3.3 Random Numbers: Exponential, Rayleigh, and Poisson

A. Purpose
Generate pseudorandom numbers from the exponential and Rayleigh distributions and pseudorandom integers from the Poisson distribution.

B. Usage

B.1 Generating exponential pseudorandom numbers
The density function for the exponential distribution with mean and standard deviation, \( \mu \), has the value zero for \( x < 0 \) and \( \mu^{-1} \exp(-x/\mu) \) for \( x \geq 0 \). The cumulative distribution function has the value zero for \( x < 0 \) and \( 1 - \exp(-x/\mu) \) for \( x \geq 0 \). If \( u \) is a random variable having the uniform distribution on \([0, 1]\) then \( x = -\mu \log u \) is a random variable having the exponential distribution with mean and standard deviation, \( \mu \).

B.1.a Program Prototype, Single Precision

REAL SRANE, XMEAN, X
Assign a value to XMEAN.

\[ X = \text{SRANE}(\text{XMEAN}) \]

B.1.b Argument Definitions
XMEAN [in] Specifies the mean and standard deviation of the desired exponential distribution. Require XMEAN > 0.
SRANE [out] The function returns a nonnegative pseudorandom number from the exponential distribution with mean and standard deviation equal to XMEAN.

B.2 Generating Rayleigh pseudorandom numbers
The density function for the Rayleigh distribution with scaling parameter, \( \alpha \), has the value zero for \( x < 0 \) and \( (x/\alpha^2) \exp(-x^2/2\alpha^2) \) for \( x \geq 0 \). The cumulative distribution function has the value zero for \( x < 0 \) and \( 1 - \exp(-x^2/2\alpha^2) \) for \( x \geq 0 \). The mean and standard deviation of this distribution are

\[
\mu = \alpha \sqrt{\pi/2} \approx 1.25331 \alpha
\]

and

\[
\sigma = \alpha \sqrt{2 - \pi/2} \approx 0.65516 \alpha
\]

If \( u \) is a random variable having the uniform distribution on \([0, 1]\) then \( x = \alpha \sqrt{-2 \log u} \) is a random variable having the Rayleigh distribution with scaling parameter, \( \alpha \).

B.2.a Program Prototype, Single Precision

REAL SRANR, ALPHA, X
Assign a value to ALPHA.

\[ X = \text{SRANR}(\text{ALPHA}) \]

B.2.b Argument Definitions
ALPHA [in] Specifies the scaling of the desired Rayleigh distribution. Require ALPHA > 0. The distribution will have mean = \( \alpha \sqrt{\pi/2} \) and variance = \( \alpha^2 \times (2 - \pi/2) \).
SRANR [out] The function returns a nonnegative pseudorandom number from the Rayleigh distribution with scaling parameter, ALPHA.

B.3 Generating Poisson pseudorandom integers
The Poisson distribution with mean and variance, \( \mu \), is defined over nonnegative integers. The nonnegative integer, \( k \), occurs with probability \( p_k \) given by

\[
p_k = e^{-\mu} \frac{\mu^k}{k!}
\]

B.3.a Program Prototype, Single Precision

REAL XMEAN
INTEGER ISRANP, K
Assign a value to XMEAN.

\[ K = \text{ISRANP}(\text{XMEAN}) \]

B.3.b Argument Definitions
XMEAN [in] Specifies the mean and variance of the desired Poisson distribution. XMEAN must be positive and not so large that \( \exp(-\text{XMEAN}) \) would underflow. For example if the underflow limit is \( 10^{-38} \), XMEAN must not exceed 87. This subprogram requires more computing time for larger values of XMEAN or if XMEAN is changed frequently. See Section D.

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The function returns a nonnegative pseudorandom integer from the Poisson distribution with mean XMEAN.

**B.4 Modifications for Double Precision**

Change the names SRANE, SRANR, and ISRANP to DRANE, DRANR, and IDRANP respectively, and change the REAL type statements above to DOUBLE PRECISION. Note particularly that if either of the function names DRANE or DRANR is used it must be typed DOUBLE PRECISION either explicitly or via an IMPLICIT statement.

