Basic Linear Algebra Subprograms Technical (BLAST) Forum
Standard

Basic Linear Algebra Subprograms Technical (BLAST) Forum

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Suggestions for Reading

This document is divided into chapters, appendices, a journal of development, and an index of routine names. It is large, and it is not necessary for a user to read it in its entirety. A user may choose to not read certain chapters or sections within this document, depending upon his/her areas of interest. Chapters 2–4 contain a functionality discussion and language bindings for dense and band, sparse, and mixed and extended precision BLAS, respectively. Thus, these chapters may be read independently, referring to Chapter 1 and the Appendix for notation and implementation details common to all chapters. Refer to section 1.3 for a more detailed description of the organization of this document.
Chapter 1

Introduction

1.1 Introduction

This document defines the BLAS Technical Forum standard, a specification of a set of kernel
routines for linear algebra, historically called the Basic Linear Algebra Subprograms and commonly
known as the BLAS. In addition to this publication, the complete standard can be found on the
BLAS Technical Forum webpage (http://www.netlib.org/blas/blast-forum/).

Numerical linear algebra, particularly the solution of linear systems of equations, linear least
squares problems, eigenvalue problems and singular value problems, is fundamental to most calcula-
tions in scientific computing, and is often the computationally intense part of such calculations.
Designers of computer programs involving linear algebraic operations have frequently chosen to
implement certain low level operations, such as the dot product or the matrix vector product, as
separate subprograms. This may be observed both in many published codes and in codes written
for specific applications at many computer installations.

This approach encourages structured programming and improves the self-documenting quality
of the software by specifying basic building blocks and identifying these operations with unique
mnemonic names. Since a significant amount of execution time in complicated linear algebraic
programs may be spent in a few low level operations, reducing the execution time spent in these
operations leads to an overall reduction in the execution time of the program. The programming
of some of these low level operations involves algorithmic and implementation subtleties that need
care, and can be easily overlooked. If there is general agreement on standard names and parameter
lists for some of these basic operations, then portability and efficiency can also be achieved.

The first major concerted effort to achieve agreement on the specification of a set of linear algebra
kernels resulted in the Level 1 Basic Linear Algebra Subprograms (BLAS)\(^1\) [39] and associated test
suite. The Level 1 BLAS are the specification and implementation in Fortran of subprograms for
scalar and vector operations. This was the result of a collaborative project in 1973-77. Following
the distribution of the initial version of the specifications to people active in the development of
numerical linear algebra software, a series of open meetings were held at conferences and, as a result,
extensive modifications were made in an effort to improve the design and make the subprograms
more robust. The Level 1 BLAS were extensively and successfully exploited by LINPACK [23],
a software package for the solution of dense and banded linear equations and linear least squares
problems.

With the advent of vector machines, hierarchical memory machines and shared memory parallel
machines, specifications for the Level 2 and 3 BLAS [26, 25], concerned with matrix-vector and
\(^1\)Originally known just as the BLAS, but in the light of subsequent developments now known as the Level 1 BLAS
matrix-matrix operations respectively, were drawn up in 1984-86 and 1987-88. These specifications made it possible to construct new software to utilize the memory hierarchy of modern computers more effectively. In particular, the Level 3 BLAS allowed the construction of software based upon block-partitioned algorithms, typified by the linear algebra software package LAPACK [6]. LAPACK is state-of-the-art software for the solution of dense and banded linear equations, linear least squares, eigenvalue and singular value problems, makes extensive use of all levels of BLAS and particularly utilizes the Level 2 and 3 BLAS for portable performance. LAPACK is widely used in application software and is supported by a number of hardware and software vendors.

To a great extent, the user community embraced the BLAS, not only for performance reasons, but also because developing software around a core of common routines like the BLAS is good software engineering practice. Highly efficient machine-specific implementations of the BLAS are available for most modern high-performance computers. The BLAS have enabled software to achieve high performance with portable code.

The original BLAS concentrated on dense and banded operations, but many applications require the solution of problems involving sparse matrices, and there have also been efforts to specify computational kernels for sparse vector and matrix operations [22, 27].

In the spirit of the earlier BLAS meetings and the standardization efforts of the MPI and HPF forums, a technical forum was established to consider expanding the BLAS in the light of modern software, language, and hardware developments. The BLAS Technical Forum meetings began with a workshop in November 1995 at the University of Tennessee. Meetings were hosted by universities, government institutions, and software and hardware vendors. Detailed minutes were taken for each of the meetings, and these minutes are available on the BLAS Technical Forum webpage (http://www.netlib.org/blas/blast-forum/).

Various working groups within the Technical Forum were established to consider issues such as the overall functionality, language interfaces, sparse BLAS, distributed-memory dense BLAS, extended and mixed precision BLAS, interval BLAS, and extensions to the existing BLAS. The rules of the forum were adopted from those used for the MPI and HPF forums. In other words, final acceptance of each of the chapters in the BLAS Technical Forum standard were decided at the meetings using Robert's Rules. Drafts of the document were also available on the BLAS Technical Forum webpage, and attendees were permitted to edit chapters, give comments, and vote on-line in “virtual meetings”, as well as to conduct discussions on the email reflector. The efforts of these working groups are summarized in this document. Most of these discussions resulted in definitive proposals which led to the specifications given in Chapters 2 - 4. Not all of the discussions resulted in definitive proposals, and such discussions are summarized in the Journal of Development in the hope that they may encourage future efforts to take those discussions to a successful conclusion.

A major aim of the standards defined in this document is to enable linear algebra libraries (both public domain and commercial) to interoperate efficiently, reliably and easily. We believe that hardware and software vendors, higher level library writers and application programmers all benefit from the efforts of this forum and are the intended end users of these standards.

The specification of the original BLAS was given in the form of Fortran 66 and subsequently Fortran 77 subprograms. In this document we provide specifications for Fortran 95, Fortran 77 and C. Reference implementations of the standard are provided on the BLAS Technical Forum webpage (http://www.netlib.org/blas/blast-forum/). Alternative language bindings for C++ and Java were also discussed during the meetings of the forum, but the specifications for these bindings were postponed for a future series of meetings.

The remainder of this chapter is organized as follows. Section 1.2 provides motivation for the

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*the current Fortran standard*
1.2. MOTIVATION

functionality. Section 1.3 outlines the organization of the document, and section 1.4 summarizes the nomenclature and conventions used in the document. Section 1.5 presents tables of functionality for the routines, and section 1.6 discusses issues concerning the numerical accuracy of the BLAS. Section 1.7 briefly describes the presentation of the specifications for the routines, and section 1.8 details the error handling mechanisms utilized within the routines.

1.2 Motivation

The motivation for the kernel operations is proven functionality. Many of the new operations are based upon auxiliary routines in LAPACK [6] (e.g., SUMSQ, GEN_GROT, GEN_HOUSE, SORT, GE_NORM, GE_COPY). Only after the LAPACK project was begun was it realized that there were operations like the matrix copy routine (GE_COPY), the computation of a norm of a matrix (GE_NORM) and the generation of Householder transformations (GEN_HOUSE) that occurred so often that it was wise to make separate routines for them.

A second group of these operations extended the functionality of some of the existing BLAS (e.g., AXPBY, WAXPBY, GER, SYR/HER, SPR/HPR, SYR2/HER2, SPR2/HPR2). For example, the Level 3 BLAS for the rank $k$ update of a symmetric matrix only allows a positive update, which means that it cannot be used for the reduction of a symmetric matrix to tridiagonal form (to facilitate the computation of the eigensystem of a symmetric matrix), or for the factorization of a symmetric indefinite matrix, or for a quasi-Newton update in an optimization routine.

Other extensions (e.g., AXPY_DOT, GE_SUM_MV, GEMVT, TRMV, GEMVER) perform two Level 1 BLAS (or Level 2 BLAS) routine calls simultaneously to increase performance by reducing memory traffic.

One important feature of the new standard is the inclusion of sparse matrix computational routines. Because there are many formats commonly used to represent sparse matrices, the Level 2 and Level 3 Sparse BLAS routines utilize an abstract representation, or handle, rather than a fixed storage description (e.g., compressed row, or skyline storage). This handle-based representation allows one to write portable numerical algorithms using the Sparse BLAS, independent of the matrix storage implementation, and gives BLAS library developers the best opportunity for optimizing and fine-tuning their kernels for specific architectures or application domains.

The original Level 2 BLAS included, as an appendix, the specification of extended precision subprograms. With the widespread adoption of hardware supporting the IEEE extended arithmetic format [37], as well as other forms of extended precision arithmetic, together with the increased understanding of algorithms to successfully exploit such arithmetic, it was felt to be timely to include a complete specification for a set of extra precise BLAS.

1.3 Organization of the Document

This document is divided into chapters, appendices, a journal of development, and an index. It is large, and it is not necessary for a user to read it in its entirety. A user may choose to not read certain chapters or sections within this document, depending upon his/her areas of interest. Chapters 2–4 contain a functionality discussion and language bindings for dense and band, sparse, and mixed and extended precision BLAS, respectively. The Journal of Development presents areas of research that are not yet mature enough to be considered as chapters, but were nevertheless discussed at the meetings of the forum. A Bibliography is also provided, as well as an Index of routine names.
CHAPTER 1. INTRODUCTION

All users are encouraged to frequently refer to the list of notation denoted in sections 1.4, 2.3, and 3.4.

- **Chapter 1: Introduction** provides a brief overview of the background, motivation, and history of the BLAS Technical Forum effort. It also outlines the structure of the document, conventions in notation, and overall functionality contained in the chapters.

- **Chapter 2: Dense and Banded BLAS** presents the functionality and language bindings for proposed “new” dense and banded BLAS routines for serial and shared memory computing.

- **Chapter 3: Sparse BLAS** presents the functionality and language bindings for proposed “new” sparse BLAS routines for serial and shared memory computing.

- **Chapter 4: Extended and Mixed Precision BLAS** presents the functionality and language bindings for proposed extended-precision and mixed-precision BLAS routines for serial and shared memory computing.

- **Appendix** contains pertinent definitions and implementation details for the chapters.

- **Legacy BLAS** contains alternative language bindings for the legacy Level 1, 2, and 3 BLAS for dense and band matrix computations.

- **Journal of Development** contains separate proposals for environmental query routines, distributed-memory dense BLAS, Fortran 95 Thin BLAS, and Interval BLAS.

1.4 Nomenclature and Conventions

This section addresses mathematical notation and definitions, as well as the numerical accuracy for the BLAS routines. Language-independent issues are also presented.

1.4.1 Notation

The following notation is used throughout the document.

- $A$, $B$, $C$ – matrices
- $D$, $D_L$, $D_R$ – diagonal matrices
- $H$ – Householder matrix
- $J$ – symmetric tridiagonal matrix (including $2 \times 2$ blocked diagonal)
- $P$ – permutation matrix
- $T$ – triangular matrix
- $\text{op}(A)$ – denotes $A$, or $A^T$ or $A^H$ where $A$ is a matrix.
- transpose – denotes $A^T$ where $A$ is a matrix.
- conjugate-transpose – denotes $A^H$ where $A$ is a complex Hermitian matrix.
1.4. NOMENCLATURE AND CONVENTIONS

• $u$, $v$, $w$, $x$, $y$, $z$ – vectors
• $\bar{x}$ – specifies the conjugate of the complex vector $x$
• $incu$, $incv$, $inew$, $incx$, $incy$, $incz$ – stride between successive elements of the respective vector
• Greek letters - scalars (but not exclusively Greek letters)
• $x_i$ - an element of a one-dimensional array
• $y|_{x}$ – refers to the elements of $y$ that have common indices with the sparse vector $x$.
• $\epsilon$ - machine epsilon
• $\leftarrow$ – assignment statement
• $\leftrightarrow$ – swap (assignment) statement
• $|| \cdot ||_p$ – the p-norm of a vector or matrix

Additional notation for sparse matrices can be found in 3.4.

For the mathematical formulation of the operations, as well as their algorithmic presentation, we have chosen to index the vector and matrix operands starting from zero. This decision was taken to simplify the presentation of the document but has no impact on the convention a particular language binding may choose.

1.4.2 Operator Arguments

Some BLAS routines take input-only arguments that are called “operator” arguments. These arguments allow for the specification of multiple related operations to be performed by a single function.

The operator arguments used in this document are $\text{norm}$, $\text{sort}$, $\text{side}$, $\text{uplo}$, $\text{trans}$, $\text{conj}$, $\text{diag}$, $\text{jrot}$, $\text{order}$, $\text{index\_base}$, and $\text{prec}$. Their possible meanings are defined as follows:

$\text{norm}$: this argument is used by the routines computing the norm of a vector or matrix. Eight possible distinct values are valid that specify the norm to be computed, namely the one-norm, real one-norm, infinity-norm and real infinity norms for vectors and matrices, the 2-norm for vectors, and the Frobenius-norm, max-norm and real max-norm for matrices.

$\text{sort}$: this argument is used by the sorting routines. Two possible distinct values are valid that specify whether the data should be sorted in increasing or decreasing order.

$\text{side}$: this argument is used only by functions computing the product of two matrices $A$ and $B$. Two possible distinct values are valid, that specify whether $A \cdot B$ or $B \cdot A$ should be computed.

$\text{uplo}$: this argument refers to triangular and symmetric (Hermitian) matrices. Two possible distinct values are valid distinguishing whether the matrix, or its storage representation, is upper or lower triangular.

$\text{trans}$: this argument is used by the routines applying a matrix, say $A$, to another vector or another matrix. Three possible distinct values are valid that specify whether the matrix $A$, its transpose $A^T$ or its conjugate transpose $A^H$ should be applied. We use the notation $\text{op}(A)$ to refer to $A$, $A^T$ or $A^H$ depending on the input value of the $\text{trans}$ operator argument.
conj: this argument is used by the complex routines operating with \( \bar{x} \) or \( x \).

diag: this argument refers exclusively to triangular matrices. Two possible distinct values are valid distinguishing whether the triangular matrix has unit-diagonal or not.

jrot: this argument is used by the routine to generate Jacobi rotations. Three possible distinct values are valid and specify whether the rotation is an inner rotation, an outer rotation, or a sorted rotation.

order: this argument is used by the C bindings to specify if elements within a row of an array are contiguous, or if elements within a column of an array are contiguous (see section 2.6.6).

index_base: this argument is used by Chapter 3 to specify either one-based or zero-based indexing (see section 3.4.1).

prec: this argument is used in Chapter 4 and specifies the internal precision to be used by an extended precision routine. Four distinct values are valid and specify whether the internal precision is single precision, double precision, indigenous, or extra. Details on these settings can be found in section 4.3.1.

All possible meanings for each operator are listed in section A.3. Their representation is defined in the interface issues for the specific programming language: sections 2.4, 3.6.1, and 4.4.1 for Fortran 95; sections 2.5, 3.6.2, and 4.4.2 for Fortran 77; and sections 2.6, 3.6.3, and 4.4.3 for C. The values of the Fortran 95 derived types (for Chapters 2 and 4) are defined in the Fortran 95 module blas_operator_arguments, and the values of the Fortran 95 named constants (for Chapter 3) are defined in blas_sparse_namedconstants, see section A.4. Similarly, the values of the Fortran 77 named constants are defined in the Fortran 77 include file blas_namedconstants.h, in section A.5. And finally, the values of the C enumerated types are defined in the C include file blas_enum.h, in section A.6.

Rationale. The intent is to provide each language binding with the opportunity to choose the most appropriate form these arguments should take. For example, in Fortran 95, derived types with named constants have been selected for Chapters 2 and 4, whereas derived types could not be used in Chapter 3 (see section 3.6.1 for details). In Fortran 77, integers with named constants have been chosen. And finally, in C, operator arguments are represented by enumerated types. (End of rationale.)

1.4.3 Scalar Arguments

Many scalar arguments are used in the specifications of the BLAS routines. For example, the size of a vector or matrix operand is determined by the integer argument(s) \( m \) and/or \( n \). Note that it is permissible to call the routines with \( m \) or \( n \) equal to zero, in which case the routine exits immediately without referencing its vector/matrix elements. Some routines return a displacement denoted by the integer argument \( k \). The scaling of a vector or matrix is often denoted by the arguments alpha and beta.

The following symbols are used: \( a, b, c, d, r, s, t, \alpha, \beta \) and \( \tau \).

1.4.4 Vector Operands

A \( n \)-length vector operand \( x \) is specified by two arguments – \( x \) and \( \text{incx} \). \( x \) is an array that contains the entries of the \( n \)-length vector \( x \). \( \text{incx} \) is the stride within \( x \) between two successive elements of the vector \( x \).
The following lowercase letters are used to denote a vector: u, v, w, x, y, and z. The corresponding strides are respectively denoted incu, incv, incw, incx, incy, and incz.

Advice to implementors. The increment arguments incu, incv, incw, incx, incy and incz may not be zero. (End of advice to implementors.)

Example: The mathematical function returning the inner-product \( r \) of two real \( n \)-length vectors \( x \) and \( y \) can be defined by:

\[
    r = x^T y = \sum_{i=0}^{n-1} x_i y_i.
\]

Rationale. The arguments incx, and incy do not play a role in the mathematical formulation of the operation. These arguments allow for the specification of subvector operands in various language bindings. Therefore, some of these arguments may not be present in all language-dependent specifications. (End of rationale.)

1.4.5 Matrix Operands

A \( m \)-by-\( n \) matrix operand \( A \) is specified by the argument \( A \). \( A \) is a language-dependent data structure containing the entries of the matrix operand \( A \). The representation of the matrix entry \( a_{i,j} \) in \( A \) is denoted by \( A(i,j) \) for all \((i,j)\) in the interval \([0 \ldots m - 1] \times [0 \ldots n - 1] \).

Capital letters are used to denote a matrix. The functions involving matrices use only four symbols, namely \( A, B, C, \) and \( T \).

1.4.6 Naming Conventions

Language bindings are specified for Fortran 95, Fortran 77, and C.

The Fortran 95 language bindings have routine names of the form \(<\text{name}>\), where \(<\text{name}>\) is in lowercase letters and indicates the computation performed. These bindings use generic interfaces to manipulate the data type of the routine, and thus their names do not contain a letter to denote the data type.

The Fortran 77 and C language bindings have routine names of the form \( \text{BLAS}_x<\text{name}>\), where the letter \( x \), indicates the data type as follows:

<table>
<thead>
<tr>
<th>Data type</th>
<th>( x )</th>
<th>Fortran 77</th>
<th>( x )</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>s.p. real</td>
<td>S</td>
<td>REAL</td>
<td>s</td>
<td>float</td>
</tr>
<tr>
<td>d.p. real</td>
<td>D</td>
<td>DOUBLE PRECISION</td>
<td>d</td>
<td>double</td>
</tr>
<tr>
<td>s.p. complex</td>
<td>C</td>
<td>COMPLEX</td>
<td>c</td>
<td>float</td>
</tr>
<tr>
<td>d.p. complex</td>
<td>Z</td>
<td>COMPLEX*16 or DOUBLE COMPLEX</td>
<td>z</td>
<td>double</td>
</tr>
</tbody>
</table>

The suffix \(<\text{name}>\) in the routine name indicates the computation performed. In the matrix-vector and matrix-matrix routines of Chapters 2 and 4 (and Appendix C.4), the type of the matrix (or of the most significant matrix) is also specified as part of this \(<\text{name}>\) name of the routine. Most of these matrix types apply to both real and complex matrices; a few apply specifically to one or the other, as indicated below. Note that for Appendix C.4, these matrix types apply to interval matrices.
GB general band
GE general (i.e., unsymmetric, in some cases rectangular)
HB (complex) Hermitian band
HE (complex) Hermitian
HP (complex) Hermitian, packed storage
SB (real) symmetric band
SP symmetric, packed storage
SY symmetric
TB triangular band
TP triangular, packed storage
TR triangular (or in some cases quasi-triangular)
US unstructured sparse

For Fortran 77, routine names are in uppercase letters; however, for the C interfaces all routine names are in lowercase letters. To avoid possible name collisions, programmers are strongly advised not to declare variables or functions with names beginning with these prefixes.

A detailed discussion of the format of the <name> naming convention is contained in each respective chapter of the document.

1.5 Overall Functionality

This section summarizes, in tabular form, the functionality of the proposed routines. Issues such as storage formats or data types are not addressed. The functionality of the existing Level 1, 2 and 3 BLAS [39, 22, 26, 25] is a subset of the functionality proposed in this document.

In the original BLAS, each level was categorized by the type of operation; Level 1 addressed scalar and vector operations, Level 2 addressed matrix-vector operations, while Level 3 addressed matrix-matrix operations. The functionality tables in this document are categorized in a similar manner, with additional categories to cover operations which were not addressed in the original BLAS.

Unless otherwise specified, the operations apply to both real and complex arguments. For the sake of compactness the complex operators are omitted, so that whenever a transpose operation is given the conjugate transpose should also be assumed for the complex case.

The last column of each table denotes in which chapter of this document the functionality occurs. Specifically,

- "D" denotes dense and banded BLAS (Chapter 2),
- "S" denotes sparse BLAS (Chapter 3), and
- "E" denotes extended and mixed precision BLAS (Chapter 4).

1.5.1 Scalar and Vector Operations

This section lists scalar and vector operations. The functionality tables are organized as follows. Table 1.1 lists the scalar and vector reduction operations, Table 1.2 lists the vector rotation operations, Table 1.3 lists the vector operations, and Table 1.4 lists those vector operations that involve only data movement.

For the Sparse BLAS, $x$ is a compressed sparse vector and $y$ is a dense vector. Details of data structures are in Section 3.4.1.

