only one process failure. However, it is straightforward to extend the result here to the multiple simultaneous process failures case by simply using a weighted checksum scheme [11].

4.1 Two-Dimensional Block-Cyclic Distribution

It is well-known [4] that the layout of an application's data within the hierarchical memory of a concurrent computer is critical in determining the performance and scalability of the parallel code. By using two-dimensional block-cyclic data distribution [4], ScaLAPACK seeks to maintain load balance and reduce the frequency with which data must be transferred between processes.

For reasons described above, ScaLAPACK organizes the one-dimensional process array representation of an abstract parallel computer into a two-dimensional rectangular

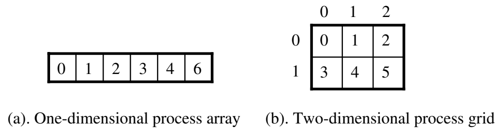


Fig. 4. Process grid in ScaLAPACK

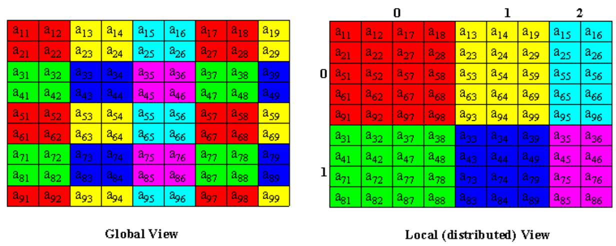


Fig. 5. Two-dimensional block-cyclic matrix distribution

process grid. Therefore, a process in ScaLAPACK can be referenced by its row and column coordinates within the grid. An example of such an organization is shown in Fig. 4.

The two-dimensional block-cyclic data distribution scheme is a mapping of the global matrix onto the rectangular process grid. There are two pairs of parameters associated with the mapping. The first pair of parameters is (mb, nb), where mb is the row block size and nb is the column block size. The second pair of parameters is (P, Q), where P is the number of process rows in the process grid and Q is the number of process columns in the process grid. Given an element a_{ij} in the global matrix A, the process coordinate (p_i, q_j) that a_{ij} resides can be calculated by

$$\begin{cases} p_i = \lfloor \frac{i}{mb} \rfloor \bmod P, \\ q_j = \lfloor \frac{j}{nb} \rfloor \bmod Q, \end{cases}$$

The local coordinate (i_{p_i}, j_{q_j}) which a_{ij} resides in the process (p_i, q_j) can be calculated according to the following formula

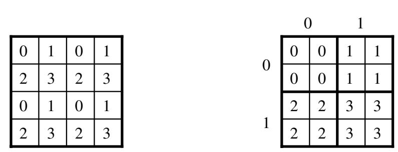
$$\begin{cases} i_{p_i} = \lfloor \frac{\lfloor \frac{i}{mb} \rfloor}{P} \rfloor . mb + (i \mod mb), \\ j_{q_j} = \lfloor \frac{\lfloor \frac{j}{nb} \rfloor}{Q} \rfloor . nb + (j \mod nb), \end{cases}$$

Fig. 5 is an example of mapping a 9-by-9 matrix onto a 2-by-3 process grid according two-dimensional block-cyclic data distribution with $mb = nb = 2$.

4.2 Encoding Two-Dimensional Block Cyclic Matrices

In this section, we will construct different encoding schemes which can be used to design checkpoint-free fault tolerant matrix computation algorithms in ScaLAPACK. The purpose of encoding is to creat the checksum relationship proposed in the step 1 of Section 3.2.

Assume a matrix M is originally distributed in a P-by-Q process grid according to the two dimensional block cyclic data distribution. For the convenience of presentation, assume the size of the local matrices in each process is the same. We will explain different coding schemes for the matrix M with the help of the example matrix in Fig. 6. Fig. 6 (a) shows the global view of all the elements of the example matrix. After the matrix is mapped onto a 2-by-2 process grid with $mb = nb = 1$, the distributed view of this matrix is shown in Fig. 6 (b).

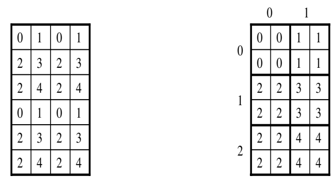


(a). Original matrix from global view (b). Original matrix from distributed view

Fig. 6. Two-dimensional block cyclic distribution of an example matrix

Suppose we want to tolerate a single process failure. We dedicate another $P + Q + 1$ additional processes and organize the total $PQ + P + Q + 1$ process as a $P + 1$-by-$Q + 1$ process grid with the original matrix M distributed onto the first P rows and Q columns of the process grid.

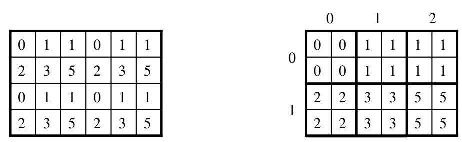
The *distributed column checksum matrix* M^c of the matrix M is the original matrix M plus the part of data on the $(P + 1)^{th}$ process row which can be obtained by adding all local matrices on the first P process rows. Fig. 7 (b) shows the distributed view of the column checksum matrix of the example matrix from Fig. 6. Fig. 7 (a) is the global view of the column checksum matrix.