**C. Examples and Remarks**

The programs DRSRANE, DRISRANP, and DRSRANR demonstrate, respectively, the use of SRANE, ISRANP, and SRANR. These programs use SSTAT1 and SSTAT2, or ISSTA1 and ISSTA2 to compute and print statistics and a histogram based on a sample of 10000 numbers each.

To fetch or set the seed used in the underlying pseudorandom integer sequence use the subroutines described in Chapter 3.1.

**D. Functional Description**

**Method**

The exponential random number is computed as \( x = -\mu \log u \) where \( u \) is a random number from the uniform distribution on \([0, 1]\).

The Rayleigh random number is computed as \( x = \alpha \sqrt{-2\log u} \) where \( u \) is a random number from the uniform distribution on \([0, 1]\).

The Poisson subprogram uses ideas from \([1]\). The method begins by obtaining a random number, \( u \), from the uniform distribution on \([0, 1]\). Then the probabilities \( p_0, p_1, \ldots \), defined above in Section B.3, are summed until the sum reaches or exceeds \( u \). The index of the last term in the sum is then returned as the Poisson random integer.

To improve efficiency on the assumption that the subprogram may be referenced many successive times with the value of XMEAN remaining unchanged, the partial sums through at most the term \( p_{84} \) are stored in an internal array as they are computed. On subsequent references, if the value of XMEAN has not been changed, previously computed partial sums can be tested without the need to recompute them. The testing starts at the index nearest to the value, XMEAN, since these indices have the highest probabilities of being selected.

These subprograms obtain uniform pseudorandom numbers by calling SRANUA or DRANUA, using the array in common block /RANCMS/ or /RANCMD/ as a buffer as described in Chapter 3.1.

Values returned as double-precision random numbers will have random bits throughout the word, however the quality of randomness should not be expected to be as good in a low-order segment of the word as in a high-order part.

**References**


**E. Error Procedures and Restrictions**

In subprograms SRANE, DRANE, SRANR, and DRANR the input parameter should be positive, however no test is made of this. The input parameter is simply used as a multiplicative factor.

Subprogram ISRANP will issue an error message and return the value \(-1\) if XMEAN \(\leq 0\) or if XMEAN \(\geq -0.5 \times \log (\text{underflow limit})\).

**F. Supporting Information**

**Entry Required Files**

<table>
<thead>
<tr>
<th>Subprogram</th>
<th>Required Files</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRANE</td>
<td>DRANE, ERFIN, ERMSG, RANPK1, RANPK2</td>
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<td>DRANR</td>
<td>DRANR, ERFIN, ERMSG, RANPK1, RANPK2</td>
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<td>IDRANP</td>
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<tr>
<td>SRANR</td>
<td>ERFIN, ERMSG, RANPK1, RANPK2, SRANR</td>
</tr>
</tbody>
</table>

DRSRANE

program DRSRANE
\begin{verbatim}
2001-05-22 DRSRANE  Krogh Minor change for making .f90 version.
1994-10-19 DRSRANE  Krogh Changes to use M77CON
1987-12-09 DRSRANE  Lawson Initial Code.
\end{verbatim}

Driver to demonstrate use of SRANE to generate random numbers
from the exponential distribution with standard deviation, STDDEV.
Program computes histogram for N numbers

integer  I, NCELLS
parameter(NCELLS = 12+2)
integer  IHIST(NCELLS), N
external SRANE
real  SRANE, STATS(5), STDDEV, Y1, Y2, YTAB(1), ZERO

parameter(ZERO = 0.0E0)
data  N / 10000 /
data  Y1, Y2 / 0.0E0, 6.0E0 /
data  STDDEV / 1.0E0 /

STATS(1) = ZERO
\begin{verbatim}
do 20 1 = 1, N

  YTAB(1) = SRANE(STDDEV)
  Accumulate statistics and histogram.

  call SSTAT1(YTAB(1), 1, STATS, IHIST, NCELLS, Y1, Y2)
\end{verbatim}

continue

print  ' (11x,a)', 'Exponential random numbers from SRANE,'
print  ' (11x,a,g12.4,/1x)', 'with STDDEV = ', STDDEV