For further details of vector norm notation, refer to section 2.1.1.
### 1.5. OVERALL FUNCTIONALITY

| Dot product | $r \leftarrow \beta r + \alpha x^T y$ | D,E |
| Vector norms | $r \leftarrow x^T y$ | S |
| | $r \leftarrow ||x||_1$ | D |
| | $r \leftarrow ||x||_1 R_i$ | D |
| | $r \leftarrow ||x||_2$ | D |
| | $r \leftarrow ||x||_\infty$ | D |
| Sum | $r \leftarrow \sum_i x_i$ | D,E |
| Min value & location | $k, x_k; k = \arg \min_i x_i$ | D |
| Min abs value & location | $k, x_k, k = \arg \min_i (|\text{Re}(x_i)| + |\text{Im}(x_i)|)$ | D |
| Max value & location | $k, x_k; k = \arg \max_i x_i$ | D |
| Max abs value & location | $k, x_k, k = \arg \max_i (|\text{Re}(x_i)| + |\text{Im}(x_i)|)$ | D |
| Sum of squares | $(\text{scl}, ssq) \leftarrow \sum x_i^2$, $\text{ssq} \cdot \text{scl}^2 = \sum x_i^2$ | D |

Table 1.1: Reduction Operations

| Generate Givens rotation | $(c, s, r) \leftarrow \text{rot}(a, b)$ | D |
| Generate Jacobi rotation | $(a, b, c, s) \leftarrow \text{jrot}(x, y, z)$ | D |
| Generate Householder transform | $(\alpha, x, \tau) \leftarrow \text{house}(\alpha, x)$, $H = I - \alpha uu^T$ | D |

Table 1.2: Generate Transformations

| Reciprocal Scale | $x \leftarrow x/\alpha$ | D |
| Scaled vector accumulation | $y \leftarrow \alpha x + \beta y$, $y \leftarrow \alpha x + y$ | D,E,S |
| Scaled vector addition | $w \leftarrow \alpha x + \beta y$ | D,E |
| Combined axpy & dot product | $\begin{cases} \bar{w} \leftarrow w - \alpha v \\ r \leftarrow \bar{w}^T u \end{cases}$ | D |
| Apply plane rotation | $(x, y) \leftarrow (x, y)R$ | D |

Table 1.3: Vector Operations

| Copy | $y \leftarrow x$ | D |
| Swap | $y \leftrightarrow x$ | D |
| Sort vector | $x \leftarrow \text{sort}(x)$ | D |
| Sort vector & return index vector | $(p, x) \leftarrow \text{sort}(x)$ | D |
| Permute vector | $x \leftarrow Px$ | D |
| Sparse gather | $x \leftarrow y|x$ | S |
| Sparse gather and zero | $x \leftarrow y|x; y|x \leftarrow 0$ | S |
| Sparse scatter | $y|x \leftarrow x$ | S |

Table 1.4: Data Movement with Vectors
1.5.2 Matrix-Vector Operations

This section lists matrix-vector operations in table 1.5. The matrix arguments $A$, $B$ and $T$ are dense or banded or sparse. In addition, where appropriate, the matrix $A$ can be symmetric (Hermitian) or triangular or general. The matrix $T$ represents an upper or lower triangular matrix, which can be unit or non-unit triangular. For the Sparse BLAS, the matrix $A$ is sparse, the matrix $T$ is sparse triangular, and the vectors $x$ and $y$ are dense.

Details of the data structures are discussed in sections 2.2, and 3.4.1.

<table>
<thead>
<tr>
<th>Matrix-vector product</th>
<th>$y \leftarrow \alpha Ax + \beta y$, $y \leftarrow \alpha A^T x + \beta y$</th>
<th>$x \leftarrow \alpha Tx$, $x \leftarrow \alpha T^T x$</th>
<th>$y \leftarrow \alpha Ax + y$, $y \leftarrow \alpha A^T x + y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summed matrix-vector multiplies</td>
<td>$y \leftarrow \alpha Ax + \beta Bx$</td>
<td>$D$, $E$</td>
<td></td>
</tr>
<tr>
<td>Multiple matrix-vector multiplies</td>
<td>$x \leftarrow T^T y$</td>
<td>$D$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w \leftarrow Tz$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x \leftarrow \beta A^T y + z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w \leftarrow \alpha Ax$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiple matrix-vector multiplies and low rank updates</td>
<td>$A \leftarrow A + u_1 v_1^T + u_2 v_2^T$</td>
<td>$D$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$x \leftarrow \beta A^T y + z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$w \leftarrow \alpha Ax$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular solve</td>
<td>$x \leftarrow \alpha T^{-1} x$, $x \leftarrow \alpha T^{-T} x$</td>
<td>$D$, $S$, $E$</td>
<td></td>
</tr>
<tr>
<td>Rank one updates and symmetric ($A = A^T$)</td>
<td>$A \leftarrow \alpha xy^T + \beta A$</td>
<td>$D$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A \leftarrow \alpha xx^T + \beta A$</td>
<td>$D$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A \leftarrow (\alpha x)y^T + y(\alpha x)^T + \beta A$</td>
<td>$D$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1.5: Matrix-Vector Operations

1.5.3 Matrix Operations

This section lists a variety of matrix operations. The functionality tables are organized as follows. Table 1.6 lists single matrix operations and matrix operations that involve $O(n^2)$ operations, Table 1.7 lists the $O(n^3)$ matrix-matrix operations and Table 1.8 lists those matrix operations that involve only data movement. Where appropriate one or more of the matrices can also be symmetric (Hermitian) or triangular or general. The matrix $T$ represents an upper or lower triangular matrix, which can be unit or non-unit triangular. $D$, $D_L$, and $D_R$ represent diagonal matrices, and $J$ represents a symmetric tridiagonal matrix (including $2 \times 2$ block diagonal).

Details of the data structures are discussed in sections 2.2, and 3.4.1.

For further details of matrix norm notation, refer to section 2.1.3.
### 1.5. OVERALL FUNCTIONALITY

| Matrix norms | $r \leftarrow ||A||_{1}, r \leftarrow ||A||_{1R}$ | D |
| Diagonal scaling | $r \leftarrow ||A||_{F}, r \leftarrow ||A||_{\infty}, r \leftarrow ||A||_{\infty R}$ | D |
|        | $r \leftarrow ||A||_{\max}, r \leftarrow ||A||_{\max R}$ | D |
|        | $A \leftarrow D A, A \leftarrow A D, A \leftarrow D_{L} A D_{R}$ | D |
|        | $A \leftarrow D A D$ | D |
|        | $A \leftarrow A + B D$ | D |
| Matrix acc and scale | $C \leftarrow \alpha A + \beta B$ | D |
| Matrix add and scale | $B \leftarrow \alpha A + \beta B, B \leftarrow \alpha A^{T} + \beta B$ | D |

Table 1.6: Matrix Operations – $O(n^2)$ floating point operations

| Matrix-matrix product | $C \leftarrow \alpha A B + \beta C, C \leftarrow \alpha A^{T} B + \beta C$ | D, E |
| Triangular multiply | $B \leftarrow \alpha T B, B \leftarrow \alpha B T$ | D, E |
| Triangular solve | $B \leftarrow \alpha T^{-1} B, B \leftarrow \alpha B T^{-1}$ | D, S, E |
| Symmetric rank $k$ & $2k$ updates ($C = C^{T}$) | $C \leftarrow \alpha A A^{T} + \beta C, C \leftarrow \alpha A^{T} A + \beta C$ | D, E |
|        | $C \leftarrow \alpha A J A^{T} + \beta C, C \leftarrow \alpha A^{T} J A + \beta C$ | D |
|        | $C \leftarrow (\alpha A) B^{T} + B (\alpha A)^{T} + \beta C$, | D, E |
|        | $C \leftarrow (\alpha A)^{T} B + B^{T} (\alpha A) + \beta C$ | D |
|        | $C \leftarrow (\alpha A J) B^{T} + B (\alpha A J)^{T} + \beta C$, | D |
|        | $C \leftarrow (\alpha A J)^{T} B + B^{T} (\alpha A J) + \beta C$ | D |

Table 1.7: Matrix-Matrix Operations - $O(n^3)$ floating point operations

| Matrix copy | $B \leftarrow A, B \leftarrow A^{T}$ | D |
| Matrix transpose | $A \leftarrow A^{T}$ | D |
| Permute Matrix | $A \leftarrow P A, A \leftarrow A P$ | D |

Table 1.8: Data Movement with Matrices
1.6 Numerical Accuracy and Environmental Enquiry

To understand the numerical behavior of the routines proposed here, certain floating point parameters are necessary. Detailed error bounds and limitations due to overflow and underflow are discussed in individual chapters (see sections 2.7, 3.7, 4.3.3, and C.4.4) but all of them depend on details of how floating point numbers are represented. These details are available by calling an environmental enquiry function called FPINFO.

Floating point numbers are represented in scientific notation as follows. This discussion follows the IEEE Floating Point Arithmetic Standard 754 [7].

\[ x = \pm d.d \cdots d \times BASE^E \]

where \( d.d \cdots d \) is a number represented as a string of \( T \) significant digits in base \( BASE \) with the “point” to the right of the leftmost digit, and \( E \) is an integer exponent. \( E \) ranges from \( EMIN \) up to \( EMAX \). This means that the largest representable number, which is also called the overflow threshold or OV, is just less than \( BASE^{EMAX+1} \). This also means that the smallest positive “normalized” representable number (i.e. where the leading digit of \( d.d \cdots d \) is nonzero) is \( BASE^{EMIN} \), which is also called the underflow threshold or UN.

When overflow occurs (because a computed quantity exceeds OV in absolute value), the result is typically \( \pm \infty \), or perhaps an error message. When underflow occurs (because a computed quantity is less than UN in absolute magnitude) the returned result may be either 0 or a tiny number less than UN in magnitude, with minimal exponent EMIN but with a leading zero (0.d.d \cdots d). Such tiny numbers are often called denormalized or subnormal, and floating point arithmetic which returns them instead of 0 is said to support gradual underflow.

The relative machine precision (or machine epsilon) of a basic operation \( \oplus \in \{+,-,\times,\div\} \) is defined as the smallest \( EPS > 0 \) satisfying

\[ fl(a \oplus b) = (a \oplus b) \times (1 + \delta) \text{ for some } |\delta| \leq EPS \]

for all arguments \( a \) and \( b \) that do not cause underflow, overflow, division by zero, or an invalid operation. When \( fl(a \oplus b) \) is a closest floating point number to the true result \( a \oplus b \) (with ties broken arbitrarily), then rounding is called “proper” and \( EPS = .5 \times BASE^{1-T} \). Otherwise typically \( EPS = BASE^{1-T} \), although it can sometimes be worse if arithmetic is not implemented carefully. We further say that rounding is “IEEE style” if ties are broken by rounding to the nearest number whose least significant digit is even (i.e. whose bottom bit is 0).

The function FPINFO returns the above floating point parameters, among others, to help the user understand the accuracy to which results are computed. FPINFO can return the values for either single precision or double precision. The way the precision is specified is language dependent, as is the choice of floating point parameter to return, and described in section 2.7. The names single and double may have different meanings on different machines: We have long been accustomed to single precision meaning 32-bits on all IEEE and most other machines [7], except for Cray and its emulators where single is 64-bits. And there are historical examples of 60-bit formats on some old CDC machines, etc. Nonetheless, we all agree on single precision as a phrase with a certain system-dependent meaning, and double precision too, meaning at least twice as many significant digits as single.

\(^3\text{We ignore implementation details like “hidden bits”, as well as unusual representations like logarithmic arithmetic and double-double.}\)
1.7. LANGUAGE BINDINGS

The values returned by FPINFO are as follows, including the values returned for IEEE single and IEEE double, the most common cases. The floating point parameters in column 1 have analogous meanings as the like-named character arguments of the LAPACK subroutine xLAMCH.4

<table>
<thead>
<tr>
<th>Floating point parameter</th>
<th>Description</th>
<th>Value in IEEE single</th>
<th>Value in IEEE double</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASE</td>
<td>base of the machine</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>number of digits</td>
<td>24</td>
<td>53</td>
</tr>
<tr>
<td>RND</td>
<td>1 when proper rounding occurs in addition</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>IEEE</td>
<td>1 when rounding in addition is IEEE style</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>EMIN</td>
<td>minimum exponent before (gradual) underflow</td>
<td>-126</td>
<td>-1022</td>
</tr>
<tr>
<td>EMAX</td>
<td>maximum exponent before overflow</td>
<td>127</td>
<td>1023</td>
</tr>
<tr>
<td>EPS</td>
<td>machine epsilon</td>
<td>$2^{-24} \approx 5 \times 10^{-8}$</td>
<td>$2^{-53} \approx 10^{-16}$</td>
</tr>
<tr>
<td>PREC</td>
<td>EPS*BASE</td>
<td>$2^{-23}$</td>
<td>$2^{-52}$</td>
</tr>
<tr>
<td>UN</td>
<td>underflow threshold</td>
<td>$2^{-126} \approx 10^{-38}$</td>
<td>$2^{-1022} \approx 10^{-308}$</td>
</tr>
<tr>
<td>OV</td>
<td>overflow threshold</td>
<td>$\sim 2^{128} \approx 10^{38}$</td>
<td>$\sim 2^{1024} \approx 10^{308}$</td>
</tr>
<tr>
<td>SFMIN</td>
<td>safe minimum, such that 1/SFMIN does not overflow</td>
<td>$2^{-126} \approx 10^{-38}$</td>
<td>$2^{-1022} \approx 10^{-308}$</td>
</tr>
</tbody>
</table>

Table 1.9: Values returned by FPINFO

Chapter 4 defines an additional FPINFO-like function to supplement this one with additional information needed for error bounds.

1.7 Language Bindings

Each specification of a routine corresponds to an operation outlined in the functionality tables. Operations are organized analogous to the order in which they are presented in the functionality tables. The specification has the form:

NAME (**multi-word description of operation**) < **mathematical representation**>

---

4Here are the differences: In xLAMCH, UN was called RMIN and OV was called RMAX. The value of IEEE was computed by xLAMCH but not returned. xLAMCH returned EMIN+1 and EMAX+1 instead of EMIN and EMAX, respectively (this corresponds to a different choice of where to put the "point" in $d.d\ldots d * BASE^p$).
Optional brief textual description of the functionality including any restrictions that apply to all language bindings.

- Fortran 95 binding
- Fortran 77 binding
- C binding

Alternative language bindings for C++ and Java were also discussed during the meetings of the forum, but the specifications for these bindings were postponed for a future series of meetings.

1.8 Error Handling

This document supports two types of error-handling capabilities: an error handler and error return codes. Each chapter of this document, and thus each flavor of BLAS, has the choice of using either capability, whichever is more appropriate. Chapters 2 and 4 rely on an error handler, and Chapter 3 provides error return codes.

One error handler, `BLAS_ERROR`, is defined. A series of error return codes are also defined. Each function in this document determines when and if an error-handling mechanism is called, and its function specification must document the conditions (if any) which trigger the error handling mechanism.

1.8.1 Return Codes

Routines in the Sparse BLAS chapter utilize return codes since many of the operations need to be recoverable. In Fortran 95 and 77, the error return code of a BLAS routine is returned in the parameter `istat`, usually the last argument in the parameter list. In C, the error code is the return value of the function. In either case, the value of the error code is the integer 0 if the operation was successful. In the event of an error detection, a nonzero value is returned and control returns back to the calling program, as usual. The application is not aborted or halted, and it is the responsibility of the caller to check error status of these BLAS operations.

1.8.2 Error Handlers

The error handler defines some minimal scalar input argument checking.

*Advice to implementors.* A BLAS supplier is free to provide multiple interfaces to the libraries, so that a second interface may perform no error checking. *(End of advice to implementors.)*

Additional error checking may be performed (for instance, checking that there are no zeros on the diagonal of a triangular solve), but these kinds of tests are too implementation-constraining to be mandated by the standard. Any additional error checking must not abort execution.

When any of the mandated scalar input argument checks fail, if the BLAS error handler is used, it must use the API given below. The default behavior of the BLAS-compliant error handler is to print an informative error message and abort execution. However, the API of this error handler is mandated by this document specifically so that a user can override the default error handler with a user-defined routine, so that this behavior can be changed. It is therefore necessary that the implementor not assume that the error handler stops execution, but rather must return explicitly before altering the routine’s operands in the event of an error.
1.8. ERROR HANDLING

The following are defined as errors by this standard. All Fortran 95, Fortran 77, and C routines
must perform the following error check.

- Any value of the operator arguments whose meaning is not specified in section A.3 is invalid.
Additionally, all Fortran 77 and C routines must perform the following error checks, unless otherwise
noted in the specification of the routine.

- Any problem dimension or bandwidth (e.g., m, n, k, kl, ku) less than zero
- Any vector increment (e.g., incw, incx, incy, incz) equal to zero
- Any leading dimension (e.g. lda, ldb, ldc, ldt) less than one
- Any leading dimension (e.g. lda, ldb, ldc, ldt) less than the relevant dimension of the problem.
  The relevant dimension of the problem is:
  
  - n, for a square, symmetric, or triangular matrix
  - m, for a m x n general, non-transposed matrix
  - n, for a m x n general, transposed matrix
  - kl + ku + 1 for a m x n general band matrix
  - k + 1 for a n x n symmetric or triangular band matrix with k super- or subdiagonals

Each language binding possesses its own unique error handler. However, all error handlers
minimally pass three pieces of information:

1. RNAME, the name of the routine in which the error occurred.

2. IFLAG, an integer flag which, if negative, means that parameter number -IFLAG caused the
   error, and if set to nonnegative, is an implementation-specific error code

3.IVAL, the value of parameter number -IFLAG.

Each language’s BLAS error handler should print an informative error message describing the error,
and halt execution. The API of the error handler is explicitly spelled out in each section, so that if
this behavior is not desired by the user or higher level library provider, it may be changed by the
BLAS user, overriding the BLAS’s error handler with one which performs as required.

The API for each language binding is mandated in the following sections; as an advice to the
implementor, an example of a BLAS-2000 compliant error handler is included as well.

F95 error handler

The Fortran 95 BLAS do not need to test the option arguments, since these are derived types and
hence invalid arguments are flagged by the compiler. The only case where array dimensions are
arguments to the Fortran 95 BLAS are the nonsymmetric band routines where m and kl are passed
as arguments. The other array dimensions can be determined in the BLAS routines using the
intrinsic function SIZE, and arrays should be checked for conformance according to the operation
being performed. For example in the operation AB the second dimension of A must equal the first
dimension of B. Note that, for consistency, m is included in all of the nonsymmetric band routines
although in some cases it is redundant; in those cases it should be tested against the relevant array
dimension.

The mandated API of the routine is:
MODULE blas_error_handler

INTERFACE blas_error
    SUBROUTINE blas_error(rname,iflag,ival)
        INTEGER, INTENT (IN) :: iflag
        INTEGER, OPTIONAL, INTENT (IN) :: ival
        CHARACTER (*), INTENT (IN) :: rname
    END SUBROUTINE blas_error
END INTERFACE
END MODULE blas_error_handler

A possible implementation would be:

SUBROUTINE blas_error(rname,iflag,ival)
    ! .. Scalar Arguments ..
    ! The optional argument ival must be present when iflag is in (-98,-1)
    INTEGER, INTENT (IN) :: iflag
    INTEGER, OPTIONAL, INTENT (IN) :: ival
    CHARACTER (*), INTENT (IN) :: rname
    ! ..
    SELECT CASE (iflag)
    CASE (-99)
        WRITE (*,1000) rname
    CASE (-98:-1)
        WRITE (*,2000) rname, -iflag, ival
    CASE DEFAULT
        WRITE (*,3000) iflag, rname
    END SELECT

    STOP

1000 FORMAT ('On entry to ',A, &
     ' two or more array argument sizes do not conform')
2000 FORMAT ('On entry to ',A,' argument number',I3, &
     ' had the illegal value of ',I5)
3000 FORMAT ('Unknown error code ',I5,' raised by routine ',A)

END SUBROUTINE blas_error

F77 error handler

The mandated API of the routine is:

SUBROUTINE BLAS_ERROR( RNAME, IFLAG, IVAL )
CHARACTER(*) RNAME
INTEGER IFLAG, IVAL

A possible implementation would be:

SUBROUTINE BLAS_ERROR( RNAME, IFLAG, IVAL )
CHARACTER(*) RNAME
1.8. ERROR HANDLING

INTEGER IFLAG, IVAL

IF ( IFLAG.LT.0 ) THEN
  WRITE(*,1000) RNAME, -IFLAG, IVAL
ELSE
  WRITE(*,2000) IFLAG, RNAME
END IF
STOP

1000 FORMAT('On entry to ',A, ' parameter number', I3,
  $ ' had the illegal value of', I)

2000 FORMAT('Unknown error code ',I,' raised by routine',A)
END

C error handler

The mandated API of the routine is:

void BLAS_error(char *rname, int iflag, int ival, char *form, ...)

A possible implementation would be:

#include <stdio.h>
#include <stdarg.h>

void BLAS_error(char *rname, int iflag, int ival, char *form, ...)
{
  va_list argptr;

  va_start(argptr, form);
  fprintf(stderr, "Error #%d from routine %s:\n", iflag, rname);
  if (form) vfprintf(stderr, form, argptr);
  else if (iflag < 0)
    fprintf(stderr,
      " Parameter number %d to routine %s had the illegal value %d\n",
      -iflag, rname, ival);
  else fprintf(stderr, " Unknown error code %d from routine %s\n",
      iflag, rname);
  exit(iflag);
}
Chapter 2

Dense and Banded BLAS

2.1 Overview and Functionality

This chapter defines the functionality and language bindings for the dense and banded BLAS routines, addressing mathematical operations with scalars, vectors and dense, banded, and triangular matrices but not sparse data structures.