(a). Column checksum matrix from global view (b). Column checksum matrix from distributed view

Fig. 7. Distributed column checksum matrix of the example matrix

The *distributed row checksum matrix* M^r of the matrix M is the original matrix M plus the part of data on the $(Q+1)^{th}$ process columns which can be obtained by adding all local matrices on the first Q process columns. Fig. 8 (b) shows the distributed view of the row checksum matrix of the example matrix from Fig. 6. Fig. 8 (a) is the global view of the row checksum matrix.



(a). Row checksum matrix from global view (b). Row checksum matrix from distributed view

Fig. 8. Distributed row checksum matrix of the original matrix

The *distributed full checksum matrix* M^f of the matrix M is the original matrix M, plus the part of data on the $(P+1)^{th}$ process row which can be obtained by adding all local matrices on the first P process rows, plus the part of data on the $(Q+1)^{th}$ process column which can be obtained by adding all local matrices on the first Q process columns. Fig. 9 (b) shows the distributed view of the full checksum matrix of the example matrix from Fig. 6. Fig. 9 (a) is the global view of the full checksum matrix.

4.3 Parallel Matrix Multiplication Algorithm in ScaLAPACK

To achieve high performance, the matrix-matrix multiplication in ScaLAPACK uses a blocked outer product version of the matrix matrix multiplication algorithm. Let A_j denote the j^{th} column block of the matrix A and B_j^T denote the j^{th} row block of the matrix B. Fig. 11 is the algorithm to perform the matrix matrix multiplication. Fig. 10 shows the j^{th} step of the matrix matrix multiplication algorithm.

0	1	1	0	1	1
2	3	5	2	3	5
2	4	6	2	4	6
0	1	1	0	1	1
2	3	5	2	3	5
2	4	6	2	4	6

(a). Full checksum matrix from global view

	0		1		2	
0	0	0	1	1	1	1
	0	0	1	1	1	1
1	2	2	3	3	5	5
	2	2	3	3	5	5
2	2	2	4	4	6	6
	2	2	4	4	6	6

(b). Full checksum matrix from distributed view

Fig. 9. Distributed full checksum matrix of the original matrix

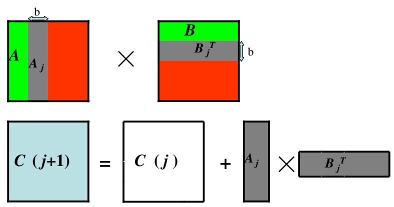


Fig. 10. The j^{th} step of the blocked outer product version parallelmatrix-matrix multiplication algorithm

4.4 Maintaining Global Consistent States by Computation

Many algorithms can be used to perform parallel matrix matrix multiplication. But, as shown in Section 2.2, the checksum relationship may not be maintained in the middle of the computation if inappropriate algorithms are chosen to perform the operation. However, if the outer product version algorithm is used to operate on the encoded checksum matrices, the redesigning of the algorithm to maintain checksum during computation (step 2 of Section 3.2) becomes a very simple task.

for $j = 0, 1, \ldots$
    row broadcast A_j;
    column broadcast B_j^T;
    local update: $C = C + A_j * B_j^T$;
end

Fig. 11. Blocked outer product version parallel matrix-matrix multiplication algorithm

Assume A, B and C are distributed matrices on a P by Q process grid with the first element of each matrix on process (0, 0). Let A^c, B^r and C^f denote the corresponding distributed checksum matrix. Let A^c_j denote the j^{th} column block of the matrix A^c and B^{rT}_j denote the j^{th} row block of the matrix B^r. We first prove the following fundamental theorem for matrix matrix multiplication with checksum matrices.

Theorem 5: Let $S_j = C^f + \sum_{k=0}^{j-1} A^c_k * B^{rT}_k$, then S_j is a distributed full checksum matrix.

Proof: It is straightforward that $A^c_k * B^{rT}_k$ is a distributed full checksum matrix and the sum of two distributed full checksum matrices is a distributed checksum matrix. S_j is the sum of $j+1$ distributed full checksum matrices, therefore is a distributed full checksum matrix. □

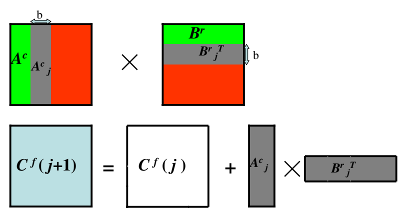


Fig. 12. The j^{th} step of the fault tolerant matrix-matrix multiplication algorithm

Theorem 5 tells us that at the end of each iteration of the blocked outer product version matrix matrix multiplication algorithm with checksum matrices, the checksum relationship of all checksum matrices are still maintained. This tells us that a coded global consistent state of the critical application data is maintained in memory at the end of each iteration of the matrix matrix multiplication algorithm if we perform the computation with related checksum matrices.

However, in a distributed environment, different process may update there local data asynchronously. Therefore, if when some process has updated their local matrix and some process is still in the communication stage, a failure happens, then the relationship of the data in the distributed matrix will not be maintained and the data on all processes would not form a consistent state. But this could be solved by simply performing a synchronization before performing local update. Therefore, in the following algorithm in Fig. 13, there will always be a coded global consistent state (i.e. the checksum relationship) of the matrix A^c, B^r and C^f in memory. Hence, a single process failure at any time during the matrix matrix multiplication would be able to be recovered from the checksum relationship. Fig. 12 shows the j^{th} step of the fault tolerate matrix matrix multiplication algorithm.

```
construct checksum matrices A^c, B^r, and C^f;
for j = 0, 1, ...
    row broadcast A_j^c;
    column broadcast B_j^{rT};
    synchronize;
    local update: C^f = C^f + A_j^c * B_j^{rT};
end
```

Fig. 13. A fault tolerant matrix-matrix multiplication algorithm

4.5 Overhead and Scalability Analysis

In this section, we analysis the overhead introduced by the algorithm-based checkpoint-free fault tolerance for matrix matrix multiplication.