Print the statistics and histogram.

\begin{verbatim}
call SSTAT2(STATS, IHIST, NCELLS, Y1, Y2)
stop
end
\end{verbatim}
## ODSRANE

Exponential random numbers from SRANE with STDDEV = 1.000

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<th>PLOT OF COUNT</th>
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<td>1.00</td>
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Count | Minimum | Maximum | Mean  | Std. Deviation |
<table>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
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<td>8.1793</td>
<td>1.0001</td>
<td>0.98840</td>
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</table>

3.3–4 Random Numbers: Exponential, Rayleigh, and Poisson

July 11, 2015
DRSRANR

program DRSRANR

> 2001-05-22 DRSRANR Krogh Minor change for making .f90 version.
> 1994-10-19 DRSRANR Krogh Changes to use M77CON
> 1987-12-09 DRSRANR Lawson Initial Code.
—S replaces "?": DR?RANR, ?RANR, ?STAT1, ?STAT2

c Driver to demonstrate use of SRANR to generate random numbers
from the Rayleigh distribution with parameter, ALPHA.
Program computes histogram for N numbers

c integer NCELLS
parameter(NCELLS = 13+2)
external SRANR
real ALPHA, SRANR, ONE, PIOV2
real STATS(5), TWO, Y1, Y2, YTAB(1), ZERO
integer I, IHIST(NCELLS), N
c
parameter(ONE = 1.0E0, TWO = 2.0E0, ZERO = 0.0E0)
data N / 10000/
data Y1, Y2 / 0.0E0, 4.33333E0/
data ALPHA / 1.0E0 /

PIOV2 = TWO * atan(ONE)
STATS(1) = ZERO
do 20 I =1,N
Get random number
YTAB(1) = SRANR(ALPHA)
Accumulate statistics and histogram.
call SSTAT1(YTAB(1), 1, STATS, IHIST, NCELLS, Y1, Y2)
continue
Print the statistics and histogram.
print '(13x,a)', 'Rayleigh random numbers from SRANR'
print '(13x,a,g12.4)', 'with ALPHA = ',ALPHA
print '(1x,a/13x,g13.5,a,g13.5/1x)',
* 'The Mean and Std. Dev. of the theoretical distribution are',
* ALPHA * sqrt(PIOV2), ' and ',ALPHA * sqrt(TWO - PIOV2)
call SSTAT2(STATS, IHIST, NCELLS, Y1, Y2)
stop
end
The mean and standard deviation of the theoretical distribution are 1.2533 and 0.65514.

<table>
<thead>
<tr>
<th>Break Point</th>
<th>Count</th>
<th>Plot of Count</th>
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<td>*</td>
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<tr>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Count Minimum Maximum Mean Std. Deviation
10000 0.13970E-01 4.0446 1.2537 0.65437
DRISRANP

Driver to demonstrate use of ISRANP to generate random integers from the Poisson distribution with mean, XMEAN.
Program computes histogram for N samples.


integer NCELLS, NI
parameter(NCELLS = 7+2, NI = 100)
external ISRANP
integer I, IHIST(NCELLS), ISRANP, ISTATS(3), ITAB(NI), ILOW, J, N
real XMEAN, XSTATS(2)

data N / 10000 /
data ILOW / 0 /
data XMEAN / 1.0e0 /

ISTATS(1) = 0
do 30 J = 1, N/NI
do 20 I = 1, NI
   ITAB(I) = ISRANP(XMEAN)
20   continue
   call ISSTA1(ITAB, NI, ISTATS, XSTATS, IHIST, ILOW, NCELLS)
30   continue
   call ISSTA2(ISTATS, XSTATS, IHIST, ILOW, NCELLS)
end

ODISRANP

Poisson random integers from ISRANP
with XMEAN = 1.000

<table>
<thead>
<tr>
<th>VALUE</th>
<th>COUNT</th>
<th>PLOT OF COUNT</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>634</td>
<td></td>
</tr>
<tr>
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<td>151</td>
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</tr>
<tr>
<td>5</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Count Minimum Maximum Mean Std. Deviation
10000  0   6  1.0018  1.0099

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