The chapter is organized as follows. Sections 2.1.1, 2.1.2, and 2.1.3 list in tabular form the functionality of the proposed routines. Unless otherwise specified, the operations apply to both real and complex arguments. For the sake of compactness the complex operators are omitted, so that whenever a transpose operation is given the conjugate transpose should also be assumed for the complex case. Section 2.2 defines the matrix storage schemes. Section 2.3 discusses general interface issues, and sections 2.4, 2.5, and 2.6 detail the interface issues for the respective language bindings – Fortran 95, Fortran 77, and C. Section 2.7 discusses issues concerning the numerical accuracy of the BLAS. And lastly, sections 2.8.2 – 2.8.10 present the language bindings for the proposed routines.

2.1.1 Scalar and Vector Operations

This section lists scalar and vector operations. The functionality tables are organized as follows. Table 2.1 lists the scalar and vector reduction operations, table 2.2 lists the rotation operations, table 2.3 lists the vector operations, and table 2.4 lists vector operations involving only data movement. Notation in the tables is defined in section 1.4, and details of the data structures are discussed in section 2.2. Vector norms are defined in Appendix A.1. The language bindings are presented in sections 2.8.2, 2.8.4, and 2.8.5.

2.1.2 Matrix-Vector Operations

This section lists the matrix-vectors operations in functionality table 2.5. Unless otherwise specified, the operations apply to both real and complex arguments. For the sake of compactness the complex operators are omitted, so that whenever a transpose operation is given both the conjugate and conjugate transpose should also be assumed for the complex case.

The matrix $T$ represents an upper or lower triangular matrix, which can be unit or non-unit triangular. $D$ represents a diagonal matrix. Notation in the tables is defined in section 1.4, and details of the data structures are discussed in section 2.2. The language bindings are presented in section 2.8.6.
1. **OVERVIEW AND FUNCTIONALITY**

| Dot product | \( r \leftarrow \beta r + \alpha x^T y \) | DOT |
| Vector norms | \( r \leftarrow ||x||_1, r \leftarrow ||x||_1 R \) | NORM |
| Sum | \( r \leftarrow ||x||_2 \) | SUM |
| Min value & location | \( k, x_k; k = \arg \min_i x_i \) | MIN.VAL |
| Min abs value & location | \( k, x_k, k = \arg \min_i (|Re(x_i)| + |Im(x_i)|) \) | AMIN.VAL |
| Max value & location | \( k, x_k; k = \arg \max_i x_i \) | MAX.VAL |
| Max abs value & location | \( k, x_k, k = \arg \max_i (|Re(x_i)| + |Im(x_i)|) \) | AMAX.VAL |
| Sum of squares | \( \sum_i x_i \) | SUMSQ |

**Table 2.1: Reduction Operations**

| Generate Givens rotation | \((c, s, r) \leftarrow \text{rot}(a, b)\) | GEN_GROT |
| Generate Jacobi rotation | \((a, b, c, s) \leftarrow \text{jrot}(x, y, z)\) | GEN_JROT |
| Generate Householder transform | \((\alpha, x, \tau) \leftarrow \text{house}(\alpha, x), H = I - \alpha uu^T\) | GEN_HOUSE |

**Table 2.2: Generate Transformations**

| Reciprocal Scale | \( x \leftarrow x/\alpha \) | RSSCALE |
| Scaled vector accumulation | \( y \leftarrow \alpha x + \beta y \) | AXPBY |
| Scaled vector addition | \( w \leftarrow \alpha x + \beta y \) | WAXPBY |
| Combined axpy & dot product | \( \begin{align*} \hat{w} & \leftarrow w - \alpha v \\ r & \leftarrow \hat{w}^T u \end{align*} \) | AXPY_DOT |
| Apply plane rotation | \( (x, y) \leftarrow (x, y) R \) | APPLY_GROT |

**Table 2.3: Vector Operations**

| Copy | \( y \leftarrow x \) | COPY |
| Swap | \( y \leftrightarrow x \) | SWAP |
| Sort vector | \( x \leftarrow \text{sort}(x) \) | SORT |
| Sort vector & return index vector | \((p, x) \leftarrow \text{sort}(x)\) | SORTV |
| Permute vector | \( x \leftarrow P x \) | PERMUTE |

**Table 2.4: Data Movement with Vectors**

1.3 **Matrix Operations**

This section lists single matrix operations, matrix-matrix operations, and matrix operations involving data movement. The functionality tables are organized as follows. Table 2.6 lists single matrix operations and matrix operations that involve \(O(n^2)\) floating point operations, Table 2.7 lists the \(O(n^3)\) matrix-matrix floating point operations and Table 2.8 lists those matrix floating point operations that involve only data movement. Unless otherwise specified, the operations apply to both real and complex arguments. For the sake of compactness the complex operators are omit-
<table>
<thead>
<tr>
<th>Operation Description</th>
<th>Expression</th>
<th>Datatypes</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Matrix vector product</strong></td>
<td>( y \leftarrow \alpha Ax + \beta y )</td>
<td>GE, GB, SY, HE, SB, HB, SP, HP</td>
<td>MV</td>
</tr>
<tr>
<td>( y \leftarrow \alpha A^T x + \beta y )</td>
<td>GE, GB</td>
<td>MV</td>
<td></td>
</tr>
<tr>
<td>( x \leftarrow \alpha T x, \ x \leftarrow \alpha T^T x )</td>
<td>TR, TB, TP</td>
<td>MV</td>
<td></td>
</tr>
<tr>
<td><strong>Summed matrix vector multiplies</strong></td>
<td>( y \leftarrow \alpha Ax + \beta B x )</td>
<td>GE</td>
<td>SUM_MV</td>
</tr>
<tr>
<td><strong>Multiple matrix vector multiplies</strong></td>
<td>( { x \leftarrow T^T y ) )</td>
<td>TR</td>
<td>MVT</td>
</tr>
<tr>
<td>( w \leftarrow T z )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( x \leftarrow \beta A^T y + z )</td>
<td>GE</td>
<td>MVT</td>
<td></td>
</tr>
<tr>
<td>( w \leftarrow \alpha Ax )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Multiple mv multiplies &amp; low rank updates</strong></td>
<td>( A \leftarrow A + u_1 v_1^T + u_2 v_2^T )</td>
<td>GE</td>
<td>MVER</td>
</tr>
<tr>
<td>( x \leftarrow \beta \hat{A}^T y + z )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( w \leftarrow \alpha \hat{A} x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Triangular solve</strong></td>
<td>( x \leftarrow \alpha T^{-1} x, \ x \leftarrow \alpha T^{-T} x )</td>
<td>TR, TB, TP</td>
<td>SV</td>
</tr>
<tr>
<td><strong>Rank one updates</strong> and symmetric ( (A = A^T) )</td>
<td>( A \leftarrow \alpha x y^T + \beta A )</td>
<td>GE</td>
<td>R</td>
</tr>
<tr>
<td>( A \leftarrow \alpha x x^T + \beta A )</td>
<td>SY, HE, SP, HP</td>
<td>R</td>
<td></td>
</tr>
<tr>
<td><strong>Rank one &amp; two updates</strong></td>
<td>( A \leftarrow (\alpha x) y^T + y(\alpha x)^T + \beta A )</td>
<td>SY, HE, SP, HP</td>
<td>R2</td>
</tr>
</tbody>
</table>

Table 2.5: Matrix-Vector Operations

Note that whenever a transpose operation is given both the conjugate and conjugate transpose should also be assumed for the complex case. The matrix \( T \) represents an upper or lower triangular matrix, which can be unit or non-unit triangular. \( D, D_L, \) and \( D_R \) represent diagonal matrices, and \( J \) is a symmetric tridiagonal matrix. Notation in the tables is defined in section 1.4, and details of the data structures are discussed in section 2.2. Matrix norms are defined in Appendix A.2. The language bindings are listed in sections 2.8.6, 2.8.7, 2.8.8, and 2.8.9.

<table>
<thead>
<tr>
<th>Operation Description</th>
<th>Expression</th>
<th>Datatypes</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Matrix norms</strong></td>
<td>( r \leftarrow |A|_1, \ r \leftarrow |A|_R, \ r \leftarrow |A|_F, )</td>
<td>GE, GB, SY, HE, SB, HB, SP, HP, TR, TB, TP</td>
<td>NORM</td>
</tr>
<tr>
<td>( r \leftarrow |A|<em>{\infty}, \ r \leftarrow |A|</em>{\infty R} )</td>
<td>GE, GB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r \leftarrow |A|<em>{\text{max}}, \ r \leftarrow |A|</em>{\text{max} R} )</td>
<td>GE, GB</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Diagonal scaling</strong></td>
<td>( A \leftarrow D A, \ A \leftarrow A D )</td>
<td>GE, GB</td>
<td>DIAG_SCALE</td>
</tr>
<tr>
<td>( A \leftarrow D_L A D_R )</td>
<td>GE, GB</td>
<td>DIAGONAL</td>
<td></td>
</tr>
<tr>
<td>( A \leftarrow D A D )</td>
<td>SY, HE, SB, HB, SP, HP</td>
<td>LRSCALE</td>
<td></td>
</tr>
<tr>
<td>( A \leftarrow A + B D )</td>
<td>GE, GB</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Matrix acc and scale</strong></td>
<td>( B \leftarrow \alpha A + \beta B, \ B \leftarrow \alpha A^T + \beta B )</td>
<td>GE, GB, SY, SB, SP, TR, TB, TP</td>
<td>ACC</td>
</tr>
<tr>
<td><strong>Matrix add and scale</strong></td>
<td>( C \leftarrow \alpha A + \beta B )</td>
<td>GE, GB, SY, SB, SP, TR, TB, TP</td>
<td>ADD</td>
</tr>
</tbody>
</table>

Table 2.6: Matrix Operations – \( O(n^2) \) floating point operations
### 2.1.**Overview and Functionality**

<table>
<thead>
<tr>
<th>Matrix matrix product</th>
<th>$C \leftarrow \alpha A B + \beta C$, $C \leftarrow \alpha A^T B + \beta C$</th>
<th>GE</th>
<th>MM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C \leftarrow \alpha A B^T + \beta C$, $C \leftarrow \alpha A^T B^T + \beta C$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C \leftarrow \alpha A B + \beta C$, $C \leftarrow \alpha B A + \beta C$</td>
<td>SY,HE</td>
<td>MM</td>
</tr>
<tr>
<td>Triangular multiply</td>
<td>$B \leftarrow \alpha T B$, $B \leftarrow \alpha B T$</td>
<td>TR</td>
<td>MM</td>
</tr>
<tr>
<td></td>
<td>$B \leftarrow \alpha T^T B$, $B \leftarrow \alpha B T^T$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular solve</td>
<td>$B \leftarrow \alpha T^{-1} B$, $B \leftarrow \alpha B T^{-1}$</td>
<td>TR</td>
<td>SM</td>
</tr>
<tr>
<td></td>
<td>$B \leftarrow \alpha T^{-T} B$, $B \leftarrow \alpha B T^{-T}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Symmetric rank $k$ &amp; $2k$ updates ($C = C^T$)</td>
<td>$C \leftarrow \alpha A A^T + \beta C$, $C \leftarrow \alpha A^T A + \beta C$</td>
<td>SY,HE</td>
<td>RK</td>
</tr>
<tr>
<td></td>
<td>$C \leftarrow \alpha A J A^T + \beta C$, $C \leftarrow \alpha A^T J A + \beta C$</td>
<td>SY,HE</td>
<td>_TRIDIAG_RK</td>
</tr>
<tr>
<td></td>
<td>$C \leftarrow (\alpha A)^T B + B (\alpha A)^T + \beta C$, $C \leftarrow (\alpha A)^T B + B^T (\alpha A) + \beta C$</td>
<td>SY,HE</td>
<td>R2K</td>
</tr>
<tr>
<td></td>
<td>$C \leftarrow (\alpha A J)^T B + B (\alpha A J)^T + \beta C$, $C \leftarrow (\alpha A J)^T B + B^T (\alpha A J) + \beta C$</td>
<td>SY,HE</td>
<td>_TRIDIAG_R2K</td>
</tr>
</tbody>
</table>

Table 2.7: Matrix-Matrix Operations – $O(n^3)$ floating point operations

<table>
<thead>
<tr>
<th>Matrix copy</th>
<th>$B \leftarrow A$</th>
<th>GE,GB,SY,HE,SB,HB,SP,HP,TR,TB,TP</th>
<th>_COPY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B \leftarrow A^T$</td>
<td>GE,GB</td>
<td>_COPY</td>
</tr>
<tr>
<td>Matrix transpose</td>
<td>$A \leftarrow A^T$</td>
<td>GE</td>
<td>_TRANS</td>
</tr>
<tr>
<td>Permute Matrix</td>
<td>$A \leftarrow P A$, $A \leftarrow A P$</td>
<td>GE</td>
<td>_PERMUTE</td>
</tr>
</tbody>
</table>

Table 2.8: Data Movement with Matrices
2.2 Matrix Storage Schemes

The following matrix storage schemes are used:

- column-based and row-based storage in a contiguous array;
- packed storage for symmetric, Hermitian or triangular matrices;
- band storage for band matrices;

In the examples below, * indicates an array element that need not be set and is not referenced by the BLAS routines. Elements that “need not be set” are never read, written to, or otherwise accessed by the BLAS routines. The examples illustrate only the relevant part of the arrays; array arguments may of course have additional rows or columns, according to the usual rules for passing array arguments in C or Fortran.

2.2.1 Conventional Storage

The default scheme for storing matrices in the Fortran 95 and Fortran 77 interfaces is the one described in subsection 2.5.3: a matrix $A$ is stored in a two-dimensional array $A$, with matrix element $a_{ij}$ stored in array element $A(i, j)$, assuming one-based indexing.

For the C language interfaces, matrices may be stored column-wise or row-wise as described in subsection 2.6.6: a matrix $A$ is stored in a one-dimensional array $A$, with matrix element $a_{ij}$ stored column-wise in array element $A(i + j \times lda)$ or row-wise in array element $A(j + i \times lda)$, assuming zero-based indexing.

If a matrix is triangular (upper or lower, as specified by the argument uplo), only the elements of the relevant triangle are accessed. The remaining elements of the array need not be set. Such elements are indicated by * in the examples below. For example, assuming zero-based indexing and $n = 3$:

<table>
<thead>
<tr>
<th>order</th>
<th>uplo</th>
<th>Triangular matrix $A$</th>
<th>Storage in array $A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>blas_colmajor</td>
<td>blas_upper</td>
<td>$\begin{pmatrix} a_{00} &amp; a_{01} &amp; a_{02} \ a_{11} &amp; a_{12} &amp; \ast \ast \ast a_{11} \ast a_{02} a_{12} a_{22} \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>blas_rowmajor</td>
<td>blas_upper</td>
<td>$\begin{pmatrix} a_{00} &amp; a_{01} &amp; a_{02} \ a_{11} &amp; a_{12} &amp; \ast a_{12} a_{22} \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>blas_colmajor</td>
<td>blas_lower</td>
<td>$\begin{pmatrix} a_{00} \ a_{10} a_{11} \ a_{20} a_{21} a_{22} \end{pmatrix}$</td>
<td></td>
</tr>
<tr>
<td>blas_rowmajor</td>
<td>blas_lower</td>
<td>$\begin{pmatrix} a_{00} \ a_{10} a_{11} \ a_{20} a_{21} a_{22} \end{pmatrix}$</td>
<td></td>
</tr>
</tbody>
</table>

Routines that handle symmetric or Hermitian matrices allow for either the upper or lower triangle of the matrix (as specified by uplo) to be stored in the corresponding elements of the array; the remaining elements of the array need not be set. For example, when $n = 3$:
2.2. MATRIX STORAGE SCHEMES

<table>
<thead>
<tr>
<th>order</th>
<th>uplo</th>
<th>Hermitian matrix A</th>
<th>Storage in array A</th>
</tr>
</thead>
</table>
| blas_colmajor | blas_upper| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{01} & a_{02} \\
|               |          | a_{01} & a_{11} & a_{12} \\
|               |          | a_{02} & a_{12} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} * a_{01} \ast a_{02} * a_{12} * a_{22} \] |
| blas_rowmajor | blas_upper| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{01} & a_{02} \\
|               |          | a_{01} & a_{11} & a_{12} \\
|               |          | a_{02} & a_{12} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{01} a_{02} * a_{11} \ast a_{12} * a_{22} \] |
| blas_colmajor | blas_lower| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{10} & a_{20} \\
|               |          | a_{10} & a_{11} & a_{21} \\
|               |          | a_{20} & a_{21} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{10} a_{20} * a_{11} \ast a_{21} * a_{22} \] |
| blas_rowmajor | blas_lower| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{10} & a_{20} \\
|               |          | a_{10} & a_{11} & a_{21} \\
|               |          | a_{20} & a_{21} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{10} a_{20} * a_{11} \ast a_{21} * a_{22} \] |

2.2.2 Packed Storage

Symmetric, Hermitian or triangular matrices may be stored more compactly, if the relevant triangle (again as specified by uplo) is packed by columns or rows in a one-dimensional array. In the BLAS, arrays that hold matrices in packed storage, have names ending in ‘P’. So, in the case of zero-based addressing as in C, we have the following formulas (For one-based addressing, as in Fortran, replace $i$ by $i - 1$ and $j$ by $j - 1$ in these formulas).

- if uplo = blas_upper then
  - if order = blas_colmajor, $a_{ij}$ is stored in $\text{AP}(i + j(j + 1)/2)$ for $i \leq j$;
  - if order = blas_rowmajor, $a_{ij}$ is stored in $\text{AP}(j + i(2n - i - 1)/2)$ for $i \leq j$;

- if uplo = blas_lower then
  - if order = blas_colmajor, $a_{ij}$ is stored in $\text{AP}(i + j(2n - j - 1)/2)$ for $j \leq i$.
  - if order = blas_rowmajor, $a_{ij}$ is stored in $\text{AP}(j + i(i + 1)/2)$ for $j \leq i$.

For example, assuming zero-based indexing:

<table>
<thead>
<tr>
<th>order</th>
<th>uplo</th>
<th>Triangular matrix A</th>
<th>Packed storage in array ap</th>
</tr>
</thead>
</table>
| blas_colmajor | blas_upper| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{01} & a_{02} \\
|               |          | a_{11} & a_{12} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{01} a_{02} a_{11} a_{12} a_{22} \] |
| blas_rowmajor | blas_upper| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{01} & a_{02} \\
|               |          | a_{11} & a_{12} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{01} a_{02} a_{11} a_{12} a_{22} \] |
| blas_colmajor | blas_lower| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{10} & a_{20} \\
|               |          | a_{11} & a_{21} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{10} a_{20} a_{11} a_{21} a_{22} \] |
| blas_rowmajor | blas_lower| \[
|               |          | \begin{pmatrix}
|               |          | a_{00} & a_{10} & a_{20} \\
|               |          | a_{11} & a_{21} & a_{22} \\
|               |          | \end{pmatrix} \] | \[ a_{00} a_{10} a_{20} a_{11} a_{21} a_{22} \] |
Note that for real or complex symmetric matrices, packing the upper triangle by columns is equivalent to packing the lower triangle by rows; packing the lower triangle by columns is equivalent to packing the upper triangle by rows. For complex Hermitian matrices, packing the upper triangle by columns is equivalent to packing the conjugate of the lower triangle by rows; packing the lower triangle by columns is equivalent to packing the conjugate of the upper triangle by rows.

2.2.3 Band Storage

For Fortran (column-major storage), an \(m\)-by-\(n\) band matrix with \(kl\) subdiagonals and \(ku\) superdiagonals may be stored compactly in a two-dimensional array with \(kl + ku + 1\) rows and \(n\) columns. Columns of the matrix are stored in corresponding columns (contiguous storage dimension) of the array, and diagonals of the matrix are stored in rows (non-contiguous or strided dimension) of the array. This storage scheme should be used in practice only if \(kl, ku \ll \min(m, n)\), although BLAS routines work correctly for all values of \(kl\) and \(ku\). In the BLAS, arrays that hold matrices in band storage have names ending in 'B'.

To be precise, for column-major storage, \(a_{ij}\) is stored in \(AB(ku+i-j,j)\) for \(\max(0,j-ku) \leq i \leq \min(m-1,j+kl)\). For row-major storage, \(a_{ij}\) is stored in \(AB(i,kl+j-i)\) for \(\max(0,j-ku) \leq i \leq \min(n-1,j+kl)\). For example, assuming column-major storage, when \(m = n = 5\), \(kl = 2\) and \(ku = 1:\)

<table>
<thead>
<tr>
<th>Band matrix (A)</th>
<th>Band storage in array (AB)</th>
</tr>
</thead>
</table>
| \[
\begin{pmatrix}
  a_{00} & a_{01} \\
  a_{10} & a_{11} & a_{12} \\
  a_{20} & a_{21} & a_{22} & a_{23} \\
  a_{31} & a_{32} & a_{33} & a_{34} \\
  a_{42} & a_{43} & a_{44}
\end{pmatrix}
\] | \[
\begin{pmatrix}
  * & a_{01} & a_{12} & a_{23} & a_{34} \\
  a_{00} & a_{11} & a_{22} & a_{33} & a_{44} \\
  a_{10} & a_{21} & a_{32} & a_{43} & * \\
  a_{20} & a_{31} & a_{42} & * & *
\end{pmatrix}
\] |

The elements marked * in the upper left and lower right corners of the array \(AB\) need not be set, and are not referenced by BLAS routines.

For C (row-major storage), \(\text{order} = \text{blas_rowmajor}\), the rows of the matrix are stored in corresponding rows (contiguous storage dimension) of the array, and diagonals of the matrix are stored in columns (non-contiguous or strided dimension) of the array. The \(m\)-by-\(n\) band matrix with \(kl\) subdiagonals and \(ku\) superdiagonals is stored in a one-dimensional array with \(n\) rows and \(kl+ku+1\) columns, strided by \(lda\). The padding with elements marked * is now shifted to ensure that rows of the matrix are stored contiguously. Refer to section B.2.12 for full details.