For the simplicity of presentation, we assume all three matrices A, B, and C are square. Assume all three matrices are distributed onto a P by P process grid with m by m local matrices on each process. The size of the global matrices is Pm by Pm. Assume all elements in matrices are 8-byte double precision floating-point numbers. Assume every process has the same speed and disjoint pairs of processes can communicate without interfering each other. Assume it takes $\alpha + \beta k$ seconds to transfer a message of k bytes regardless which processes are involved, where α is the latency of the communication and $\frac{1}{\beta}$ is the bandwidth of the communication. Assume a process can concurrently send a message to one partner and receive a message from a possibly different partner. Let γ denote the time it takes for a process to perform one floating-point arithmetic operation.

4.5.1 Time Complexity for Parallel Matrix Matrix Multiplication

Note that the sizes of all three global matrices A, B, and C are all Pm, therefore, the total number of floating-point arithmetic operations in the matrix matrix multiplication is $2P^3m^3$. There are P^2 processes with each process executing the same number of floating-point arithmetic operations. Hence, the total number of floating-point arithmetic operations on each process is $2Pm^3$. Therefore, the time T_{matrix_comp} for the computation in matrix matrix multiplication is

$$T_{matrix\_comp} = 2Pm^3\gamma.$$

In the parallel matrix matrix multiplication algorithm in Fig. 11, the columns of A and the rows of B also need to broadcast to other column and row processes respectively. To broadcast one block columns of A using a simple binary tree broadcast algorithm, it takes $2(\alpha + 8bm\beta)\log_2 P$, where b is the row block size in the two dimensional block cyclic distribution. Therefore, the time T_{matrix_comm} for the communication in matrix matrix multiplication is

$$T_{matrix\_comm} = 2\alpha\frac{Pm}{b}\log_2 P + 16\beta Pm^2\log_2 P.$$

Therefore, the total time to perform parallel matrix matrix multiplication is

$$\begin{aligned} T_{matrix\_mult} &= T_{matrix\_comp} + T_{matrix\_comm} \\ &= 2Pm^3\gamma + 2\alpha\frac{Pm}{b}\log_2 P \\ &\quad +16\beta Pm^2\log_2 P. \end{aligned} \tag{3}$$

4.5.2 Overhead for Calculating Encoding

To make matrix matrix multiplication fault tolerant, the first type of overhead introduced by the algorithm-based checkpoint-free fault tolerance technique is (1) constructing the distributed column checksum matrix A^c from A; (2) constructing the distributed row checksum matrix B^r from B; (3) constructing the distributed full checksum matrix C^f from C;

The distributed checksum operation involved in constructing all these checksum matrices performs the summation of P local matrices from P processes and saves the result into the $(P+1)^{th}$ process. Let T_{each_encode} denote the time for one checksum operation and T_{total_encode} denote the time for constructing all three checksum matrices A^c, B^r, and C^f, then

$$T_{total\_encode} = 4T_{each\_encode}$$

By using a fractional tree reduce style algorithm [25], the time complexity for one checksum operation can be expressed as

$$\begin{aligned} T_{each\_encode} &= 8m^2\beta\left(1+O\left(\left(\frac{\log_2 P}{m^2}\right)^{1/3}\right)\right) \\ &\quad +O(\alpha\log_2 P)+O(m^2\gamma) \end{aligned}$$

Therefore, the time complexity for constructing all three checksum matrices is

$$\begin{aligned} T_{total\_encode} &= 32m^2\beta\left(1+O\left(\left(\frac{\log_2 P}{m^2}\right)^{1/3}\right)\right) \\ &\quad +O(\alpha\log_2 P)+O(m^2\gamma). \end{aligned} \quad (4)$$

In practice, unless the size of the local matrices m is very small or the size of the process grid P is extremely large, the total time for constructing all three checksum matrices is almost independent of the size of the process grid P.

The overhead (%) R_{total_encode} for constructing checksum matrices for matrix matrix multiplication is

$$\begin{aligned} R_{total\_encode} &= \frac{T_{total\_encode}}{T_{matrix\_mult}} \\ &= O(\frac{1}{Pm}) \end{aligned} \quad (5)$$

From (5), we can conclude

1) If the size of the data on each process is fixed (m is fixed), then as the number of processes increases to infinite (that is $P \to \infty$), the overhead (%) for constructing the checksum matrices decreases to zero with a speed of $O(\frac{1}{P})$
2) If the number of processes is fixed (P is fixed), then as the size of the data on each process increases to infinite (that is $m \to \infty$) the overhead (%) for constructing the checksum matrices decreases to zero with a speed of $O(\frac{1}{m})$

4.5.3 Overhead for Performing Computations on Encoded Matrices

The fault tolerant matrix matrix multiplication algorithm in Fig. 13 performs computations using checksum matrices which have larger size than the original matrices. However, the total number of processes devoted to computation also increases. A more careful analysis of the algorithm in Fig. 13 indicates that the number of floating-point arithmetic operations on each process in the fault tolerant algorithm (Fig. 13) is actually the same as that of the original non-fault tolerant algorithm (Fig. 11).