Triangular band matrices are stored in the same format, with either \(kl = 0\) if upper triangular, or \(ku = 0\) if lower triangular.

For Fortran 77, and symmetric or Hermitian band matrices with \(kd\) subdiagonals or superdiagonals, only the upper or lower triangle (as specified by \(\text{uplo}\)) need be stored:

- if \(\text{uplo} = \text{blas_upper}\), \(a_{ij}\) is stored in \(AB(kd+i-j,j)\) for \(\max(0,j-kd) \leq i \leq j\);
- if \(\text{uplo} = \text{blas_lower}\), \(a_{ij}\) is stored in \(AB(i-j,j)\) for \(j \leq i \leq \min(n-1,j+kd)\).

For example, assuming zero-based indexing and \(n = 5\) and \(kd = 2:\)
2.2. **MATRIX STORAGE SCHEMES**

<table>
<thead>
<tr>
<th>uplo</th>
<th>Hermitian band matrix $A$</th>
<th>Band storage in array $AB$</th>
</tr>
</thead>
</table>
| blas_upper | \[
a_{00} & a_{01} & a_{02} \\
\bar{a}_{01} & a_{11} & a_{12} & a_{13} \\
\bar{a}_{02} & \bar{a}_{12} & a_{22} & a_{23} & a_{24} \\
\bar{a}_{13} & \bar{a}_{23} & a_{33} & a_{34} \\
\bar{a}_{24} & \bar{a}_{34} & a_{44}
\] | * * a_{02} a_{13} a_{24} |
|           |                           | * a_{01} a_{12} a_{23} a_{34} |
|           |                           | a_{00} a_{11} a_{22} a_{33} a_{44} |
| blas_lower| \[
a_{00} & \bar{a}_{10} & \bar{a}_{20} \\
a_{10} & a_{11} & a_{21} & a_{31} \\
a_{20} & a_{21} & a_{22} & a_{32} & a_{42} \\
a_{31} & a_{32} & a_{33} & a_{43} \\
a_{42} & a_{43} & a_{44}
\] | a_{00} a_{11} a_{22} a_{33} a_{44} |
|           |                           | a_{10} a_{21} a_{32} a_{43} * |
|           |                           | a_{20} a_{31} a_{42} * * |

Similarly, for C (row-major storage), `order = blas_rowmajor`, the contiguous dimension (rows) of the matrix is stored in the contiguous dimension (rows) of the array, strided by `lda`. And pictorially, the one-dimensional array is the transpose of the AB storage as depicted above. The padding with elements marked * is now shifted to ensure that rows of the matrix are stored contiguously. Refer to section B.2.12 for full details.

### 2.2.4 Unit Triangular Matrices

Some BLAS routines have an option to handle unit triangular matrices (that is, triangular matrices with diagonal elements = 1). This option is specified by an argument `diag`. If `diag = blas_unit_diag` (Unit triangular), the array elements corresponding to the diagonal elements of the matrix are not referenced by the BLAS routines. The storage scheme for the matrix (whether conventional, packed or band) remains unchanged, as described in subsection 2.2.1.

#### 2.2.5 Representation of a Householder Matrix

An elementary reflector (or elementary **Householder matrix**) $H$ of order $n$ is a unitary matrix of the form

$$H = I - \tau vv^H$$  \hspace{1cm} (2.1)

where $\tau$ is a scalar, and $v$ is an $n$-vector, with $|\tau|^2 ||v||^2 = 2 \text{Re}(\tau)$; $v$ is often referred to as the **Householder vector**. Often $v$ has several leading or trailing zero elements, but for the purpose of this discussion assume that $H$ has no such special structure.

This representation agrees with what is used in LAPACK [6] (which differs from those used in LINPACK [23] or EISPACK [48, 32]) sets $v_1 = 1$; hence $v_1$ need not be stored. In real arithmetic, $1 \leq \tau \leq 2$, except that $\tau = 0$ implies $H = I$.

In complex arithmetic, $\tau$ may be complex, and satisfies $1 \leq \text{Re}(\tau) \leq 2$ and $|\tau - 1| \leq 1$. Thus a complex $H$ is not Hermitian (as it is in other representations), but it is unitary, which is the important property. The advantage of allowing $\tau$ to be complex is that, given an arbitrary complex vector $x$, $H$ can be computed so that

$$H^T x = \beta(1, 0, \ldots, 0)^T$$

with real $\beta$. This is useful, for example, when reducing a complex Hermitian matrix to real symmetric tridiagonal matrix, or a complex rectangular matrix to real bidiagonal form.
2.2.6 Representation of a Permutation Matrix

An \(n\)-by-\(n\) permutation matrix \(P\) is represented as a product of at most \(n\) interchange permutations. An interchange permutation \(E\) is a permutation obtained by swapping two rows of the identity matrix. An efficient way to represent a general permutation matrix \(P\) is with an integer vector \(p\) of length \(n\). In other words, \(P = E_n \ldots E_1\) and each \(E_i\) is the identity with rows \(i\) and \(p_i\) interchanged.

\[
\begin{align*}
\text{Do } i &= 0 \text{ to } n - 1 & \text{or Do } i &= n - 1 \text{ to } 0 \\
x(i) &\leftrightarrow x( p(i) ) & x(i) &\leftrightarrow x( p(i) ) \\
\text{End do} & & \text{End do}
\end{align*}
\]

2.3 Interface Issues

2.3.1 Naming Conventions

The naming conventions adopted for the routines are as defined in section 1.4.6.

2.3.2 Argument Aliasing

Correctness is only guaranteed if output arguments are not aliased with any other arguments.

2.4 Interface Issues for Fortran 95

Some of the functions in the tables of this chapter can be replaced by simple array expressions and assignments in Fortran 95, without loss of convenience or performance (assuming a reasonable degree of optimization by the compiler). Fortran 95 also allows groups of related functions to be merged together, each group being covered by a single interface.

The following sections discuss the indexing base for vector and matrix operands, the features of the Fortran 95 language that are used, the matrix storage schemes that are supported, and error handling.

We strongly recommend that optional arguments be supplied by keyword, not by position, since the order in which they are described may differ from the order in which they appear in the argument list.

2.4.1 Fortran 95 Modules

Refer to Appendix A.4 for the Fortran 95 module \texttt{blas_dense}. The module \texttt{blas_operator_arguments} contains the derived type values, and separate modules are supplied with explicit interfaces to the routines. If the module \texttt{blas_dense} is accessed by a \texttt{USE} statement in any program which makes calls to these BLAS routines, then those calls can be checked by the compiler for errors in the numbers or types of arguments.

2.4.2 Indexing

The Fortran 95 interface returns indices in the range \(1 \leq I \leq N\) (where \(N\) is the number of entries in the dimension in question, and \(I\) is the index). This allows functions returning indices to be directly used to index standard arrays.
Likewise, for routines returning an index within a vector or matrix operand, this reference point is indexed starting at one.

2.4.3 Design of the Fortran 95 Interfaces

The proposed design utilizes the following features of the Fortran 95 language.

**Generic interfaces:** all procedures are accessed through *generic* interfaces. A single generic name covers several specific instances whose arguments may differ in the following properties:

- **data type** (real or complex).
- **precision** (or equivalently, kind type parameter “kind-value”). However, all real or complex arguments must have the same precision. We allow both single and double precision.
- **rank** Some arguments may either have rank 2 (to store a matrix) or rank 1 (to store a vector). In other cases an argument may be either a rank 1 array or a scalar.
- **different argument lists** Some of the arguments are optional. If one of these arguments does not appear in the calling sequence, a predefined value or a predefined action is assumed. Table 2.9 contains the predefined value or action for these arguments.

**Assumed-shape arrays:** all array arguments are *assumed-shape* arrays, which must have the exact shape required to store the corresponding matrix or vector. Hence arguments to specify array-dimensions or problem-dimensions are not required. The procedures assume that the supplied arrays have valid and consistent shapes. Zero dimensions (implying empty arrays) are allowed.

This means that, for a vector operand, the offset and stride are not needed as arguments. The actual argument corresponding to a n-length vector dummy argument could be:

<table>
<thead>
<tr>
<th>actual argument</th>
<th>comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>x(ix)</td>
<td>ix \neq 1 and incx \neq 1</td>
</tr>
<tr>
<td>x(1:1+(n-1)*incx)</td>
<td>ix= 1 and incx \neq 1</td>
</tr>
<tr>
<td>x(0:(n-1)*incx)</td>
<td>ix= 0 and incx \neq 1</td>
</tr>
<tr>
<td>x(ixix+n-1)</td>
<td>ix \neq 1 and incx= 1</td>
</tr>
<tr>
<td>x(1:n)</td>
<td>ix= 1 and incx= 1</td>
</tr>
<tr>
<td>x</td>
<td>if x is declared with shape (n), i.e. x(n)</td>
</tr>
<tr>
<td>x(ix)</td>
<td>where ix is an integer vector of ( n ) elements containing valid indices of x</td>
</tr>
<tr>
<td>a(:,j)</td>
<td>column ( j ) of a two-dimension array assuming that it has ( n ) rows (SIZE(a,1) = n)</td>
</tr>
<tr>
<td>a(i,:)</td>
<td>row ( i ) of a two-dimension array assuming that it has ( n ) columns (SIZE(a,2) = n)</td>
</tr>
</tbody>
</table>

**Derived types:** In the Fortran 95 bindings, we use dummy arguments whose actual argument must be a named constant of a derived type, which is defined within the BLAS module (and accessible via the BLAS module).
2.4.4 Matrix Storage Schemes

The matrix storage schemes for the Fortran 95 interfaces are as described in section 2.2. As with the Fortran 77 interfaces, only column-major storage is permitted. However, assumed-shape arrays are used instead of assumed-size arrays.

For a general banded matrix, \( a \), three arguments \( a, m \) and \( kl \) are used to define the matrix since \( ku \) is defined from the shape of the matrix and \( kl \) (\( ku = \text{SIZE}(a,1) - kl - 1 \)). For a symmetric banded matrix, a Hermitian banded matrix or triangular banded matrix, \( a \), only \( a \) is used as an argument to define the matrix as the band width is defined from the shape of the matrix and is equal to \( \text{SIZE}(a,1) - 1 \) and \( m = n \).

2.4.5 Format of the Fortran 95 bindings

Each interface is summarized in the form of a SUBROUTINE statement (or in few cases a FUNCTION statement), in which all of the potential arguments appear. Arguments which need not be supplied are grouped after the mandatory arguments and enclosed in square brackets, for example:

```
SUBROUTINE axpby( x, y [, alpha] [, beta] )
<type>(<wp>), INTENT (IN) :: x(:)
<type>(<wp>), INTENT (INOUT) :: y(:)
<type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
```

The default value for \( \beta \) is 1.0 or \((1.0,0,0)\).

As generic interfaces are used, floating point variables that can be REAL or COMPLEX are denoted by the keyword \( \text{<type>} \) which designates the data type for the operand

\[ \text{<type>} ::= \text{REAL} | \text{COMPLEX} \]

In some routines, however, some of the floating point arguments must be of a specific data type. If this is the case, then the argument type REAL or COMPLEX is used.

The precision of the floating point variable is denoted by \( \text{<wp>} \) (i.e., “working precision”) where

\[ \text{<wp>} ::= \text{KIND}(1.0) | \text{KIND}(1.0D0) \]

and \( \text{KIND}(1.0) \) and \( \text{KIND}(1.0D0) \) represent single precision and double precision, respectively.

Some arguments may either have rank 2 (to store a matrix) or rank 1 (to store a vector). In this case, the following notation is used:

\[ \text{<bb>} ::= b(:,:) | b(:) \]

The same notation is used in the case of an argument that may either have rank 1 or is a scalar.

\[ \text{<bb>} ::= b(:) | b \]

Fortran 95 bindings use assumed shape arrays. The actual arguments must have the correct dimension. For all the procedures that contain array arguments the shape of the array arguments is given in detail after the specification. For example the specification of the SUBROUTINE axpby given above is followed by:

\[ x \text{ and } y \text{ have shape } (n) \]

which indicates that both arrays \( x \) and \( y \) must be rank 1 with the same number of elements.

The calling sequence may be followed by a table which lists the different variants of the operation, depending either on the ranks of some of the arguments or on the optional arguments. The scalar values \( \alpha \) and \( \beta \) take the defaults given in the following table:
<table>
<thead>
<tr>
<th>Argument</th>
<th>default value in real case</th>
<th>default value in complex case</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>1.0</td>
<td>(1.0,0.0)</td>
</tr>
<tr>
<td>beta</td>
<td>0.0 OR 1.0</td>
<td>(0.0,0.0) OR (1.0,0.0)</td>
</tr>
</tbody>
</table>

Procedures that contain the optional scalar $\text{beta}$ state the default value for $\text{beta}$ only if it is 1.0 or (1.0,0.0), otherwise the default is assumed to be 0.0 or (0.0,0.0).

The following table shows the notation that is used for the values of optional arguments (since $\text{alpha}$ and $\text{beta}$ are also optional, for example):

<table>
<thead>
<tr>
<th>Dummy argument</th>
<th>Notation in table</th>
<th>Named constant</th>
<th>Default value</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>1-norm</td>
<td>$\text{blas_one_norm}$</td>
<td>$\text{blas_one_norm}$</td>
</tr>
<tr>
<td></td>
<td>1R-norm</td>
<td>$\text{blas_real_one_norm}$</td>
<td>$\text{blas_one_norm}$</td>
</tr>
<tr>
<td></td>
<td>2-norm</td>
<td>$\text{blas_two_norm}$</td>
<td>$\text{blas_two_norm}$</td>
</tr>
<tr>
<td></td>
<td>Frobenius-norm</td>
<td>$\text{blas_frobenius_norm}$</td>
<td>$\text{blas_frobenius_norm}$</td>
</tr>
<tr>
<td></td>
<td>inf-norm</td>
<td>$\text{blas_inf_norm}$</td>
<td>$\text{blas_inf_norm}$</td>
</tr>
<tr>
<td></td>
<td>real-inf-norm</td>
<td>$\text{blas_real_inf_norm}$</td>
<td>$\text{blas_real_inf_norm}$</td>
</tr>
<tr>
<td></td>
<td>max-norm</td>
<td>$\text{blas_max_norm}$</td>
<td>$\text{blas_max_norm}$</td>
</tr>
<tr>
<td></td>
<td>real-max-norm</td>
<td>$\text{blas_real_max_norm}$</td>
<td>$\text{blas_real_max_norm}$</td>
</tr>
<tr>
<td>sort</td>
<td>sort in decreasing order</td>
<td>$\text{blas_decreasing_order}$</td>
<td>$\text{blas_increasing_order}$</td>
</tr>
<tr>
<td></td>
<td>sort in increasing order</td>
<td>$\text{blas_increasing_order}$</td>
<td>$\text{blas_increasing_order}$</td>
</tr>
<tr>
<td>side</td>
<td>L</td>
<td>$\text{blas_left_side}$</td>
<td>$\text{blas_left}$</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>$\text{blas_right_side}$</td>
<td>$\text{blas_right}$</td>
</tr>
<tr>
<td>uplo</td>
<td>U</td>
<td>$\text{blas_upper}$</td>
<td>$\text{blas_upper}$</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>$\text{blas_lower}$</td>
<td>$\text{blas_lower}$</td>
</tr>
<tr>
<td>trans</td>
<td>N</td>
<td>$\text{blas_no_trans}$</td>
<td>$\text{blas_no_trans}$</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>$\text{blas_trans}$</td>
<td>$\text{blas_trans}$</td>
</tr>
<tr>
<td></td>
<td>H</td>
<td>$\text{blas_conj_trans}$</td>
<td>$\text{blas_conj_trans}$</td>
</tr>
<tr>
<td>conj</td>
<td></td>
<td>$\text{blas_no_conj}$</td>
<td>$\text{blas_no_conj}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\text{blas_conj}$</td>
<td>$\text{blas_conj}$</td>
</tr>
<tr>
<td>diag</td>
<td>N</td>
<td>$\text{blas_non_unit_diag}$</td>
<td>$\text{blas_non_unit_diag}$</td>
</tr>
<tr>
<td></td>
<td>U</td>
<td>$\text{blas_unit_diag}$</td>
<td>$\text{blas_unit_diag}$</td>
</tr>
<tr>
<td>jrot</td>
<td>inner rotation</td>
<td>$\text{blas_jrot_inner}$</td>
<td>$\text{blas_jrot_inner}$</td>
</tr>
<tr>
<td></td>
<td>outer rotation</td>
<td>$\text{blas_jrot_outer}$</td>
<td>$\text{blas_jrot_outer}$</td>
</tr>
<tr>
<td></td>
<td>sorted rotation</td>
<td>$\text{blas_jrot_sorted}$</td>
<td>$\text{blas_jrot_sorted}$</td>
</tr>
</tbody>
</table>

Table 2.9: Default values of Operator Arguments

### 2.4.6 Error Handling

The Fortran 95 interface must supply an error-handling routine $\text{blas\_error}$. The API for this error-handling routine is defined in section 1.8. By default, this routine will print an error message and stop execution. The user may modify the action performed by the error-handling routine, and this modification must be documented.

The following values of arguments are invalid and will be flagged by the error-handling routine:

- Any value of the operator arguments whose meaning is not specified in the language-dependent section is invalid;
Routine-specific error conditions are listed in the respective language bindings.

2.5 Interface Issues for Fortran 77

Unless explicitly stated, the Fortran 77 binding is consistent with ANSI standard Fortran 77. There are several points where this standard diverges from the ANSI Fortran 77 standard. In particular:

- Subroutine names are not limited to six significant characters.
- Subroutine names contain an underscore.
- Subroutines may use the INCLUDE statement for include files.

Section 2.5.2 discusses the indexing of vector and matrix operands. Section A.5 defines the operator arguments, section 2.5.3 defines array arguments, and section 2.2 lists the matrix storage schemes that are supported. Section 2.5.5 details the format of the language binding, and section 2.5.6 discusses error handling.

2.5.1 Fortran 77 Include File

Refer to Appendix A.5 for details of the Fortran 77 include file `blas_namedconstants.h`.

2.5.2 Indexing

The Fortran 77 interface returns indices in the range $1 \leq I \leq N$ (where $N$ is the number of entries in the dimension in question, and $I$ is the index). This allows functions returning indices to be directly used to index standard arrays.

Likewise, for routines returning an index within a vector or matrix operand, this reference point is indexed starting at one.

2.5.3 Array Arguments

Vector arguments are permitted to have a storage spacing between elements. This spacing is specified by an increment argument. For example, suppose a vector $x$ having components $x_i$, $i = 1, \ldots, N$, is stored in an array $X()$ with increment argument $INCX$. If $INCX > 0$ then $x_i$ is stored in $X(1 + (i - 1) \cdot INCX)$. If $INCX < 0$ then $x_i$ is stored in $X(1 + (N - i) \cdot |INCX|)$. This method of indexing when $INCX < 0$ avoids negative indices in the array $X()$ and thus permits the subprograms to be written in Fortran 77. $INCX = 0$ is an illegal value.

Each two-dimensional array argument is immediately followed in the argument list by its leading dimension, whose name has the form `LD<array-name>`. If a two-dimensional array $A$ of dimension $(LDA,N)$ holds an $m$-by-$n$ matrix $A$, then $A(i,j)$ holds $a_{ij}$ for $i = 1, \ldots, m$ and $j = 1, \ldots, n$ ($LDA$ must be at least $m$). See Section 2.2 for more about storage of matrices.

Note that array arguments are usually declared in the software as assumed-size arrays (last dimension *), for example:

```
REAL A ( LDA, * )
```

although the documentation gives the dimensions as $(LDA,N)$. The latter form is more informative since it specifies the required minimum value of the last dimension. However an assumed-size array declaration has been used in the software in order to overcome some limitations in the Fortran 77 standard. In particular it allows the routine to be called when the relevant dimension ($N$, in this
2.5. INTERFACE ISSUES FOR FORTRAN 77

case) is zero. However actual array dimensions in the calling program must be at least 1 (LDA in
this example).

2.5.4 Matrix Storage Schemes

The matrix storage schemes for the Fortran 77 interfaces are as described in section 2.2. Only
column-major storage is permitted, and all two-dimensional arrays are assumed-size arrays.

2.5.5 Format of the Fortran 77 bindings

Each interface is summarized in the form of a SUBROUTINE statement (or a FUNCTION statement).
The declarations of the arguments are listed in alphabetical order. For example,

```
SUBROUTINE BLAS_xAXPY( N, ALPHA, X, INCX, BETA, Y, INCY )
  INTEGER      INCX, INCY, N
  <type>      ALPHA, BETA
  <type>      X( * ), Y( * )
```

Floating point variables are denoted by the keyword <type> which designates the data type for
the operand (REAL, DOUBLE PRECISION, COMPLEX, or COMPLEX*16). This data type will agree with
the x letter in the naming convention of the routine. In some routines, however, not all floating
point variables will be of the same data type. If this is the case, then a variable may be denoted by
the keyword <ctype> to restrict the data type to COMPLEX or COMPLEX*16, or <rtype> to restrict
the data type to REAL or DOUBLE PRECISION.

The language binding will be followed by any restrictions dictated for this interface.

2.5.6 Error Handling

The Fortran 77 interface supplies an error-handling routine BLAS_ERROR, as defined in section 1.8.
By default, this routine will print an error message and stop execution. The user may modify the
action performed by the error-handling routine, and this modification must be documented.

The following values of arguments are invalid and will be flagged by the error-handling routine:

- Any value of the operator arguments whose meaning is not specified in the language-dependent
  section is invalid;
- incw=0 or incx=0 or incy=0 or incz=0;
- lda, ldb, ldc, or ldt < 1;
- lda < m if the matrix is an $m \times n$ general matrix and trans = blas_no_trans;
- lda < n if the matrix is an $m \times n$ general matrix and trans = blas_trans;
- lda < n if the matrix is an $n \times n$ square, symmetric, or triangular matrix;
- lda < kl + ku + 1, if the matrix is an $m \times n$ general band matrix;
- lda < k+1, if the matrix is an $n \times n$ symmetric or triangular band matrix with k super- or
  subdiagonals;

Routine-specific error conditions are listed in the respective language bindings.
2.6 Interface Issues for C

The interface is expressed in terms of ANSI/ISO C. Most platforms provide ANSI/ISO C compilers, and if this is not the case, free ANSI/ISO C compilers are available (e.g., gcc).

Section 2.6.2 discusses the indexing of vector and matrix operands. Section A.6 defines the operator arguments, section 2.6.3 discusses the handling of complex data types, section 2.6.4 defines return values of complex functions, and section 2.6.5 provides the rule for argument aliasing. Section 2.6.6 defines array arguments, and section 2.6.7 lists the matrix storage schemes that are supported. Section 2.6.8 details the format of the language binding, and section 2.6.9 discusses error handling.

2.6.1 C Include File

The C interface to the BLAS has a standard include file, called blas dense.h, which minimally contains the values of the enumerated types and ANSI/ISO C prototypes for all BLAS routines. Refer to Appendix A.6 for details of the C include files pertaining to Chapters 2–4.

Advice to implementors. Note that the vendor is not constrained to using precisely this include file; only the enumerated type definitions are fully specified. The implementor is free to make any other changes which are not apparent to the user. For instance, all matrix dimensions might be accepted as size_t instead of int, or the implementor might choose to make some routines in-line. (End of advice to implementors.)

2.6.2 Indexing

The C interface returns indices in the range $0 \leq I \leq N - 1$ (where $N$ is the number of entries in the dimension in question, and $I$ is the index). This allows functions returning indices to be directly used to index standard arrays.

Likewise, for routines returning an index within a vector or matrix operand, this reference point is indexed starting at zero.

2.6.3 Handling of complex data types

All complex arguments are accepted as void *. A complex element consists of two consecutive memory locations of the underlying data type (i.e., float or double), where the first location contains the real component, and the second contains the imaginary component.

An ISO/IEC committee (WG14/X3J11) [38] is presently working on an extension to ANSI/ISO C which defines complex data types. This extension is one of several additions to the C language, commonly referred to as the C9X standard. The definition of a complex element is the same as given above, and so the handling of complex types by this interface will not need to be changed when ANSI/ISO C standard is extended.

2.6.4 Return values of complex functions

BLAS routines which return complex values in Fortran 77 are instead recast as subroutines in the C interface, with the return value being an output parameter added to the end of the argument list. This allows the output parameter to be accepted as a void pointer, as discussed above.
2.6.5 Aliasing of arguments

Unless specified otherwise, only input-only arguments (specified with the const qualifier), may be legally aliased on a call to the C interface to the BLAS.

2.6.6 Array arguments

Arrays are constrained to being contiguous in memory. They are accepted as pointers, not as arrays of pointers. Note that this means that two-dimensional array arguments in C are not permitted.

All BLAS routines which take one or more two dimensional arrays as arguments receive one extra parameter as their first argument. This argument is an enumerated type (see Appendix A). If this parameter is set to blas_rowmajor, it is assumed that elements within a row of the array(s) are contiguous in memory, while elements within array columns are separated by a constant stride given in the stride parameter (this parameter corresponds to the leading dimension [e.g. LDA] in the Fortran 77 interface).

If the order is given as blas_colmajor, elements within array columns are assumed to be contiguous, with elements within array rows separated by stride memory elements.

Note that there is only one blas_order_type parameter to a given routine: all array operands are required to use the same ordering.

2.6.7 Matrix Storage Schemes

The matrix storage schemes for the C interfaces are as described in section 2.2. Column-major storage and row-major storage in a contiguous array are permitted.

2.6.8 Format of the C bindings

Each routine is summarized in the form of an ANSI/ISO C prototype. For example:

```c
void BLAS_xaxpby( int n, SCALAR_IN alpha, const ARRAY x, int incx,
                 SCALAR_IN beta, ARRAY y, int incy );
```

Floating point variables are denoted by the keywords SCALAR and ARRAY to denote scalar arguments and array arguments respectively.

<table>
<thead>
<tr>
<th>SCALAR_IN</th>
<th>ARRAY or SCALAR_INOUT</th>
<th>operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>float or double</td>
<td>float * or double *</td>
<td>real arithmetic</td>
</tr>
<tr>
<td>const void *</td>
<td>void *</td>
<td>complex arithmetic</td>
</tr>
</tbody>
</table>

This data type will agree with the x letter in the naming convention of the routine. In some routines, however, not all floating point variables will be of the same data type. If this is the case, then a variable may be denoted by the keyword rscalar_inout, cscalar_inout, rarray, or carray, to restrict the data type to real or complex arithmetic, respectively.

The language binding will be followed by any restrictions dictated for this interface.

2.6.9 Error Handling

The C interface must supply an error-handling routine BLAS_error. This error-handling routine will accept as input a character string, specifying the name of the routine where the error occurred.
CHAPTER 2. DENSE AND BANDED BLAS

By default, this routine will print an error message and stop execution. The user may modify the action performed by the error-handling routine, and this modification must be documented.

The following values of arguments are invalid and will be flagged by the error-handling routine:

- Any value of the operator arguments whose meaning is not specified in the language-dependent section is invalid;
- incw=0 or incx=0 or incy=0 or incz=0;
- lda, ldb, ldc, or ldt < 1;
- lda < m if the matrix is an \( m \times n \) general matrix;
- lda < n if the matrix is an \( m \times n \) square, symmetric, or triangular matrix;
- lda < kl + ku + 1, if the matrix is an \( m \times n \) general band matrix;
- lda < k+1, if the matrix is an \( n \times n \) symmetric or triangular band matrix with k super- or subdiagonals;

Routine-specific error conditions are listed in the respective language bindings.

2.7 Numerical Accuracy and Environmental Enquiry

With a few exceptions that are explicitly described below, no particular computational order is mandated by the function specifications. In other words, any algorithm that produces results “close enough” to the usual algorithms presented in a standard book on matrix computations [33, 19, 35] is acceptable. For example, Strassen’s algorithm may be used for matrix multiplication, even though it can be significantly less accurate than conventional matrix multiplication for some pairs of matrices [35]. Also, matrix multiplication may be implemented either as \( C = (\alpha \cdot A) \cdot B + (\beta \cdot C) \) or \( C = \alpha \cdot (A \cdot B) + (\beta \cdot C) \) or \( C = A \cdot (\alpha \cdot B) + (\beta \cdot C) \), whichever is convenient.

To use the error bounds in [33, 19, 35] and elsewhere, certain machine parameters are needed to describe the accuracy of the arithmetic.

These are described in detail in section 1.6, and returned by function xFINFO. Its calling sequence in C or Fortran 77 is

\[
\text{result} = \text{xFINFO}(\text{CMACH})
\]

where x=S for single precision and x=D for double precision. In Fortran 95, its calling sequence is

\[
\text{result} = \text{FPINFO}(\text{CMACH}, \text{float})
\]

where the “kind” of float (single or double) is used to determine the kind of the result. The argument CMACH can take on the following named constant values (the exact representations are language dependent, with CMACH available as a derived type in Fortran 95, named integer constants in Fortran 77, and an enumerated type in C). The named constant values are defined in sections A.4, A.5, and A.6. CMACH has the analogous meaning (see footnote 4 in section 1.6 for a discussion) as the like-named character argument of the LAPACK auxiliary routine xLAMCH:
### 2.8. LANGUAGE BINDINGS

<table>
<thead>
<tr>
<th>Value of CMACH</th>
<th>Name of floating point parameter (see Table 1.9 in section 1.6 for details)</th>
</tr>
</thead>
<tbody>
<tr>
<td>blas_base</td>
<td>BASE</td>
</tr>
<tr>
<td>blas_t</td>
<td>T</td>
</tr>
<tr>
<td>blas_rnd</td>
<td>RND</td>
</tr>
<tr>
<td>blas_ieee</td>
<td>IEEE</td>
</tr>
<tr>
<td>blas_emin</td>
<td>EMIN</td>
</tr>
<tr>
<td>blas_emax</td>
<td>EMAX</td>
</tr>
<tr>
<td>blas_eps</td>
<td>EPS</td>
</tr>
<tr>
<td>blas_prec</td>
<td>PREC</td>
</tr>
<tr>
<td>blas_underflow</td>
<td>UN</td>
</tr>
<tr>
<td>blas_overflow</td>
<td>OV</td>
</tr>
<tr>
<td>blas_sfmin</td>
<td>SFMIN</td>
</tr>
</tbody>
</table>

Here are the exceptional routines where we ask for particularly careful implementations to avoid unnecessary over/underflows, that could make the output unnecessarily inaccurate or unreliable. The details of each routine are described with the language dependent calling sequences. Model implementations that avoid unnecessary over/underflows are based on corresponding LAPACK auxiliary routines, NAG routines, or cited reports.

1. Reduction Operations (Section 2.8.2)
   - NORM (Vector norms)
   - SUMSQ (Sum of squares)

2. Generate Transformations (Section 2.8.3)
   - GEN_GROT (Generate Givens rotation)
   - GEN_JROT (Generate Jacobi rotation)
   - GEN_HOUSE (Generate Householder transform)

3. Vector Operations (Section 2.8.4)
   - RSCALE (Reciprocal scale)

4. Matrix Operations (Section 2.8.7)
   - \{GE,GB,SY,HE,SB,SP,HP,TR,TB,TP\}._NORM (Matrix norms)

### 2.8 Language Bindings

Each specification of a routine will correspond to an operation outlined in the functionality tables. Operations are organized analogous to the order in which they are presented in the functionality tables. The specification will have the form:

**NAME** *(multi-word description of operation) < mathematical representation>*

*Optional brief textual description of the functionality including any restrictions that apply to all language bindings.*
• Fortran 95 binding
• Fortran 77 binding
• C binding

2.8.1 Overview

• Reduction Operations (section 2.8.2)
  – DOT (Dot product)
  – NORM (Vector norms)
  – SUM (Sum)
  – MIN_VAL (Min value & location)
  – AMIN_VAL (Min absolute value & location)
  – MAX_VAL (Max value & location)
  – AMAX_VAL (Max absolute value & location)
  – SUMSQ (Sum of squares)

• Generate Transformations (section 2.8.3)
  – GEN_GROT (Generate Givens rotation)
  – GEN_JROT (Generate Jacobi rotation)
  – GEN HOUSE (Generate Householder transform)

• Vector Operations (section 2.8.4)
  – RSSCALE (Reciprocal Scale)
  – AXPBY (Scaled vector accumulation)
  – WAXPBY (Scaled vector addition)
  – AXPYD_DOT (Combined AXPY and DOT)
  – APPLY_GROT (Apply plane rotation)

• Data Movement with Vectors (section 2.8.5)
  – COPY (Vector copy)
  – SWAP (Swap)
  – SORT (Sort vector)
  – SORTV (Sort vector & return index vector)
  – PERMUTE (Permute vector)

• Matrix-Vector Operations (section 2.8.6)
  – {GE,GB}MV (Matrix vector product)
  – {SY,SB,SP}MV (Symmetric matrix vector product)
  – {HE,HB,HP}MV (Hermitian matrix vector product)
2.8. LANGUAGE BINDINGS

- \{TR, TB, TP\}MV (Triangular matrix vector product)
- GE_SUM_MV (Summed matrix vector multiplies)
- GEMVT (Combined matrix vector product)
- TRMVT (Combined triangular matrix vector product)
- GEMVER (Combined matrix vector product with a rank 2 update)
- \{TR, TB, TP\}SV (Triangular solve)
- GER (Rank one update)
- \{SY, SP\}R (Symmetric rank one update)
- \{HE, HP\}R (Hermitian rank one update)
- \{SY, SP\}R2 (Symmetric rank two update)
- \{HE, HP\}R2 (Hermitian rank two update)

- Matrix Operations (section 2.8.7)
  - \{GE, GB, SY, HE, SB, HB, SP, HP, TR, TB, TP\}_NORM (Matrix norms)
  - \{GE, GB\}_DIAG_SCALE (Diagonal scaling)
  - \{GE, GB\}_LSCALE (Two-sided diagonal scaling)
  - \{SY, SB, SP\}_LSCALE (Two-sided diagonal scaling of a symmetric matrix)
  - \{HE, HB, HP\}_LSCALE (Two-sided diagonal scaling of a Hermitian matrix)
  - \{GE, GB\}_DIAG_SCALE_ACC (Diagonal scaling and accumulation)
  - \{GE, GB, SY, SB, SP, TR, TB, TP\}_ACC (Matrix accumulation and scale)
  - \{GE, GB, SY, SB, SP, TR, TB, TP\}_ADD (Matrix add and scale)

- Matrix-Matrix Operations (section 2.8.8)
  - GEMM (General Matrix Matrix product)
  - SYMM (Symmetric matrix matrix product)
  - HEMM (Hermitian matrix matrix product)
  - TRMM (Triangular matrix matrix multiply)
  - TRSM (Triangular solve)
  - SYRK (Symmetric rank-k update)
  - HERK (Hermitian rank-k update)
  - SY_TRIDIAK_RK (Symmetric rank-k update with tridiagonal matrix)
  - HE_TRIDIAK_RK (Hermitian rank-k update with tridiagonal matrix)
  - SYR2K (Symmetric rank-2k update)
  - HER2K (Hermitian rank-2k update)
  - SY_TRIDIAK_R2K (Symmetric rank-2k update with tridiagonal matrix)
  - HE_TRIDIAK_R2K (Hermitian rank-2k update with tridiagonal matrix)

- Data Movement with Matrices (section 2.8.9)
  - \{GE, GB, SY, SB, SP, TR, TB, TP\}_COPY (Matrix copy)
\[
\begin{align*}
\{\text{HE},{\textsc{hb}},{\textsc{hp}}\}_{\text{COPY}} & \text{ (Matrix copy)} \\
\{\text{ge}\}_{\text{TRANS}} & \text{ (Matrix transposition)} \\
\{\text{ge}\}_{\text{PERMUTE}} & \text{ (Permute matrix)} \\
\end{align*}
\]

- Environmental Enquiry (section 2.8.10)
- FPINFO (Environmental enquiry)

### 2.8.2 Reduction Operations

**DOT** (Dot Product)

\[
x, y \in \mathbb{R}^n, r \leftarrow \beta r + \alpha x^T y = \beta r + \alpha \sum_{i=0}^{n-1} x_i y_i
\]

\[
x, y \in \mathbb{C}^n, r \leftarrow \beta r + \alpha x^T y = \beta r + \alpha \sum_{i=0}^{n-1} x_i y_i \quad \text{or} \quad r \leftarrow \beta r + \alpha x^H y = \beta r + \alpha \sum_{i=0}^{n-1} \bar{x}_i \bar{y}_i
\]

The routine **DOT** adds the scaled dot product of two vectors \(x\) and \(y\) into a scaled scalar \(r\). The routine returns immediately if \(n\) is less than zero, or, if beta is equal to one and either alpha or \(n\) is equal to zero. If alpha is equal to zero then \(x\) and \(y\) are not read. Similarly, if beta is equal to zero, \(r\) is not read. As described in section 2.5.3, the value \(\text{incx}\) or \(\text{incy}\) less than zero is permitted. However, if \(\text{incx}\) or \(\text{incy}\) is equal to zero, an error flag is set and passed to the error handler.

When \(x\) and \(y\) are complex vectors, the vector components \(x_i\) are used un conjugated or conjugated as specified by the operator argument \(\text{conj}\). If \(x\) and \(y\) are real vectors, the operator argument \(\text{conj}\) has no effect.

- Fortran 95 binding:

```fortran
SUBROUTINE dot( x, y, r [, conj [, alpha [, beta] ] ] )
  <type>(<wp>), INTENT(IN) :: x(:,), y(:)
  <type>(<wp>), INTENT(INOUT) :: r
  TYPE (blas_conj_type), INTENT(IN), OPTIONAL :: conj
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
  x and y have shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xDOT( CONJ, N, ALPHA, X, INCX, BETA, Y, INCY, R )
INTEGER CONJ, INCX, INCY, N
<type> ALPHA, BETA, R
<type> X(*), Y(*)
```

- C binding:

```c
void BLAS_xdot( enum blas_conj_type conj, int n, SCALAR_IN alpha,
               const ARRAY x, int incx, SCALAR_IN beta, const ARRAY y,
               int incy, SCALAR_INOUT r );
```
2.8. LANGUAGE BINDINGS

NORM (Vector norms) \( r \leftarrow ||x||_1, ||x||_R, ||x||_2, ||x||_\infty, \text{ or } ||x||_{\infty R} \)

The routine NORM computes the \( ||\cdot||_1, ||\cdot||_R, ||\cdot||_2, ||\cdot||_\infty, \text{ or } ||\cdot||_{\infty R} \) of a vector \( x \) depending on the value passed as the norm operator argument.

If \( \text{norm} = \text{blas_frobenius_norm} \), an error flag is not raised, and the two-norm is returned to the user. If \( n \) is less than or equal to zero, this routine returns immediately with the output scalar \( r \) set to zero. The resulting scalar \( r \) is always real and its value is as defined in section 2.1.1. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:
  
  ```fortran
  REAL(<wp>) FUNCTION norm( x [, norm] )
  <type>(<wp>), INTENT (IN) :: x(:)
  TYPE (blas_norm_type), INTENT (IN), OPTIONAL :: norm
  where
  x has shape (n)
  ```

- Fortran 77 binding:

  ```fortran
  <type> FUNCTION BLAS_xNORM( NORM, N, X, INCX )
  INTEGER INCX, N, NORM
  <type> X( * )
  ```

- C binding:

  ```c
  void BLAS_xnorm( enum blas_norm_type norm, int n, const ARRAY x,
  int incx, RSCALAR_INOUT r );
  ```

SUM (Sum) \( r \leftarrow \sum_{i=0}^{n-1} x_i \)

The routine SUM computes the sum of the entries of a vector \( x \). If \( n \) is less than or equal to zero, this routine returns immediately with the output scalar \( r \) set to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  <type>(<wp>) FUNCTION sum( x )
  <type>(<wp>), INTENT (IN) :: x(:)
  where
  x has shape (n)
  ```

  This is the same as the Fortran 95 intrinsic function SUM.

- Fortran 77 binding:
\[<\text{type}>\text{ FUNCTION BLAS}_x\text{SUM( N, X, INCX )}\]
\[
\text{INTEGER INCX, N}\]
\[
<\text{type}>\ x( \ast )\]

- C binding:

\[\text{void BLAS}_x\text{sum( int n, const ARRAY x, int incx, SCALAR_INOUT sum );}\]

**MIN\_VAL** (Min value & location) \( k, x_k \) such that \( k = \arg \min_{0 \leq i < n} x_i \)

The routine **MIN\_VAL** finds the smallest component of a real vector \( x \) and determines the smallest offset or index \( k \) such that \( x_k = \min_{0 \leq i < n} x_i \). This value \( x_k \) is returned by the routine and denoted by \( \arg \min_{0 \leq i < n} x_i \) below. When the value of the \( n \) argument is less than or equal to zero, the routine should initialize the output scalars \( k \) to the largest invalid index or offset value (negative one or zero) and \( r \) to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

*Advice to users.* The routine **MIN\_VAL** strictly operates on real vectors. This routine is not defined for complex vectors. (*End of advice to users.*)

- Fortran 95 binding:

\[
\text{SUBROUTINE min_val( x, k, r )}\]
\[
\text{REAL(<wp>), INTENT (IN) :: x(:)}\]
\[
\text{INTEGER, INTENT (OUT) :: k}\]
\[
\text{REAL(<wp>), INTENT (OUT) :: r}\]
\[
\text{where}\]
\[
\text{x has shape (n)}\]

- Fortran 77 binding:

\[
\text{SUBROUTINE BLAS}_x\text{MIN\_VAL( N, X, INCX, K, R )}\]
\[
\text{INTEGER INCX, K, N}\]
\[
<\text{rtype}>\ R\]
\[
<\text{rtype}>\ x( \ast )\]

- C binding:

\[\text{void BLAS}_x\text{min_val( int n, const RARRAY x, int incx, int k,}\]
\[
\text{RSCALAR_INOUT r );}\]

**AMIN\_VAL** (Min absolute value & location) \( k, x_k \) such that \( k = \arg \min_{0 \leq i < n} (|Re(x_i)| + |Im(x_i)|) \)

The routine **AMIN\_VAL** finds the offset or index of the smallest component of a vector \( x \) and also returns the smallest component of the vector \( x \) with respect to the absolute value. When the value of the \( n \) argument is less than or equal to zero, the routine should initialize the output scalars \( k \) to the largest invalid index or offset value (negative one or zero) and \( r \) to zero. The resulting scalar \( r \) is always real. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.
• Fortran 95 binding:

```fortran
SUBROUTINE amin_val(x, k, r)
  <type>(<wp>), INTENT(IN) :: x(:)
  INTEGER, INTENT(OUT) :: k
  REAL(<wp>), INTENT(OUT) :: r
where
  x has shape (n)
```

A Fortran 95 interface was defined for this routine since it would have been too expensive using Fortran 95 intrinsics.

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_amin_val(N, X, INCX, K, R)
  INTEGER INCX, K, N
  <rtype> R
  <type> X(*)
```

• C binding:

```c
void BLAS_amin_val( int n, const ARRAY x, int incx, int k,
                    RSCALAR_INOUT r );
```

MAX_VAL (Max value & location) \[ k, x_k \] such that \[ k = \arg \max_{0 \leq i < n} x_i \]

The routine MAX_VAL finds the largest component of a real vector \( x \) and determines the smallest offset or index \( k \) such that \( x_k = \max_{0 \leq i < n} x_i \). This value \( x_k \) is returned by the routine and denoted by \( k \) below. When the value of the \( n \) argument is less than or equal to zero, the routine should initialize the output scalars \( k \) to the largest invalid index or offset value (negative one or zero) and \( r \) to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

Advice to users. The routine MAX_VAL strictly operates on real vectors. This routine is not defined for complex vectors. (End of advice to users.)