As far as the communication is concerned, in the original algorithm (in Fig. 11), the column (and row) blocks are broadcast to P processes. In the fault tolerant algorithms (in Fig. 13), the column (and row) blocks now have to be broadcast to $P+1$ processes.

Therefore, the total time to perform matrix matrix multiplication with checksum matrices is

$$\begin{aligned} T_{matrix\_mult\_checksum} &= 2Pm^3\gamma + 2\alpha\frac{Pm}{b}\log_2(P+1) \\ &\quad +16\beta Pm^2\log_2(P+1). \end{aligned}$$

Therefore, the overhead (time) to perform computations with checksum matrices is

$$\begin{aligned} T_{overhead\_matrix\_mult} &= T_{matrix\_mult\_checksum} \\ &\quad -T_{matrix\_mult} \\ &= 2\alpha\frac{Pm}{b}\log_2(1+\frac{1}{P}) \\ &\quad +16\beta Pm^2\log_2(1+\frac{1}{P}). \end{aligned} \tag{6}$$

The overhead (%) $R_{overhead\_matrix\_mult}$ for performing computations with checksum matrices in fault tolerant matrix matrix multiplication is

$$\begin{aligned} R_{overhead\_matrix\_mult} &= \frac{T_{overhead\_matrix\_mult}}{T_{matrix\_mult}} \\ &= O(\frac{1}{Pm}) \end{aligned} \tag{7}$$

From (7), we can conclude that

1) If the size of the data on each process is fixed (m is fixed), then as the number of processes increases to infinite (that is $P \to \infty$), the overhead (%) for performing computations with checksum matrices decreases to zero with a speed of $O(\frac{1}{P})$
2) If the number of processes is fixed (P is fixed), then as the size of the data on each process increases to infinite (that is $m \to \infty$) the overhead (%) for performing computations with checksum matrices decrease to zero with a speed of $O(\frac{1}{m})$

4.5.4 Overhead for Recovery

The failure recovery contains two steps: (1) recover the programming environment; (2) recover the application data.

The overhead for recovering the programming environment depends on the specific programming environment. For FT-MPI [12] which we perform all our experiment on, it introduce a negligible overhead (refer to Section 5).

The procedure to recover the three matrices A, B, and C is similar to calculating the checksum matrices. Except for matrix C, it can be recovered from either the row checksum or the column checksum relationship. Therefore, the overhead to recover data is

$$\begin{aligned} T_{recover\_data} &= 24m^2\beta\left(1 + O\left(\left(\frac{\log_2 P}{m^2}\right)^{1/3}\right)\right) \\ &\quad + O(\alpha \log_2 P) + O(m^2\gamma) \end{aligned} \tag{8}$$

In practice, unless the size of the local matrices m is very small or the size of the process grid P is extremely large, the total time for recover all three checksum matrices is almost independent of the size of the process grid P.

The overhead (%) $R_{recover\_data}$ for constructing checksum matrices for matrix matrix multiplication is

$$\begin{aligned} R_{recover\_data} &= \frac{T_{recover\_data}}{T_{matrix\_mult}} \\ &= O(\frac{1}{Pm}) \end{aligned} \tag{9}$$

which decreases with the speed of $O(\frac{1}{Pm})$.

5 Experimental Evaluation

In this section, we experimentally evaluate the performance overhead of applying the algorithm-based checkpoint-free fault tolerance technique to the ScaLAPACK matrix-matrix multiplication kernel. We performed four sets of experiments to answer the following five questions:

1) What is the performance overhead of constructing checksum matrices?

2) What is the performance overhead of performing computations with checksum matrices?
3) What is the performance overhead of recovering FT-MPI programming environments?
4) What is the performance overhead of recovering checksum matrices?

For each set of experiments, the size of the problems and the number of computation processes used are listed in TABLE 1.

TABLE 1
Experiment Configurations

Process grid w/out FT	Process grid w/ FT	Size of the original matrix	Size of the checksum matrix
2 by 2	3 by 3	12,800	19,200
3 by 3	4 by 4	19,200	25,600
4 by 4	5 by 5	25,600	32,000
5 by 5	6 by 6	32,000	38,400
6 by 6	7 by 7	38,400	44,800
7 by 7	8 by 8	44,800	51,200
8 by 8	9 by 9	51,200	57,600
9 by 9	10 by 10	57,600	64,000
10 by 10	11 by 11	64,000	70,400

All experiments were performed on a cluster of 64 dual-processor nodes with AMD Opteron(tm) Processor 240. Each node of the cluster has 2 GB of memory and runs a Linux operating system. The nodes are connected with Myrinet. The timer we used in all measurements is MPI_Wtime.

The programming environment we used is FT-MPI [12]. A process failure is simulated by killing one process in the middle of the computation. After a process failure is detected, MPI_Comm_dup() is called to recover the communication environment. The lost data on the failed process is then recovered by solving the checksum equation in Section 3.2.

Fig. 14 shows the flow of control and the state transition diagram for our fault tolerant program. Upon failure, the system error recovery module replaces dead processes and transfers control to a recovery point in the surviving processes More details on how to write fault tolerant applications using FT-MPI can be found in reference [10], [12].

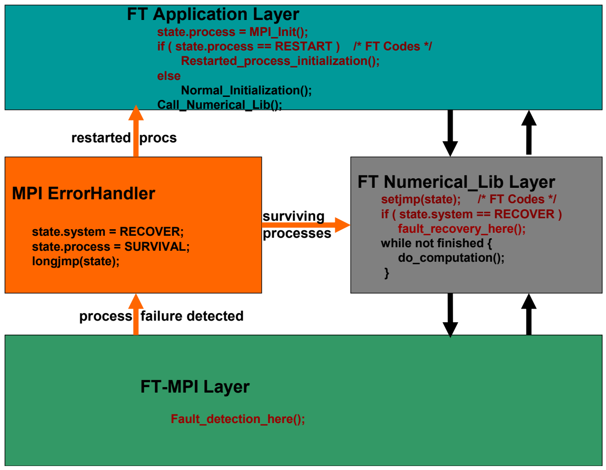


Fig. 14. Fault tolerant application control flow

5.1 Overhead for Constructing Checksum Matrices

The first set of experiments is designed to evaluate the performance overhead of constructing checksum matrices. We keep the amount of data in each process fixed (that is the size of local matrices m fixed), and increase the size of the test matrices (hence the size of process grid).

Fig. 15 reports the time for performing computations on original matrices and the time for constructing the three checksum matrices A^c, B^r, and C^f. Fig. 16 reports the overhead (%) for constructing the three checksum matrices.

From Fig. 15, we can see that, as the size of the global matrices increases, the time for constructing checksum matrices increases only slightly. This is because, in the formula (4), when the size of process grid P is small, $32m^2\beta$ is the dominate factor in the time to constructing checksum matrices.

Fig. 16 indicates that the overhead (%) for constructing checksum matrices decreases as the number of processes increases, which is consistent with our theoretical formula (5)

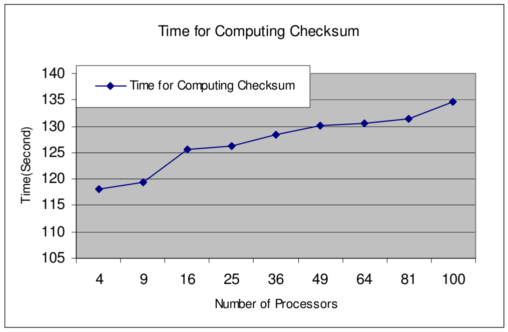


Fig. 15. The overhead (time) for constructing checksum matrices

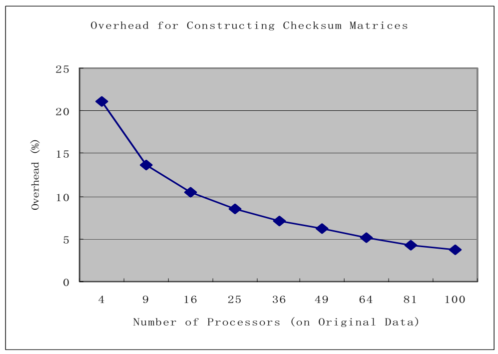


Fig. 16. The overhead (%) for constructing checksum matrices

for the overhead for constructing checksum matrices in Section 4.5.2.

5.2 Overhead for Performing Computations on Encoded Matrices

The algorithm-based checkpoint-free fault tolerance technique involve performing computations with checksum matrices, which introduces some overhead into the fault tolerance scheme. The purpose of this experiment is to evaluate the performance overhead of performing computations with checksum matrices.

Fig. 17 reports the execution time for performing computations on original matrices and the execution time for performing computations on checksum matrices for different size of matrices. Fig. 18 reports the overhead (%) for performing computations with checksum matrices.

Fig. 17 indicates the amount of time increased for performing computations with

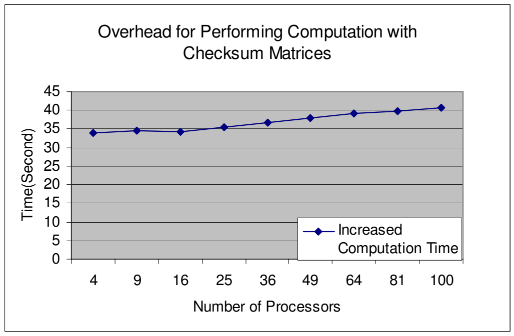


Fig. 17. The overhead (time) for performing computations with encoded matrices

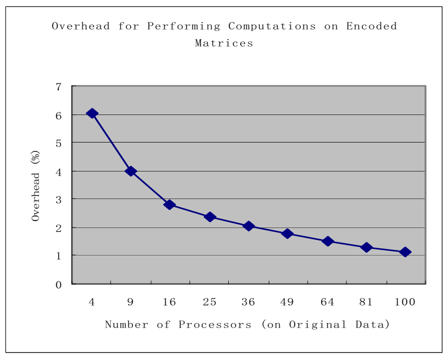


Fig. 18. The overhead (%) for performing computations with encoded matrices

checksum matrices increases slightly as the size of matrices increases. The reason for this increase is that, when perform computations with checksum matrices, column blocks of A^c (and row blocks of B^r) have to be broadcast to one more process. The dominate time for parallel matrix matrix multiplication is the time for computation which is the same for both fault tolerant algorithm and non-fault tolerant algorithm. Therefore, the amount of time increased for fault tolerant algorithm increases only slightly as the size of matrices increases. This experimental results agree with our previous theretically analysis in Section 4.5.3.