• Fortran 95 binding:

```fortran
SUBROUTINE max_val(x, k, r)
  REAL(<wp>), INTENT(IN) :: x(:)
  INTEGER, INTENT(OUT) :: k
  REAL(<wp>), INTENT(OUT) :: r
where
  x has shape (n)
```

• Fortran 77 binding:
SUBROUTINE BLAS_xMAX_VAL( N, X, INCX, K, R )
INTEGER       INCX, K, N
<rtype>        R
<rtype>        X( * )

- C binding:

void BLAS_xmax_val( int n, const ARRAY x, int incx, int k,
RSCALAR_INOUT r );

AMAX_VAL (Max absolute value & location)  \( k, x_k \) such that

\[ k = \arg \max_{0 \leq i < n} \left( |Re(x_i)| + |Im(x_i)| \right) \]

The routine AMAX_VAL finds the offset or index of the largest component of a vector \( x \) and also
returns the largest component of the vector \( x \) with respect to the absolute value. When the value
of the \( n \) argument is less than or equal to zero, the routine should initialize the output scalars \( k \) to
the largest invalid index or offset value (negative one or zero) and \( r \) to zero. The resulting scalar \( r \)
is always real. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if
\( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

SUBROUTINE amax_val( x, k, r )
<type>(<wp>), INTENT (IN) :: x(:)
INTEGER, INTENT (OUT) :: k
REAL(<wp>), INTENT (OUT) :: r
where
x has shape (n)

- Fortran 77 binding:

SUBROUTINE BLAS_xAMAX_VAL( N, X, INCX, K, R )
INTEGER       INCX, K, N
<rtype>        R
<type>         X( * )

- C binding:

void BLAS_xmax_val( int n, const ARRAY x, int incx, int k,
RSCALAR_INOUT r );

SUMSQ (Sum of squares)

\[(sd, sq) \leftarrow \sum x_i^2,\]

The routine SUMSQ returns the values \( sd \) and \( sq \) such that

\[ sd^2 \times sq = scale^2 \times \text{sumsq} + \sum_{i=0}^{n-1} (Re(x_i)^2 + Im(x_i)^2), \]
The value of \( \text{sumsq} \) is assumed to be at least unity and the value of \( \text{ssq} \) will then satisfy \( 1.0 \leq \text{ssq} \leq (\text{sumsq} + n) \) when \( x \) is a real vector and \( 1.0 \leq \text{ssq} \leq (\text{sumsq} + 2n) \) when \( x \) is a complex vector. \( \text{scale} \) is assumed to be non-negative and \( \text{scl} \) returns the value

\[
\text{scl} = \max_{0 \leq i < n} (\text{scale}, \text{abs}(\text{Re}(x_i)), \text{abs}(\text{Im}(x_i))).
\]

\( \text{scale} \) and \( \text{sumsq} \) must be supplied on entry in \( \text{scl} \) and \( \text{ssq} \) respectively. \( \text{scl} \) and \( \text{ssq} \) are overwritten by \( \text{scl} \) and \( \text{ssq} \) respectively. The arguments \( \text{scl} \) and \( \text{ssq} \) are therefore always real scalars. If \( \text{scl} \) is less than zero or \( \text{ssq} \) is less than one, an error flag is set and passed to the error handler. If \( n \) is less than or equal to zero, this routine returns immediately with \( \text{scl} \) and \( \text{ssq} \) unchanged. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

\textit{Advice to implementors.} High-quality implementations of this routine \texttt{SUMSQ} should be accurate. The subroutine \texttt{SLASSQ} of the LAPACK [6] software library is an example of such an accurate implementation. High-quality implementations should document the accuracy of the algorithms used in this routine so as to alleviate the portability problems this represents. (\textit{End of advice to implementors.})

- **Fortran 95 binding:**

  ```fortran
  SUBROUTINE sumsq( x, ssq, scl )
  <type>(<wp>), INTENT (IN) :: x(:)
  REAL(<wp>), INTENT (INOUT) :: ssq, scl
  where
  x has shape (n)
  ```

- **Fortran 77 binding:**

  ```fortran
  SUBROUTINE BLAS_xSUMSQ( N, X, INCX, SSQ, SCL )
  INTEGER INCX, N
  <rtype> SCL, SSQ
  <type> X( * )
  ```

- **C binding:**

  ```c
  void BLAS_xsumsq( int n, const ARRAY x, int incx, RSCALAR_INOUT ssq,
                   RSCALAR_INOUT scl );
  ```

\textbf{2.8.3 Generate Transformations}

\texttt{GEN_GROT} (Generate Givens rotation) \( (c, s, r) \leftarrow \text{rot}(a, b) \)

The routine \texttt{GEN_GROT} constructs a Givens plane rotation so that

\[
\begin{pmatrix}
  c & s \\
  -\bar{s} & c
\end{pmatrix}
\begin{pmatrix}
  a \\
  b
\end{pmatrix}
= \begin{pmatrix}
  r \\
  0
\end{pmatrix},
\]
where $c$ is always a real scalar and $c^2 + |s|^2$ is equal to one. The scalars $a$ and $b$ are unchanged on exit. $c$, $s$ and $r$ are defined precisely as follows, where we use the function

$$
\text{sign}(x) = \begin{cases} 
\frac{x}{|x|} & \text{if } x \neq 0 \\
1 & \text{if } x = 0
\end{cases}
$$

Defining Givens rotations:

if $b = 0$ (includes the case $a = b = 0$)

\begin{align*}
  & c = 1 \\
  & s = 0 \\
  & r = a
\end{align*}

else if $a = 0$ (b must be nonzero)

\begin{align*}
  & c = 0 \\
  & s = \text{sign}(b) \\
  & r = |b|
\end{align*}

else (a and b both nonzero)

\begin{align*}
  & c = \frac{|a|}{\sqrt{|a|^2 + |b|^2}} \\
  & s = \text{sign}(a) \frac{b}{\sqrt{|a|^2 + |b|^2}} \\
  & r = \text{sign}(a) \sqrt{|a|^2 + |b|^2}
\end{align*}

eendif

When $a$ and $b$ are real, $\bar{b}$ may be replaced by $b$.

Advice to implementors. High-quality implementations of this routine GEN_GROT should be accurate. We recommend one of the implementations described in [9]. We note that the above definition of Givens rotations matches the one in the subroutine CLARTG of the LAPACK [6] software library, but differs slightly from the definitions used in LAPACK routines SLARTG, SLARGV and CLARGV. LAPACK routines use these slightly different Givens rotations continue to function correctly [9]. (End of advice to implementors.)

- Fortran 95 binding:

  ```fortran
  SUBROUTINE gen_grot( a, b, c, s, r )
  <type>(<wp>), INTENT(IN) :: a, b
  REAL(<wp>), INTENT(OUT) :: c
  <type>(<wp>), INTENT(OUT) :: s, r
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xGEN_GROT( A, B, C, S, R )
  <rtype>
  <type> C
  <type> A, B, R, S
  ```

- C binding:

  ```c
  void BLAS_xgen_grot( SCALAR_IN a, SCALAR_IN b, RSCALAR_INOUT c,
                      SCALAR_INOUT s, SCALAR_INOUT r );
  ```
2.8. LANGUAGE BINDINGS

GEN_JROT (Generate Jacobi rotation) \[(a, b, c, s) \leftarrow \text{jrot}(x, y, z)\]

The routine GEN_JROT constructs a Jacobi rotation so that

\[
\begin{pmatrix}
  a & 0 \\
  0 & b
\end{pmatrix} =
\begin{pmatrix}
  c & s \\
  -s & c
\end{pmatrix} \cdot
\begin{pmatrix}
  x & y \\
  y & z
\end{pmatrix} \cdot
\begin{pmatrix}
  c & -s \\
  s & c
\end{pmatrix},
\]

If \( \text{JROT} = \text{blas\_inner\_rotation} \), then the rotation is chosen so that \( c \geq \frac{1}{\sqrt{2}} \).
If \( \text{JROT} = \text{blas\_outer\_rotation} \), then the rotation is chosen so that \( 0 \leq c \leq \frac{1}{\sqrt{2}} \).
If \( \text{JROT} = \text{blas\_sorted\_rotation} \), then the rotation is chosen so that \( \text{abs}(a) \geq \text{abs}(b) \).

On entry, the argument \( x \) contains the value \( x \), and on exit it contains the value \( a \). On entry, the argument \( y \) contains the value \( y \). On entry, the argument \( z \) contains the value \( z \), and on exit it contains the value \( b \). The arguments \( x \) and \( z \) are real scalars, and argument \( c \) is always a real scalar and \( c^2 + |s|^2 \) is equal to one.

Advice to implementors. High-quality implementations of this routine GEN_JROT should document the accuracy of the algorithms used in those functions so as to alleviate the portability problems this represents. (See NAG routine F06BEF). (End of advice to implementors.)

- Fortran 95 binding:

```
SUBROUTINE gen_jrot( x, y, z, c, s [, jrot] )
    REAL(<wp>), INTENT (INOUT) :: x, z
    <type>(<wp>), INTENT (IN) :: y
    REAL(<wp>), INTENT (OUT) :: c
    <type>(<wp>), INTENT (OUT) :: s
    TYPE (blas_jrot_type), INTENT(IN), OPTIONAL :: jrot
```

- Fortran 77 binding:

```
SUBROUTINE BLAS_xGEN_JROT( JROT, X, Y, Z, C, S )
    INTEGER JROT
    <rtype> C, X, Z
    <type> S, Y
```

- C binding:

```
void BLAS_xgen_jrot( enum blas_jrot_type jrot, RSCALAR_INOUT x,
                    SCALAR_IN y, RSCALAR_INOUT z, RSCALAR_INOUT c,
                    SCALAR_INOUT s );
```

GEN_HOUSE (Generate Householder transform) \[(\alpha, x, \tau) \leftarrow \text{house}(\alpha, x), \]

The routine GEN_HOUSE generates an elementary reflector \( H \) of order \( n \), such that

\[
H(\begin{array}{c}
\alpha \\
\hline
x
\end{array}) = (\begin{array}{c}
\beta \\
\hline
0
\end{array}) \text{ and } H^HH = I,
\]
where $\alpha$ and $\beta$ are scalars, and $x$ is an $(n - 1)$-element vector. $\beta$ is always a real scalar. $H$ is represented in the form

$$H = I - \tau \left( \begin{array}{c} 1 \\ v \end{array} \right) \left( \begin{array}{c} 1 \\ v^T \end{array} \right),$$

where $\tau$ is a scalar and $v$ is a $(n - 1)$-element vector. $\tau$ is called the Householder scalar and $\left( \begin{array}{c} 1 \\ v \end{array} \right) \left( \begin{array}{c} 1 \\ v^T \end{array} \right)$ the Householder vector. Note that when $x$ is a complex vector, $H$ is not Hermitian. If the elements of $x$ are zero, and $\alpha$ is real, then $\tau$ is equal to zero and $H$ is taken to be the unit matrix. Otherwise, the real part of $\tau$ is greater than or equal to one and less than or equal to two. Moreover, the absolute value of the quantity $\tau - 1$ is less than or equal to one.

On exit, the scalar argument alpha is overwritten with the value of the scalar $\beta$. Similarly, the vector argument $x$ is overwritten with the vector $v$. If $n$ is less than or equal to zero, this function returns immediately with the output scalar tau set to zero. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler.

Advice to implementors. High-quality implementations of this routine GEN_HOUSE should be accurate. The subroutines SLARFG and CLARFG of the LAPACK [6] software library are examples of such an accurate implementation. High-quality implementations should document the accuracy of the algorithms used in those functions so as to alleviate the portability problems this represents. (End of advice to implementors.)

Advice to users. Routines to apply Householder transformations are not provided. The subroutines xORMyy of the LAPACK [6] software library are examples of such implementations. (End of advice to users.)

- Fortran 95 binding:

```fortran
SUBROUTINE gen_house( alpha, x, tau )
    <type>(<wp>), INTENT (INOUT) :: alpha
    <type>(<wp>), INTENT (INOUT) :: x(:)
    <type>(<wp>), INTENT (OUT) :: tau

where
    x has shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xGEN_HOUSE( N, ALPHA, X, INCX, TAU )
    INTEGER        INCX, N
    <type>          ALPHA, TAU
    <type>          X( * )
```

- C binding:

```c
void BLAS_xgen_house( int n, SCALAR_INOUT alpha, ARRAY x, int incx,
                      SCALAR_INOUT tau );
```
2.8. LANGUAGE BINDINGS

2.8.4 Vector Operations

RScale (Reciprocal Scale)

\[ x \leftarrow x / \alpha \]

The routine RScale scales the entries of a vector \( x \) by the real scalar \( 1/\alpha \). The scalar \( \alpha \) is always real and supposed to be nonzero. This should be done without overflow or underflow as long as the final result \( x/\alpha \) does not overflow or underflow. If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero or if \( \text{alpha} \) is equal to zero, an error flag is set and passed to the error handler.

Advice to implementors. High-quality implementations of this routine RScale should be accurate. The subroutine \xRSC\ of the LAPACK [6] software library is an example of such an accurate implementation. High-quality implementations should document the accuracy of the algorithms used in those functions so as to alleviate the portability problems this represents.

(End of advice to implementors.)

- Fortran 95 binding:

```fortran
SUBROUTINE rscale( alpha, x )
    REAL(<wp>), INTENT (IN) :: alpha
    <type>(<wp>), INTENT (INOUT) :: x(:)
    where
    x has shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xRScale( N, ALPHA, X, INCX )
    INTEGER INCX, N
    <rtype> ALPHA
    <type> X( * )
```

- C binding:

```c
void BLAS_xrscale( int n, RSCALAR_IN alpha, ARRAY x, int incx );
```

AXPBY (Scaled vector accumulation)

\[ y \leftarrow \alpha x + \beta y \]

The routine AXPBY scales the vector \( x \) by \( \alpha \) and the vector \( y \) by \( \beta \), adds these two vectors to one another and stores the result in the vector \( y \). If \( n \) is less than or equal to zero, or if \( \alpha \) is equal to zero and \( \beta \) is equal to one, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE axpby( x, y [, alpha] [, beta ] )
    <type>(<wp>), INTENT (IN) :: x(:)
```
### Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xAXPYB( N, ALPHA, X, INCX, BETA, Y, INCY )
INTEGER INCX, INCY, N
<type> ALPHA, BETA
<type> X( * ), Y( * )
```

### C binding:

```c
void BLAS_xaxpyb( int n, SCALAR_IN alpha, const ARRAY x, int incx,
                SCALAR_IN beta, ARRAY y, int incy );
```

### WAXPY (Scaled vector addition)

\[ w \leftarrow \alpha x + \beta y \]

The routine **WAXPY** scales the vector \( x \) by \( \alpha \) and the vector \( y \) by \( \beta \), adds these two vectors to one another and stores the result in the vector \( w \). If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) is equal to zero, an error flag is set and passed to the error handler.

### Fortran 95 binding:

```fortran
SUBROUTINE waxpyb( x, y, w [, alpha] [, beta] )
<type>(<wp>), INTENT (IN) :: x(:), y(:)
<type>(<wp>), INTENT (OUT) :: w(:)
<type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
where
x, y and w have shape (n)
```

The default value for \( \beta \) is 1.0 or (1.0,0.0).

### Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xWAXPYB( N, ALPHA, X, INCX, BETA, Y, INCY, W, INCW )
INTEGER INCW, INCX, INCY, N
<type> ALPHA, BETA
<type> W( * ), X( * ), Y( * )
```

### C binding:

...
void BLAS_xwaxpby( int n, SCALAR_IN alpha, const ARRAY x, int incx, 
    SCALAR_IN beta, const ARRAY y, int incy, ARRAY w, 
    int incw );

AXPY_DOT (Combined AXPY and DOT) \[ \bar{w} \leftarrow w - \alpha w, r \leftarrow \bar{w}^T u \]

The routine combines an axpy and a dot product. \( w \) is decremented by a multiple of \( v \). A dot product is then computed with the decremented \( w \).

Advice to implementors. Note that \( \bar{w} \) may be used to update \( r \) before it is written back to memory. This optimization, which accelerates algorithms like modified Gram-Schmidt orthogonalization, is the justification for AXPY_DOT, which could otherwise be implemented by calls to AXPBY and DOT. (End of advice to implementors.)

If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value incw or incv or incu less than zero is permitted. However, if either incw or incv or incu is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  SUBROUTINE axpy_dot( \( w, v, u, r \) [, \( \alpha \) ] )
  <type>(<wp>), INTENT (IN) :: \( v(:) \), \( u(:) \)
  <type>(<wp>), INTENT (INOUT) :: \( w(:) \)
  <type>(<wp>), INTENT (OUT) :: \( r \)
  <type>(<wp>), INTENT (IN), OPTIONAL :: \( \alpha \)

  where
  \( u, v \) and \( w \) have shape (\( n \))

- Fortran 77 binding:

  SUBROUTINE BLAS_xAXPY_DOT( N, ALPHA, W, INCW, V, INCV, U, INCU, 
  $ 
  INTEGER \( \text{INCW, INCV, INCU, N} \)
  <type> \( \text{ALPHA, R} \)
  <type> \( W( * ), V( * ), U( * ) \)

- C binding:

  void BLAS_xaxpy_dot( int n, SCALAR_IN alpha, ARRAY w, int incw, 
  const ARRAY v, int incv, const ARRAY u, int incu, 
  SCALAR_INOUT r );

APPLY_GROT (Apply plane rotation) \[ ( x \ y ) \leftarrow ( x \ y ) R \]
The routine APPLY_GROT applies a plane rotation to the vectors \( x \) and \( y \). When the vectors \( x \) and \( y \) are real vectors, the scalars \( c \) and \( s \) are real scalars. When the vectors \( x \) and \( y \) are complex vectors, \( c \) is a real scalar and \( s \) is a complex scalar. This routine computes

\[
\forall i \in [0 \ldots n - 1], \left( \begin{array}{c} x_i \\ y_i \end{array} \right) = \left( \begin{array}{cc} c & s \\ -\bar{s} & c \end{array} \right) \left( \begin{array}{c} x_i \\ y_i \end{array} \right).
\]

If \( n \) is less than or equal to zero or if \( c \) is one and \( s \) is zero, the routine APPLY_GROT returns immediately. As described in section 2.5.3, the value of \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE apply_grot( c, s, x, y )
    REAL(<wp>), INTENT (IN) :: c
    <type>(<wp>), INTENT (IN) :: s
    <type>(<wp>), INTENT (INOUT) :: x(:), y(:)
where
    x and y have shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xAPPLY_GROT( N, C, S, X, INCX, Y, INCY )
INTEGER          INCX, INCY, N
<rtype>          C
<rtype>          S
<rtype>          X(*), Y(*)
```

- C binding:

```c
void BLAS_xapply_grot( int n, RSCALAR_IN c, SCALAR_IN s, ARRAY x, int incx,
    ARRAY y, int incy );
```

2.8.5 Data Movement with Vectors

COPY (Vector copy)

\( y \leftarrow x \)

The routine COPY copies the vector \( x \) into the vector \( y \). If \( n \) is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE copy( x, y )
    <type>(<wp>), INTENT (IN) :: x(:)
    <type>(<wp>), INTENT (OUT) :: y(:)
where
    x and y have shape (n)
```
This is similar to the Fortran 95 assignment \( y = x \).

- Fortran 77 binding:
  
  ```fortran
  SUBROUTINE BLAS_xCOPY( N, X, INCX, Y, INCY )
  INTEGER INCY, INCX, N
  <type> X( * ), Y( * )
  ``

- C binding:

  ```c
  void BLAS_xcopy( int n, const ARRAY x, int incx, ARRAY y, int incy );
  ```

---

**SWAP (Swap)** \( y \leftrightarrow x \)

The routine SWAP interchanges the vectors \( x \) and \( y \). If \( n \) is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE swap( x, y )
  <type>(<wp>), INTENT (INOUT) :: x(:,), y(:,)
  where
  x and y have shape (n)
  ``

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xSWAP( N, X, INCX, Y, INCY )
  INTEGER INCX, INCY, N
  <type> X( * ), Y( * )
  ``

- C binding:

  ```c
  void BLAS_xswap( int n, ARRAY x, int incx, ARRAY y, int incy );
  ```

---

**SORT (Sort vector)** \( x \leftarrow \text{sort}(x) \)

The routine SORT sorts the entries of a real vector \( x \) in increasing or decreasing order and overwrites this vector \( x \) with the sorted vector. If \( n \) is less than or equal to zero, the function returns immediately. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

Advice to users. The routine SORT strictly operates on real vectors. This routine is not defined for complex vectors. (End of advice to users.)