Fig. 18 shows that the overhead (%) for performing computations with checksum matrices decreases as the number of processes increases, which is consistent with our previous theoretical results (formula (7) in Section 4.5.3).

5.3 Overhead for Recovering FT-MPI Environment

The overhead for recovering programming environments depends on the specific programming environments. In this section, we evaluate the performance overhead of recovering FT-MPI environment.

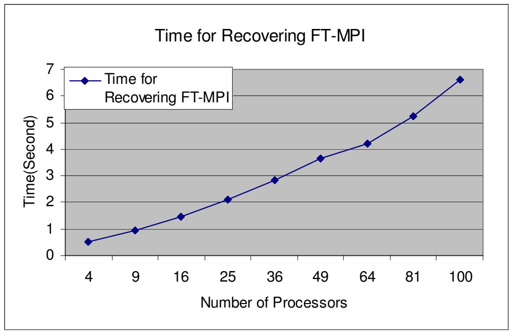


Fig. 19. The overhead (time) for recovering FT-MPI environment

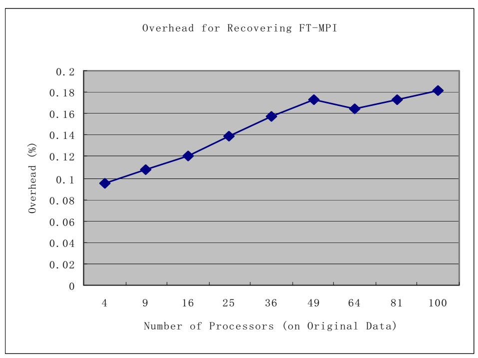


Fig. 20. The overhead (%) for recovering FT-MPI environment

Fig. 19 reports the time for recovering FT-MPI communication environment with single process failure. Fig. 20 reports the overhead (%) for recovering FT-MPI communication environment. Fig. 20 indicates that the overhead for recovering FT-MPI is less than 0.2% which is negligible in practice.

5.4 Overhead for Recovering Application Data

The purpose of this set of experiments is to evaluate the performance overhead of recovering application data from single process failure.

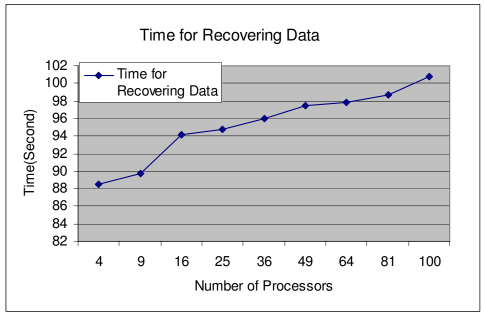


Fig. 21. The overhead (time) for recovering application data

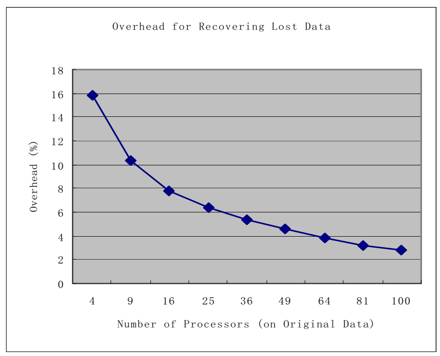


Fig. 22. The overhead (%) for recovering application data

Fig. 21 reports the time for recovering the three checksum matrices A^c, B^r, and C^f in the case of single process failure. Fig. 22 reports the overhead (%) recovering the three checksum matrices A^c, B^r, and C^f.

Fig. 21 indicates that, as the number of processes increases, the time for recovering checksum matrices increases slightly. Fig. 22 indicates that, as the number of processes increases, the overhead for recovering checksum matrices decreases, which confirmed our theoretical analysis in Section 4.5.4.

5.5 Total Overhead for Fault Tolerance

When there is no failure occurs, the total overhead equals to the overhead for calculating encoding at the begining plus the overhead of performing computation with encoded matrices. If there are failures occur, then the total performance overhead equals the overhead without failures plus the overhead for recovering FT-MPI Environment and the overhead for recovering the application data.

Fig. 23 reports the execution times of the original matrix-matrix multiplication, the fault tolerant version matrix-matrix multiplication without failures, and the fault tolerant version matrix-matrix multiplication with a single process failure. Fig. 24 reports the total overhead (%) for the proposed algorithm-based checkpoint-free fault tolerance.

Fig. 24 demonstrates that, as the number of processes increases, the total overhead (%) decreases. This is because, as the number of processors increases, except the overhead for recovering FT-MPI Environment, all other overhead decreases (as indicated in Section 5.1, Section 5.2, and Section 5.3). The overhead for recovering FT-MPI Environment is less than 0.2% which is not the dominant overhead in the total fault tolerant overhead.