Advice to implementors. The subroutine xLASRT of the LAPACK [6] software library is an example of such a routine. (End of advice to implementors.)
• Fortran 95 binding: *Refer to SORTV specification*

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSORT( SORT, N, X, INCX )
INTEGER INCX, N, SORT
<rtype> X(*)
```

• C binding:

```c
void BLAS_xsort( enum blas_sort_type sort, int n, RARRAY x, int incx);
```

SORTV (Sort vector & return index vector) \( (p, x) \leftarrow \text{sort}(x) \)

The routine SORTV sorts the entries of a real vector \( x \) in increasing or decreasing order and overwrites this vector \( x \) with the sorted vector \( x = P \times x \). If \( n \) is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incp} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incp} \) is equal to zero, an error flag is set and passed to the error handler.

The representation of the permutation vector \( p \) is described in section 2.2.6.

*Advice to users.* The routine SORTV strictly operates on real vectors. This routine is not defined for complex vectors. *(End of advice to users.)*

• Fortran 95 binding:

```fortran
SUBROUTINE sortv( x [, sort] [, p] )
REAL(<wp>), INTENT (INOUT) :: x(:)
TYPE (blas_sort_type), INTENT (IN), OPTIONAL :: sort
INTEGER, INTENT (OUT), OPTIONAL :: p(:)
where
x and p have shape (n)
```

The functionality of SORT is covered by SORTV.

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSORTV( SORT, N, X, INCX, P, INCP )
INTEGER INCP, INCX, N, SORT
INTEGER P(*)
<rtype> X(*)
```

• C binding:

```c
void BLAS_xsortv( enum blas_sort_type sort, int n, RARRAY x, int incx, 
                 int *p, int incp);
```
PERMUTE (Permute vector)\[ x \leftarrow P x \]

The routine PERMUTE permutes the entries of a vector \( x \) according to the permutation vector \( p \). If \( n \) is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incp} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incp} \) is equal to zero, an error flag is set and passed to the error handler.

The encoding of the permutation \( P \) in the vector \( p \) follows the same conventions as the ones described above for the routine SORTTV. Refer to section 2.2.6 for complete details.

• Fortran 95 binding:

```
SUBROUTINE permute( x, p )
  <type>(<up>), INTENT (INOUT) :: x(:)
  INTEGER, INTENT (IN) :: p(:)
  where
  x and p have shape (n)
```

• Fortran 77 binding:

```
SUBROUTINE BLAS_xPERMUTE( N, P, INCP, X, INCX )
  INTEGER INCP, INCX, N
  INTEGER P(*)
  <type> X(*)
```

The value of \( \text{INCP} \) may be positive or negative. A negative value of \( \text{INCP} \) applies the permutation in the opposite direction.

• C binding:

```
void BLAS_xpermute( int n, const int *p, int incp, ARRAY x, int incx );
```

The value of \( \text{incp} \) may be positive or negative. A negative value of \( \text{incp} \) applies the permutation in the opposite direction.

2.8.6 Matrix-Vector Operations

In the following section, \( op(X) \) denotes \( X \), or \( X^T \) or \( X^H \) where \( X \) is a matrix.

\{GE,GB\}MV (Matrix vector product)\[ y \leftarrow \alpha op(A)x + \beta y \]

The routines perform a matrix vector multiply \( y \leftarrow \alpha op(A)x + \beta y \) where \( \alpha \) and \( \beta \) are scalars, and \( A \) is a general (or general band) matrix. If \( m \) or \( n \) is less than or equal to zero or if \( \beta \) is equal to one and \( \alpha \) is equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler. For the routine GEMV, if \( \text{lda} \) is less than one, or \( \text{trans} = \text{blas_no_trans} \) and \( \text{lda} \) is less than \( m \), or \( \text{trans} = \text{blas_trans} \) and \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the C bindings of GEMV, if \( \text{order} = \text{blas_rowmajor} \) and if \( \text{llda} \) is less than one or \( \text{llda} \) is less than \( n \), an error flag is set and passed to the
error handler; if order = blas_colmajor and if lda is less than one or lda is less than m, an error flag is set and passed to the error handler. For the routine GBMV, if kl or ku is less than zero, or if lda is less than kl plus ku plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE gbmv( a, m, kl, x, y [, trans] [, alpha] [, beta] )
  <type>(<wp>), INTENT(IN) :: a(:,,:), x(:)
  INTEGER, INTENT(IN) :: m, kl
  <type>(<wp>), INTENT(INOUT) :: y(:)
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  where
    if trans = blas_no_trans then
      x has shape (n)
      y has shape (m)
    else if trans /= blas_no_trans then
      x has shape (m)
      y has shape (n)
    end if
```

The functionality of gemv is covered by gemm.

- Fortran 77 binding:

```fortran
General:
  SUBROUTINE BLAS_xGEMV( TRANS, M, N, ALPHA, A, LDA, X, INCX, BETA,
                          Y, INCY )
  $ General Band:
  SUBROUTINE BLAS_xGBMV( TRANS, M, N, KL, KU, ALPHA, A, LDA, X,
                          INCX, BETA, Y, INCY )
  all:
    INTEGER        INCX, INCY, KL, KU, LDA, M, N, TRANS
    <type>         ALPHA, BETA
    <type>         A( LDA, * ), X( * ), Y( * )
```

- C binding:

```c
General:
void BLAS_xgemv( enum blas_order_type order, enum blas_trans_type trans,
                 int m, int n, SCALAR_IN alpha, const ARRAY a, int lda,
                 const ARRAY x, int incx, SCALAR_IN beta, ARRAY y, int incy );

General Band:
void BLAS_xgbmv( enum blas_order_type order, enum blas_trans_type trans,
                 int m, int n, int kl, int ku, SCALAR_IN alpha, const ARRAY a,
                 int lda, const ARRAY x, int incx, SCALAR_IN beta,
                 ARRAY y, int incy );
```
\{SY,SB,SP\}MV (Symmetric matrix vector product) \hspace{1cm} y \leftarrow \alpha Ax + \beta y \text{ with } A = A^T

The routines multiply a vector \( x \) by a real or complex symmetric matrix \( A \), scales the resulting vector and adds it to the scaled vector operand \( y \). If \( n \) or \( k \) is less than or equal to zero or if \( \beta \) is equal to one and \( \alpha \) is equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler. For the routine SYMV, if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the routine SBMV, if \( \text{lda} \) is less than \( k \) plus one, an error flag is set and passed to the error handler.

- **Fortran 95 binding:**
  
  **Symmetric Band:**
  
  SUBROUTINE sbmv( a, x, y [, uplo] [, alpha] [, beta] )
  
  **Symmetric Packed:**
  
  SUBROUTINE spmv( ap, x, y [, uplo] [, alpha] [, beta] )
  
  all:
  
  \(<\text{type}>\langle<\text{wp}>\rangle, \text{INTENT(}\text{IN}\rangle :: <\text{aa}>, x(\cdot)\rangle \langle<\text{wp}>\rangle, \text{INTENT(}\text{INOUT}\rangle :: y(\cdot)\rangle
  
  \text{TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo}
  
  \langle<\text{wp}>\rangle, \text{INTENT(IN), OPTIONAL :: alpha, beta}
  
  where
  
  \( <\text{aa} > ::= a(\cdot, \cdot) \text{ or ap}(\cdot) \)

  and

  \( \text{SB} \) a has shape \((k+1,n)\)
  
  \( \text{SP} \) ap has shape \((n*(n+1)/2)\)
  
  \( x \) and \( y \) have shape \((n)\)

  \((k=\text{band width})\)

  The functionability of symv is covered by symm.

- **Fortran 77 binding:**
  
  **Symmetric:**
  
  SUBROUTINE BLAS_xSYMV( UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y, INCY )

  **Symmetric Band:**
  
  SUBROUTINE BLAS_xSBMV( UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA, Y, INCY )

  **Symmetric Packed:**
  
  SUBROUTINE BLAS_xSPMV( UPLO, N, ALPHA, AP, X, INCX, BETA, Y, INCY )

  all:

  INTEGER \hspace{.5cm} INCX, INCY, K, LDA, N, UPLO

  \( <\text{type}> \hspace{.5cm} \text{ALPHA, BETA} \)

  \( <\text{type}> \hspace{.5cm} \text{A( LDA, * ) or AP( * ), X( * ), Y( * )} \)

- **C binding:**
Symmetric:
void BLAS_xsymv( enum blas_order_type order, enum blas_uplo_type uplo,
                int n, SCALAR_IN alpha, const ARRAY a, int lda,
                const ARRAY x, int incx, SCALAR_IN beta, ARRAY y, int incy );

Symmetric Band:
void BLAS_xsbmv( enum blas_order_type order, enum blas_uplo_type uplo,
                int n, int k, SCALAR_IN alpha, const ARRAY a, int lda,
                const ARRAY x, int incx, SCALAR_IN beta, ARRAY y, int incy );

Symmetric Packed:
void BLAS_xspmv( enum blas_order_type order, enum blas_uplo_type uplo, int n,
                SCALAR_IN alpha, const ARRAY ap, const ARRAY x, int incx,
                SCALAR_IN beta, ARRAY y, int incy );

\{HE,HB,HP\}MV (Hermitian matrix vector product) \quad y \leftarrow \alpha Ax + \beta y \text{ with } A = A^H

The routines multiply a vector \( x \) by a Hermitian matrix \( A \), scales the resulting vector and adds it to the scaled vector operand \( y \). If \( n \) is less than or equal to zero or if \( \beta \) is equal to one and \( \alpha \) is equal to zero, this routine returns immediately. The imaginary part of the diagonal entries of the matrix operand are supposed to be zero and should not be referenced. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler. For the routine HEMV, if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the routine HBMV, if \( \text{lda} \) is less than \( k \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  Hermitian Band:
  SUBROUTINE hbmv( a, x, y [, uplo] [, alpha] [, beta] )
  Hermitian Packed:
  SUBROUTINE hpmv( ap, x, y [, uplo] [, alpha] [, beta] )
  all:
  COMPLEX(<wp>), INTENT(IN) :: <aa>, x(:)
  COMPLEX(<wp>), INTENT(INOUT) :: y(:)
  TYPE (blav_uplo_type), INTENT(IN), OPTIONAL :: uplo
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  where
  \(<aa> ::= a(:,:); or ap(:)\)
  and
  HB \ a has shape \( (k+1,n) \)
  HP \ ap has shape \( (n*(n+1)/2) \)
  \( x \) and \( y \) have shape \( (n) \)
  \( k=\text{band width} \)

  The functionality of hmv is covered by hemm,

- Fortran 77 binding:
2.8. LANGUAGE BINDINGS

Hermitian:

```
SUBROUTINE BLAS_xHEMV( UPLO, N, ALPHA, A, LDA, X, INCX, BETA, Y,
             $           INCY )
```

Hermitian Band:

```
SUBROUTINE BLAS_xHBMV( UPLO, N, K, ALPHA, A, LDA, X, INCX, BETA,
             $           Y, INCY )
```

Hermitian Packed:

```
SUBROUTINE BLAS_xHPMV( UPLO, N, ALPHA, AP, X, INCX, BETA, Y, INCY )
```

all:

```
INTEGER           INCX, INCY, K, LDA, N, UPLO
<ctype>            ALPHA, BETA
<ctype>            A( LDA, * ) or AP( * ), X( * ), Y( * )
```

- C binding:

```
void BLAS_xhemv( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, CSCALAR_IN alpha, const CARRAY a, int lda,
    const CARRAY x, int incx, CSCALAR_IN beta, CARRAY y,
    int incy );
```

Hermitian Band:

```
void BLAS_xhbmv( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, CSCALAR_IN alpha, const CARRAY a, int lda,
    const CARRAY x, int incx, CSCALAR_IN beta, CARRAY y,
    int incy );
```

Hermitian Packed:

```
void BLAS_xhpmv( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, CSCALAR_IN alpha, const CARRAY ap, const CARRAY x,
    int incx, CSCALAR_IN beta, CARRAY y, int incy );
```

{TR,TB,TP}MV (Triangular matrix vector product)  \( x \leftarrow \alpha T^{}x, x \leftarrow \alpha T^T x \) or \( x \leftarrow \alpha T^H x \)

The routines multiply a vector \( x \) by a general triangular matrix \( T \) or its transpose, or its
conjugate transpose, and copies the resulting vector in the vector operand \( x \). If \( n \) is less than or
equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) less
than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error
handler. For the routine TRMV, if \( \text{ldt} \) is less than one or \( \text{ldt} \) is less than \( n \), an error flag is set and
passed to the error handler. For the routine TBMV, if \( \text{ldt} \) is less than \( k \) plus one, an error flag is
set and passed to the error handler.

- Fortran 95 binding:

Triangular Band:

```
SUBROUTINE tbmv( t, x [, uplo] [, trans] [, diag] [, alpha] )
```

Triangular Packed:

```
```

all:
\[
\begin{align*}
\text{<type>(<wp>), INTENT(IN) :: <tt>} & \\
\text{<type>(<wp>), INTENT(INOUT) :: x(:)} & \\
\text{<type>(<wp>), INTENT(IN), OPTIONAL :: alpha} & \\
\text{TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag} & \\
\text{TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans} & \\
\text{TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo} & \\
\text{where} & \\
\text{<tt> ::= t(:,:)} \text{ or tp(:)} & \\
\text{and} & \\
\text{TB \ t has shape (k+1,n)} & \\
\text{TP \ tp has shape (n*(n+1)/2)} & \\
\text{x has shape (n)} & \\
(k=\text{band width}) & \\
\text{The functionality of trmv is covered by trmm.} & \\
\end{align*}
\]

- Fortran 77 binding:

Triangular:
\[
\begin{align*}
\text{SUBROUTINE BLAS_xTRMV( UPLO, TRANS, DIAG, N, ALPHA, T, LDT, X,} & \\
\text{ INCX )} & \\
\end{align*}
\]

Triangular Band:
\[
\begin{align*}
\text{SUBROUTINE BLAS_xTBMV( UPLO, TRANS, DIAG, N, K, ALPHA, T, LDT,} & \\
\text{ X, INCX )} & \\
\end{align*}
\]

Triangular Packed:
\[
\begin{align*}
\text{SUBROUTINE BLAS_xTPMV( UPLO, TRANS, DIAG, N, ALPHA, TP, X, INCX )} & \\
\end{align*}
\]

all:
\[
\begin{align*}
\text{INTEGER} & \quad \text{DIAG, INCX, K, LDT, N, TRANS, UPLO} \\
\text{<type>} & \quad \text{ALPHA} & \\
\text{<type>} & \quad \text{T( LDT, *) or TP( * ), X( * )} & \\
\end{align*}
\]

- C binding:

Triangular:
\[
\begin{align*}
\text{void BLAS_xtrmv( enum blas_order_type order, enum blas_uplo_type uplo,} & \\
\text{enum blas_trans_type trans, enum blas_diag_type diag, int n,} & \\
\text{SCALAR_IN alpha, const ARRAY t, int ldt, ARRAY x, int incx );} & \\
\end{align*}
\]

Triangular Band:
\[
\begin{align*}
\text{void BLAS_xtbmv( enum blas_order_type order, enum blas_uplo_type uplo,} & \\
\text{enum blas_trans_type trans, enum blas_diag_type diag, int n,} & \\
\text{int k, SCALAR_IN alpha, const ARRAY t, int ldt, ARRAY x,} & \\
\text{int incx );} & \\
\end{align*}
\]

Triangular Packed:
\[
\begin{align*}
\text{void BLAS_xtpmv( enum blas_order_type order, enum blas_uplo_type uplo,} & \\
\text{enum blas_trans_type trans, enum blas_diag_type diag, int n,} & \\
\text{SCALAR_IN alpha, const ARRAY tp, ARRAY x, int incx );} & \\
\end{align*}
\]
GE_SUM_MV (Summed matrix vector multiplies)  
\[ y \leftarrow \alpha Ax + \beta Bx \]

This routine adds the product of two scaled matrix vector products. It can be used to compute the residual of an approximate eigenvector and eigenvalue of the generalized eigenvalue problem \( A * x = \lambda * B * x \). If \( m \) or \( n \) is less than or equal to zero, then this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if incx or incy is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than m, or ldb is less than one or ldb is less than m, an error flag is set and passed to the error handler. For the C bindings for GE_SUM_MV, if order = blas_rowmajor and if lda is less than one or lda is less than n, or if ldb is less than one or ldb is less than n, an error flag is set and passed to the error handler; if order = blas_colmajor and if lda is less than one or lda is less than m, or if ldb is less than one or ldb is less than m, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE ge_sum_mv( a, x, b, y [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: a(:,:), b(:,:)
  <type>(<wp>), INTENT (IN) :: x(:)
  <type>(<wp>), INTENT (OUT) :: y(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta

where
  x has shape (n);
  y has shape (m);
  a and b have shape (m,n) for general matrices
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xGE_SUM_MV( M, N, ALPHA, A, LDA, X, INCX, BETA, $  B, LDB, Y, INCY )
  INTEGER INCX, INCY, LDA, LDB, M, N
  <type> ALPHA, BETA
  <type> A( LDA, * ), B( LDB, * ), X( * ), Y( * )
```

- C binding:

```c
void BLAS_xge_sum_mv( enum blas_order_type order, int m, int n, 
  SCALAR_IN alpha, const ARRAY a, int lda, 
  const ARRAY x, int incx, SCALAR_IN beta, 
  const ARRAY B, int ldb, ARRAY y, int incy );
```

GEMVT (Multiple matrix vector multiplies)  
\[ x \leftarrow \beta A^T y + z, w \leftarrow \alpha Ax \]

The routine combines a matrix vector and a transposed matrix vector multiply. It multiplies a vector \( y \) by a general matrix \( A^T \), scales the resulting vector and adds the result to \( z \), storing the result in the vector operand \( x \). It then multiplies the matrix \( A \) by \( x \), scales the resulting vector and stores it in the vector operand \( w \).
Advice to implementors. Note that $x$ and $w$ may be computed while passing $A$ through the top of the memory just once. This optimization, which accelerates algorithms like reducing a symmetric matrix to tridiagonal form, is the justification for GEMVT, which could otherwise be implemented by two calls to GEMV. (End of advice to implementors.)

If $m$ or $n$ is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value incx or incy or incw or incz less than zero is permitted. However, if either incx, incy, incw, or incz is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than $m$, an error flag is set and passed to the error handler. For the C bindings, if order = blas_rowmajor and if lda is less than one or lda is less than $n$, an error flag is set and passed to the error handler; if order = blas_colmajor and if lda is less than one or lda is less than $m$, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE gemvt( a, x, y, w, z [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: a(:,:)
  <type>(<wp>), INTENT (IN) :: y(:,), z(:)
  <type>(<wp>), INTENT (OUT) :: x(:,), w(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
  
  where
  w and y have shape (m);
  x and z have shape (n);
  a has shape (m,n) for general matrix
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xGEMVT( M, N, ALPHA, A, LDA, X, INCX, Y, INCY,
  $ INTEGER BETA, W, INCW, Z, INCZ )
  <type> ALPHAX, INCX, INCY, INCZ, LDA, M, N
  <type> A( LDA, * ), X( * ), Y( * ), W( * ), Z( * )
  ```

- C binding:

  ```c
  void BLAS_xgemvt( enum blas_order_type order, int m, int n, SCALAR_IN alpha,
  const ARRAY a, int lda, ARRAY x, int incx, const ARRAY y,
  int incy, SCALAR_IN beta, ARRAY w, int incw, const ARRAY z,
  int incz );
  ```

TRMV (Multiple triangular matrix vector product)

$x \leftarrow T^\top y$ and $w \leftarrow Tz$

The routine combines a matrix vector and a transposed matrix vector multiply. It multiplies a vector $y$ by a triangular matrix $T^\top$, storing the result as $x$. It also multiplies the matrix by the vector $z$, storing the result as $w$.

Advice to implementors. Note that $x$ and $w$ may be computed while passing $T$ through the top of the memory just once. This optimization, which accelerates algorithms like reducing a symmetric matrix to tridiagonal form, is the justification for TRMV, which could otherwise be implemented by two calls to TRMV. (End of advice to implementors.)
2.8. LANGUAGE BINDINGS

If \( n \) is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) or \( \text{incz} \) less than zero is permitted. However, if either \( \text{incx} \), \( \text{incy} \), \( \text{incw} \), or \( \text{incz} \) is equal to zero, an error flag is set and passed to the error handler. If \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE trmv( t, x, y, z [, uplo ] )
  <type>(<wp>), INTENT (IN) :: t(:,:)
  <type>(<wp>), INTENT (IN) :: y(:), z(:)
  <type>(<wp>), INTENT (OUT) :: x(:), w(:)
  TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
  where
  w, x, y and z have shape (n);
  t has shape (n,n).
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xTRMV( UPLO, N, T, LDT, X, INCX, Y, INCY, W, INCW, 
  $ Z, INCZ )
  INTEGER INCW, INCX, INCY, INCZ, LDT, N, UPLO
  <type> T( LDT, * ), W( * ), X( * ), Y( * ), Z( * )
  ```

- C binding:

  ```c
  void BLAS_xtrmv( enum blas_order_type order, enum blas_uplo_type uplo,
  int n, const ARRAY t, int ldt, ARRAY x, int incx,
  const ARRAY y, int incy, ARRAY w, int incw, const ARRAY z,
  int incz );
  ```

GEMVER (Multiple matrix vector multiply with a rank 2 update)

\[
\hat{A} \leftarrow A + u_1 v_1^T + u_2 v_2^T \quad \text{and} \quad x \leftarrow \beta \hat{A}^T y + z \quad \text{and} \quad w \leftarrow \alpha \hat{A} x
\]

The routine precedes a combined matrix vector and a transposed matrix vector multiply by a rank two update. A matrix \( A \) is updated by \( u_1 v_1^T \) and \( u_2 v_2^T \). The transpose of the updated matrix is multiplied by a vector \( y \). The resulting vector is scaled and added to the vector operand \( z \), and stored in \( x \). The operand \( x \) is multiplied by the updated matrix \( \hat{A} \). The resulting vector is scaled and stored as \( w \).

Advice to implementors. Note that \( \hat{A} \), \( x \) and \( w \) may be computed while passing \( A \) through the top of the memory just once. This optimization, which accelerates algorithms like reducing a general matrix to bidiagonal form, is the justification for GEMVER, which could otherwise be implemented by calls to other BLAS routines. (End of advice to implementors.)