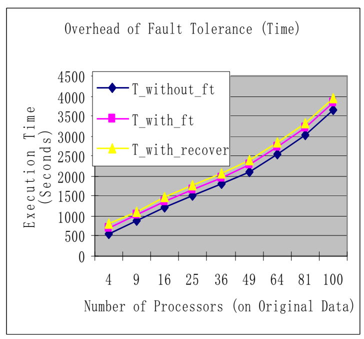


Fig. 23. The total overhead (time) for fault tolerance

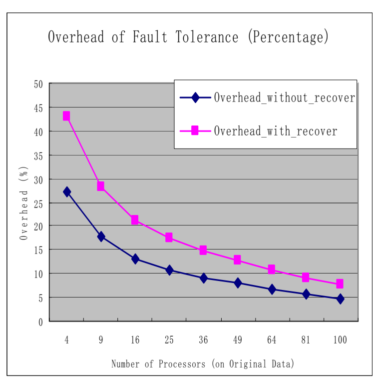


Fig. 24. The total overhead (%) for fault tolerance

6 DISCUSSION

The idea of tolerating failures by modifying applications to operate on encoded data comes from the algorithm-based fault tolerance [19]. While Huang and Abraham proved in [19] that the checksum relationship of the input checksum matrices is preserved in the final computation results at the end of computation, in this paper, we demonstrated that for many matrix matrix multiplication algorithms the checksum relationship in the input checksum matrices does not preserve in the middle of the computation. We further proved that, for the outer product version matrix matrix multiplcation algorithm, it is possible to maintain the checksum relationship in the input checksum matrices in the middle of the computation. Based on our checksum relationship in the middle of the computation, we demonstrate that fail-stop process failures (which are often tolerated by checkpointing or message logging) in ScaLAPACK matrix-matrix multiplcation can be tolerated without checkpointing or message logging.

The algorithm-based checkpoint-free fault tolerance technique presented in this paper involves solving system of linear equations to recover multiple simultaneous process failures. Therefore, the practical numerical issues involved in recovering multiple simultaneous process failures have to be addressed. Techniques proposed in [7], [8], [11] addressed part of this issue.

Compared with the typical checkpoint/restart approaches, the algorithm-based checkpoint-free fault tolerance in this paper can only tolerate partial process failures. It needs the support from programming environments to detect and locate failures. It requires the programming environments to be robust enough to survive node failures without suffering complete system failure. Both the overhead of and the additional effort to maintain a coded global consistent state of the critical application data in algorithm-based checkpoint-free fault tolerance is usually highly dependent on the specific characteristic of the application.

Unlike in typical checkpoint/restart approaches which involve periodical checkpoint, there is no checkpoint involved in this approach. Furthermore, in the algorithm-based checkpoint-free fault tolerance in this paper, whenever process failures occur, it is only necessary to recover the lost data on the failed processes. Therefore, for many applications, it is possible for this approach to achieve a much lower fault tolerant overhead than typical checkpoint/restart approaches. As shown in Section 4 and Section 5, for matrix matrix multiplication, which is one of the most fundamental operations for computational science and engineering, as the size N of the matrix increases, the fault tolerance overhead decreases toward zero with the speed of $\frac{1}{N}$.

7 Conclusion and Future Work

In this paper, we presented a checkpoint-free approach for fault tolerant matrix matrix multiplication in which, instead of taking checkpoint periodically, a coded global consistent state of the critical application data is maintained in memory by modifying applications to operate on encoded data. Because no periodical checkpoint or rollback-recovery is involved in this approach, process failures can often be tolerated with a surprisingly low overhead. We showed the practicality of this technique by applying it to the ScaLAPACK matrix-matrix multiplication kernel which is one of the most important kernels for ScaLAPACK library to achieve high performance and scalability. Experimental results demonstrated that the proposed checkpoint-free approach is able to survive process failures with a very low performance overhead.

There are many directions in which this work could be extended. The first direction is to extend this checkpoint-free approach to more applications. The second direction is to extend this technique to tolerate multiple simutaneous failures. Furthermore, it is also interesting to extend the approach to tolerate failures occured during synchronization and recovery.

Acknowledgment

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REFERENCES

[1] J. Anfinson and F. T. Luk A Linear Algebraic Model of Algorithm-Based Fault Tolerance. *IEEE Transactions on Computers*, v.37 n.12, p.1599-1604, December 1988.

[2] P. Banerjee, J. T. Rahmeh, C. B. Stunkel, V. S. S. Nair, K. Roy, V. Balasubramanian, and J. A. Abraham Algorithm-based fault tolerance on a hypercube multiprocessor. *IEEE Transactions on Computers*, vol. C-39:1132–1145, 1990.

[3] V. Balasubramanian and P. Banerjee Compiler-Assisted Synthesis of Algorithm-Based Checkingin Multiprocessors. *IEEE Transactions on Computers*, vol. C-39:436-446, 1990.

[4] L. S. Blackford, J. Choi, A. Cleary, A. Petitet, R. C. Whaley, J. Demmel, I. Dhillon, K. Stanley, J. Dongarra, S. Hammarling, G. Henry, and D. Walker. ScaLAPACK: a portable linear algebra library for distributed memory computers - design issues and performance. In *Supercomputing '96: Proceedings of the 1996 ACM/IEEE conference on Supercomputing (CDROM)*, page 5, 1996.

[5] D. L. Boley, R. P. Brent, G. H. Golub, and F. T. Luk. Algorithmic fault tolerance using the lanczos method. *SIAM Journal on Matrix Analysis and Applications*, 13:312–332, 1992.

[6] L. E. Cannon. *A cellular computer to implement the kalman filter algorithm*. Ph.D. thesis, Montana State University, Bozeman, MT, USA, 1969.

[7] Z. Chen and J. Dongarra. Numerically stable real number codes based on random matrices. In *Proceeding of the 5th International Conference on Computational Science (ICCS2005)*, Atlanta, Georgia, USA, May 22-25, 2005. LNCS 3514, Springer-Verlag.

[8] Z. Chen and J. Dongarra. Condition Numbers of Gaussian Random Matrices. *SIAM Journal on Matrix Analysis and Applications*, Volume 27, Number 3, Page 603-620, 2005.

[9] Z. Chen, G. E. Fagg, E. Gabriel, J. Langou, T. Angskun, G. Bosilca, and J. Dongarra. Fault tolerant high performance computing by a coding approach. In *Proceedings of the ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming, PPOPP 2005, June 14-17, 2005, Chicago, IL, USA*. ACM, 2005.

[10] Z. Chen, G. E. Fagg, E. Gabriel, J. Langou, T. Angskun, G. Bosilca, and J. Dongarra. Building Fault Survivable MPI Programs with FT-MPI Using Diskless Checkpointing. In *University of Tennessee Computer Science Department Technical Report*. Technical Report UT-CS-04-540, 2004.

[11] Z. Chen. *Scalable techniques for fault tolerant high performance computing*. Ph.D. thesis, University of Tennessee, Knoxville, TN, USA, 2006.

[12] G. E. Fagg, E. Gabriel, G. Bosilca, T. Angskun, Z. Chen, J. Pjesivac-Grbovic, K. London, and J. J. Dongarra. Extending the MPI specification for process fault tolerance on high performance computing systems. In *Proceedings of the International Supercomputer Conference, Heidelberg, Germany*, 2004.

[13] G. E. Fagg, E. Gabriel, Z. Chen, , T. Angskun, G. Bosilca, J. Pjesivac-Grbovic, and J. J. Dongarra. Process fault-tolerance: Semantics, design and applications for high performance computing. *Submitted to International Journal of High Performance Computing Applications*, 2004.

[14] I. Foster and C. Kesselman. *The Grid: Blueprint for a New Computing Infrastructure*. Morgan Kauffman, San Francisco, 1999.

[15] I. Foster and C. Kesselman. The GLOBUS toolkit. *The grid: blueprint for a new computing infrastructure*, pages 259–278, 1999.

[16] G. C. Fox, M. Johnson, G. Lyzenga, S. W. Otto, J. Salmon, and D. Walker. *Solving Problems on Concurrent Processors: Volum 1*. Prentice-Hall, Englewood Cliffs, NJ, 1988.

[17] E. Gabriel, G. E. Fagg, G. Bosilca, T. Angskun, J. Dongarra, J. M. Squyres, V. Sahay, P. Kambadur, B. Barrett, A. Lumsdaine, R. H. Castain, D. J. Daniel, R. L. Graham, and T. S. Woodall. Open MPI: Goals, concept, and design of a next generation MPI implementation. In *PVM/MPI*, pages 97–104, 2004.

[18] G. H. Golub and C. F. Van Loan. *Matrix Computations*. The John Hopkins University Press, , 1989.

[19] K.-H. Huang and J. A. Abraham. Algorithm-based fault tolerance for matrix operations. *IEEE Transactions on Computers*, vol. C-33:518–528, 1984.

[20] Y. Kim. *Fault Tolerant Matrix Operations for Parallel and Distributed Systems.* Ph.D. dissertation, University of Tennessee, Knoxville, June

[21] F. T. Luk and H. Park An analysis of algorithm-based fault tolerance techniques. *SPIE Adv. Alg. and Arch. for Signal Proc.*, vol. 696, 1986, pp. 222-228.

[22] J. S. Plank, Y. Kim, and J. Dongarra. Fault Tolerant Matrix Operations for Networks of Workstations Using Diskless Checkpointing. *IEEE Journal of Parallel and Distributed Computing*, 43, 125-138 (1997).

[23] J. S. Plank, K. Li, and M. A. Puening. Diskless checkpointing. *IEEE Trans. Parallel Distrib. Syst.*, 9(10):972–986, 1998.

[24] J. S. Plank. A tutorial on Reed-Solomon coding for fault-tolerance in RAID-like systems. *Software – Practice & Experience*, 27(9):995–1012, September 1997.

[25] P. Sanders and J. F. Sibeyn. A bandwidth latency tradeoff for broadcast and reduction. *Inf. Process. Lett.*, 86(1):33–38, 2003.

[26] M. Snir, S. Otto, S. Huss-Lederman, D. W. Walker, and J. Dongarra. *MPI: The Complete Reference*. Volume 1, The MIT Press, 2nd edition, 1998.

[27] V. S. Sunderam. PVM: a framework for parallel distributed computing. *Concurrency: Pract. Exper.*, 2(4):315–339, 1990.

[28] C. Wang, F. Mueller, C. Engelmann, and S. Scot. Job Pause Service under LAM/MPI+BLCR for Transparent Fault Tolerance. In *Proceedings of the 21st IEEE International Parallel and Distributed Processing Symposium, March, 2007, Long Beach, CA, USA*.