If \( m \) or \( n \) is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) or \( \text{incz} \) less than zero is permitted. However, if either \( \text{incx} \), \( \text{incy} \), \( \text{incw} \), or \( \text{incz} \) is equal to zero, an error flag is set and passed to the error handler. If \( \text{lda} \) is less than
one or lda is less than m, an error flag is set and passed to the error handler. For the C bindings, if order = blas_rowmajor and if lda is less than one or lda is less than n, an error flag is set and passed to the error handler; if order = blas_colmajor and if lda is less than one or lda is less than m, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```
SUBROUTINE gemver( a, u1, v1, u2, v2, x, y, z, w [, , alpha [, , beta] )
  <type>(<wp>), INTENT (IN) :: u1(:,), u2(:,), v1(:,), v2(:,), y(:,), z(:)
  <type>(<wp>), INTENT (INOUT) :: a(:,), x(:)
  <type>(<wp>), INTENT (OUT) :: w(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
where
  u1, u2, w and y have shape (m);
  v1, v2, x and z have shape (n);
  a has shape (m,n).
```

- Fortran 77 binding:

```
General:
SUBROUTINE BLAS_xGEMVER( M, N, A, LDA, U1, V1, U2, V2, ALPHA, X,
  $            INTEGER INCW, INCX, INCY, INCB, W, INCW, Z, INCZ )
  <type>     ALPHA, BETA
  <type>     U1( * ), V1( * ), U2( * ), V2( * )
  <type>     A( LDA, * ), W( * ), X( * ), Y( * ), Z( * )
```

- C binding:

```
void BLAS_xgemver( enum blas_order_type order, int m, int n, ARRAY a,
  int lda, const ARRAY u1, const ARRAY v1,
  const ARRAY u2, const ARRAY v2, SCALAR_IN alpha,
  ARRAY x, int incx, const ARRAY y, int incy, ARRAY w,
  int incw, SCALAR_IN beta, const ARRAY z, int incz );
```

{TR,TB,TP}SV (Triangular solve) \[ x \leftarrow \alpha T^{-1} x \]

These routines solve one of the systems of equations \( x \leftarrow \alpha T^{-1} x \) or \( x \leftarrow \alpha T^{-T} x \), where \( x \) is a vector and the matrix \( T \) is a unit, non-unit, upper or lower triangular (or triangular banded or triangular packed) matrix. If \( n \) is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler. For TRSV, if ldt is less than one or ldt is less than n, an error flag is set and passed to the error handler. For TBSV, if ldt is less than one or ldt is less than k plus one, an error flag is set and passed to the error handler.
2.8. LANGUAGE BINDINGS

Advice to implementors. Note that no check for singularity, or near singularity is specified for these triangular equation-solving routines. The requirements for such a test depend on the application, and so we felt that this should not be included, but should instead be performed before calling the triangular solver. (End of advice to implementors.)

- Fortran 95 binding:

Triangular Band:

```fortran
SUBROUTINE tbsv( t, x [, uplo] [, trans] [, diag] [, alpha] )
```

Triangular Packed:

```fortran
```

all:

```fortran
<type>(<wp>), INTENT(IN) :: <tt>
<type>(<wp>), INTENT(INOUT) :: x(:)
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
<type>(<wp>), INTENT(IN), OPTIONAL :: alpha
```

where

```fortran
<tt> ::= t(:,:) or tp(:)
```

and

```fortran
x has shape (n)
TB t has shape (k+1,n)
TP tp has shape (n*(n+1)/2)
(k=band width)
```

The functionality of trsv is covered by trsm.

- Fortran 77 binding:

Triangular:

```fortran
SUBROUTINE BLAS_xTTRSV( UPLO, TRANS, DIAG, N, ALPHA, T, LDT, X,
                         $                INCX )
```

Triangular Band:

```fortran
SUBROUTINE BLAS_xTBSV( UPLO, TRANS, DIAG, N, K, ALPHA, T, LDT,
                         $                X, INCX )
```

Triangular Packed:

```fortran
SUBROUTINE BLAS_xTPSV( UPLO, TRANS, DIAG, N, ALPHA, TP, X, INCX )
```

all:

```fortran
INTEGER        DIAG, INCX, K, LDT, N, TRANS, UPLO
<type>          ALPHA
<type>          T(LDT, *) or TP(*), X(*)
```

- C binding:

Triangular:

```c
void BLAS_xtrsv( enum blas_order_type order, enum blas_uplo_type uplo,
                 enum blas_trans_type trans, enum blas_diag_type diag,
                 int n, SCALAR_IN alpha, const ARRAY t, int ldt,
```
ARRAY x, int incx );

Triangular Band:
void BLAS_xtbsv( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, int k, SCALAR_IN alpha, const ARRAY t, int ldt,
ARRAY x, int incx );

Triangular Packed:
void BLAS_xtpsv( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, SCALAR_IN alpha, const ARRAY tp, ARRAY x,
int incx );

GER (Rank one update) $A \in \mathbb{R}^{m \times n}, A \leftarrow \alpha xy^T + \beta A$ $A \in \mathbb{C}^{m \times n}, A \leftarrow \alpha xy^T + \beta A$ or $A \leftarrow \alpha xy^H + \beta A$

This routine performs the rank 1 operation $A \leftarrow \alpha xy^T + \beta A$ where $\alpha$ and $\beta$ are scalars, $x$ and $y$ are vectors, and $A$ is a matrix. If $m \leq n$, $m$ is less than or equal to $n$ or if $\beta$ is equal to zero, an error flag is set and passed to the error handler. If $A$ is less than one or $\alpha$ is less than zero, an error flag is set and passed to the error handler. The operator argument $\text{conj}$ is only referenced when $x$ and $y$ are complex vectors. When $x$ and $y$ are complex vectors, the vector components $y_i$ are used un conjugated or conjugated as specified by the operator argument $\text{conj}$.

- Fortran 95 binding: Refer to GEMM specification
- Fortran 77 binding:

  SUBROUTINE BLAS_xGER( CONJ, M, N, ALPHA, X, INCX, Y, INCY, Beta,
  $ A, LDA )$
  INTEGER CONJ, INCX, INCY, LDA, M, N
  <type> ALPHA, BETA
  <type> A( LDA, * ), X( * ), Y( * )

- C binding:

  void BLAS_xger( enum blas_order_type order, enum blas_conj_type conj,
  int m, int n, SCALAR_IN alpha, const ARRAY x, int incx,
  const ARRAY y, int incy, SCALAR_IN beta, ARRAY a, int lda );

{SY,SP}R (Symmetric Rank One Update) $A \leftarrow \alpha xx^T + \beta A$ with $A = A^T$

The routine performs the symmetric rank-1 update $A = \alpha xx^T + \beta A$, where $\alpha$ and $\beta$ are scalars, $x$ is a vector and $A$ is a symmetric (symmetric packed) matrix. This routine returns immediately
if n is less than or equal to zero or if beta is equal to one and alpha is equal to zero. As described in
section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error
flag is set and passed to the error handler. If lda is less than one or lda is less than n, an error flag
is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routines xSYR and xSPR with added functionality
for complex symmetric matrices.

- Fortran 95 binding:

  Symmetric Packed:
  
  
  <type>(<wp>), INTENT(IN) :: x(:)
  <type>(<wp>), INTENT(INOUT) :: ap(:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta

  where
  x has shape (n)
  ap has shape (n*(n+1)/2)

  The functionality of spr is covered by syrk.

- Fortran 77 binding:

  Symmetric:
  
  SUBROUTINE BLAS_xSYR( UPLO, N, ALPHA, X, INCX, BETA, A, LDA )
  
  Symmetric Packed:
  
  SUBROUTINE BLAS_xSPR( UPLO, N, ALPHA, X, INCX, BETA, AP )

  all:
  INTEGER INCX, LDA, N, UPLO
  <type> ALPHA, BETA
  <type> A( LDA, * ) or AP( * ), X( * )

- C binding:

  Symmetric:

  void BLAS_xsyr( enum blas_order_type order, enum blas_uplo_type uplo,
  int n, SCALAR_IN alpha, const ARRAY x, int incx,
  SCALAR_IN beta, ARRAY a, int lda );

  Symmetric Packed:

  void BLAS_xspr( enum blas_order_type order, enum blas_uplo_type uplo,
  int n, SCALAR_IN alpha, const ARRAY x, int incx,
  SCALAR_IN beta, ARRAY ap );

  {HE,HP}R (Hermitian Rank One Update) \( A \leftarrow \alpha xx^H + \beta A \) with \( A = A^H \)

  The routine performs the Hermitian rank-1 update \( A = \alpha xx^H + \beta A \), where \( \alpha \) and \( \beta \) are real
  scalars, \( x \) is a complex vector and \( A \) is a Hermitian (Hermitian packed) matrix. This routine returns
immediately if \( n \) is less than or equal to zero or if \( \beta \) is equal to one and \( \alpha \) is equal to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler. If \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

Hermitian Packed:

```fortran
  COMPLEX(<wp>), INTENT(IN) :: x(:)
  COMPLEX(<wp>), INTENT(INOUT) :: ap(:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  REAL(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
  x has shape (n)
  ap has shape (n*(n+1)/2)
```

The functionality of her is covered by herk.

- Fortran 77 binding:

Hermitian:

```fortran
SUBROUTINE BLAS_xHER( UPLO, N, ALPHA, X, INCX, BETA, A, LDA )
```

Hermitian Packed:

```fortran
SUBROUTINE BLAS_xHPR( UPLO, N, ALPHA, X, INCX, BETA, AP )
```

all:

```fortran
INTEGER INCX, LDA, N, UPLO
&lt;rtype&gt; ALPHA, BETA
&lt;ctype&gt; A( LDA, * ) or AP( * ), X( * )
```

- C binding:

Hermitian:

```c
void BLAS_xher( enum blas_order_type order, enum blas_uplo_type uplo,
   int n, RSCALAR_IN alpha, const CARRAY x, int incx,
   RSCALAR_IN beta, CARRAY a, int lda );
```

Hermitian Packed:

```c
void BLAS_xhpr( enum blas_order_type order, enum blas_uplo_type uplo,
   int n, RSCALAR_IN alpha, const CARRAY x, int incx,
   RSCALAR_IN beta, CARRAY ap );
```

\{SY,SP\}R2 (Symmetric Rank two update) \( A \leftarrow \alpha xy^T + \alpha yx^T + \beta A \) with \( A = A^T \)

The routine performs the symmetric rank-2 update \( A = \alpha xy^T + \alpha yx^T + \beta A \), where \( \alpha \) and \( \beta \) are scalars, \( x \) is a vector and \( A \) is a symmetric (symmetric packed) matrix. This routine returns immediately if \( n \) is less than or equal to zero or if \( \beta \) is equal to one and \( \alpha \) is equal to zero. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either
in x or incy is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than n, an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routines xSYR2 and xSPR2 with added functionality for complex symmetric matrices.

- Fortran 95 binding:

```fortran
   <type>(<wp>), INTENT(IN) :: x(:,), y(:)
   <type>(<wp>), INTENT(INOUT) :: ap(:)
   TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
   TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
   <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
x and y have shape (n)
ap has shape (n*(n+1)/2)
```

The functionality of syr2 is covered by syc2k.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYR2( UPLO, N, ALPHA, X, INCX, Y, INCY, BETA, A, LDA )

SUBROUTINE BLAS_xSPR2( UPLO, N, ALPHA, X, INCX, Y, INCY, BETA, AP )

all:
INTEGER INCX, LDA, N, UPLO
<type> ALPHA, BETA
<type> A( LDA, * ) or AP( * ), X( * ), Y( * )
```

- C binding:

```c
void BLAS_xsyr2( enum blas_order_type order, enum blas_uplo_type uplo,
   int n, SCALAR_IN alpha, const ARRAY x, int incx,
   const ARRAY y, int incy, SCALAR_IN beta, ARRAY a, int lda );

void BLAS_xspr2( enum blas_order_type order, enum blas_uplo_type uplo,
   int n, SCALAR_IN alpha, const ARRAY x, int incx,
   const ARRAY y, int incy, SCALAR_IN beta, ARRAY ap );
```

{HE,HP}R2 (Hermitian Rank two update) \( A \leftarrow \alpha xy^H + \bar{\alpha}yx^H + \beta A \) with \( A = A^H \)

The routine performs the Hermitian rank-2 update \( A = \alpha xy^H + \bar{\alpha}yx^H + \beta A \), where \( \alpha \) is a complex scalar and and \( \beta \) is a real scalar, \( x \) and \( y \) are complex vectors and \( A \) is a Hermitian
(Hermitian packed) matrix. This routine returns immediately if n is less than or equal to zero or if beta is equal to one and alpha is equal to zero. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than n, an error flag is set and passed to the error handler.

- Fortran 95 binding:

Hermitian Packed:

```fortran
  COMPLEX (<wp>), INTENT(IN) :: x(:), y(:)
  COMPLEX (<wp>), INTENT(INOUT) :: ap(:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  COMPLEX (<wp>), INTENT(IN), OPTIONAL :: alpha, beta
WHERE
  x and y have shape (n)
  ap has shape (n*(n+1)/2)
```

The functionality of her2 is covered by her2k.

- Fortran 77 binding:

Hermitian:

```fortran
SUBROUTINE BLAS_xHER2( UPLO, N, ALPHA, X, INCX, Y, INCY, BETA, A,
                      $                      LDA )
```

Hermitian Packed:

```fortran
SUBROUTINE BLAS_xHPR2( UPLO, N, ALPHA, X, INCX, Y, INCY, BETA,
                      $                      AP )
```

all:

```fortran
INTEGER       INCX, LDA, N, UPLO
<ctype>        ALPHA
<rtype>        BETA
<ctype>        A(LDA,*) or AP(*), X(*), Y(*)
```

- C binding:

Hermitian:

```c
void BLAS_xher2( enum blas_order_type order, enum blas_uplo_type uplo,
                int n, CSCALAR_IN alpha, const CARRAY x, int incx, 
                const CARRAY y, int incy, RSCALE_IN beta, CARRAY a, 
                int lda );
```

Hermitian Packed:

```c
void BLAS_xhpr2( enum blas_order_type order, enum blas_uplo_type uplo,
                int n, CSCALAR_IN alpha, const CARRAY x, int incx, 
                const CARRAY y, int incy, RSCALE_IN beta, CARRAY ap );
```
2.8. LANGUAGE BINDINGS

2.8.7 Matrix Operations

\{GE, GB, SY, HE, SB, HB, SP, HP, TR, TB, TP\}_NORM (Matrix norms)

\[ r \leftarrow \|A\|_1, \|A\|_{1R}, \|A\|_F, \|A\|_\infty, \|A\|_{\infty R}, \|A\|_{\text{max}}, \text{ or } \|A\|_{\text{max R}} \]

These routines compute the one-norm, real one-norm, Frobenius-norm, infinity-norm, real infinity-norm, max-norm, or real max-norm of a general matrix \( A \) depending on the value passed as the norm operator argument. This routine returns immediately with the output scalar \( r \) set to zero if \( m \) (for nonsymmetric matrices) or \( n \) or \( k \) or \( k_u \) (for band matrices) or \( k \) (for symmetric band matrices) is less than or equal to zero. The resulting scalar \( r \) is always real and as defined in section 2.1.3. If \( \text{norm} = \text{blas\_two\_norm} \), requesting the two-norm of a matrix, an error flag is set and passed to the error handler. The only exception to this rule is if the matrix is a single column or a single row, whereby the Frobenius-norm is returned since the two-norm and Frobenius-norm of a vector are identical. For the routine GE_NORM, if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( m \), an error flag is set and passed to the error handler. For the C bindings of GE_NORM, if \( \text{order} = \text{blas\_row\_major} \) and if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler; if \( \text{order} = \text{blas\_col\_major} \) and if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( m \), an error flag is set and passed to the error handler. For the routine GB_NORM, if \( \text{lda} \) is less than \( k \) plus \( k_u \) plus one, an error flag is set and passed to the error handler. For the routines SY_NORM, HE_NORM, and TR_NORM, if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the routines SB_NORM, HB_NORM, and TB_NORM, if \( \text{lda} \) is less than \( k \) plus one, an error flag is set and passed to the error handler.

Advice to implementors. High-quality implementations of these routines should be accurate. The subroutines SLANGB, SLANGE, SLANTG, SLANHS, SLANSB, SLANSP, SLANST, SLANSY, SLANTB, SLANTP, and SLANTR, of the LAPACK [6] software library are examples of accurate implementations. High-quality implementations should document the accuracy of the algorithms used in this routine so as to alleviate the portability problems this represents.

(End of advice to implementors.)

- Fortran 95 binding:

  General:
  
  REAL(<wp>) FUNCTION ge_norm( a [, , norm] )

  General Band:
  
  REAL(<wp>) FUNCTION gb_norm( a, m, kl [, , norm] )

  Symmetric:
  
  REAL(<wp>) FUNCTION sy_norm( a [, , norm] [, , uplo] )

  Hermitian:
  
  REAL(<wp>) FUNCTION he_norm( a [, , norm] [, , uplo] )

  Symmetric Band:
  
  REAL(<wp>) FUNCTION sb_norm( a [, , norm] [, , uplo] )

  Hermitian Band:
  
  REAL(<wp>) FUNCTION hb_norm( a [, , norm] [, , uplo] )

  Symmetric Packed:
  
  REAL(<wp>) FUNCTION sp_norm( ap [, , norm] [, , uplo] )

  Hermitian Packed:
  
  REAL(<wp>) FUNCTION hp_norm( ap [, , norm] [, , uplo] )
Triangular:
REAL(<wp>) FUNCTION tr_norm( a [, norm] [, uplo] [, diag] )

Triangular Band:
REAL(<wp>) FUNCTION tb_norm( a [, norm] [, uplo] [, diag] )

Triangular Packed:
REAL(<wp>) FUNCTION tp_norm( ap [, norm] [, uplo] [, diag] )

all:
<type>(<wp>), INTENT (IN) :: a(:,:) | ap(:)
INTEGER, INTENT (IN) :: m, kl
TYPE (blas_norm_type), INTENT (IN), OPTIONAL :: norm
TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
TYPE (blas_diag_type), INTENT (IN), OPTIONAL :: diag

where
a has shape (m,n) for general matrix
(1,n) for general banded matrix (1 > kl)
(n,n) for symmetric, Hermitian or triangular
(k+1,n) for symmetric banded, Hermitian banded
or triangular banded (k=bond width)
ap has shape (n*(n+1)/2).

• Fortran 77 binding:

General:
<rtype> FUNCTION BLAS_xGE_NORM( NORM, M, N, A, LDA )

General Band:
<rtype> FUNCTION BLAS_xGB_NORM( NORM, M, N, KL, KU, A, LDA )

Symmetric:
<rtype> FUNCTION BLAS_xSY_NORM( NORM, UPLO, N, A, LDA )

Hermitian:
<rtype> FUNCTION BLAS_xHE_NORM( NORM, UPLO, N, A, LDA )

Symmetric Band:
<rtype> FUNCTION BLAS_xSB_NORM( NORM, UPLO, N, K, A, LDA )

Hermitian Band:
<rtype> FUNCTION BLAS_xHB_NORM( NORM, UPLO, N, K, A, LDA )

Symmetric Packed:
<rtype> FUNCTION BLAS_xSP_NORM( NORM, UPLO, N, AP )

Hermitian Packed:
<rtype> FUNCTION BLAS_xHP_NORM( NORM, UPLO, N, AP )

Triangular:
<rtype> FUNCTION BLAS_xTR_NORM( NORM, UPLO, DIAG, N, A, LDA )

Triangular Band:
<rtype> FUNCTION BLAS_xTB_NORM( NORM, UPLO, DIAG, N, K, A, LDA )

Triangular Packed:
<rtype> FUNCTION BLAS_xTP_NORM( NORM, UPLO, DIAG, N, AP )

all:
INTEGER DIAG, K, KL, KU, LDA, M, N, NORM, UPLO
<type> A( LDA, * ) or AP( * )
• C binding:

General:
void BLAS_xge_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    int m, int n, const ARRAY a, int lda, RSCALAR_INOUT r );

General Band:
void BLAS_xgb_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    int m, int n, int kl, int ku, const ARRAY a, int lda, 
                    RSCALAR_INOUT r );

Symmetric:
void BLAS_xsy_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, const ARRAY a, 
                    int lda, RSCALAR_INOUT r );

Hermitian:
void BLAS_xhe_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, const CARRAY a, 
                    int lda, RSCALAR_INOUT r );

Symmetric Band:
void BLAS_xsb_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, int k, const ARRAY a, 
                    int lda, RSCALAR_INOUT r );

Hermitian Band:
void BLAS_xhb_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, int k, const CARRAY a, 
                    int lda, RSCALAR_INOUT r );

Symmetric Packed:
void BLAS_xsp_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, const ARRAY ap, 
                    RSCALAR_INOUT r );

Hermitian Packed:
void BLAS_xhp_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, int n, const CARRAY ap, 
                    RSCALAR_INOUT r );

Triangular:
void BLAS_xtr_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, enum blas_diag_type diag, 
                    int n, const ARRAY a, int lda, RSCALAR_INOUT r );

Triangular Band:
void BLAS_xtb_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, enum blas_diag_type diag, 
                    int n, int k, const ARRAY a, int lda, RSCALAR_INOUT r );

Triangular Packed:
void BLAS_xtp_norm( enum blas_order_type order, enum blas_norm_type norm, 
                    enum blas_uplo_type uplo, enum blas_diag_type diag, 
                    int n, const ARRAY ap, RSCALAR_INOUT r );
\{GE,GB\}_DIAG\_SCALE (Diagonal scaling) \( A \leftarrow DA, AD \) with \( D \) diagonal

These routines scale a general (or banded) matrix \( A \) on the left side or the right side by a diagonal matrix \( D \). This routine returns immediately if \( m \) or \( n \) or \( k_l \) or \( k_u \) (for band matrices) is less than or equal to zero. As described in section 2.5.3, the value \( \text{incd} \) less than zero is permitted. However, if \( \text{incd} \) is equal to zero, an error flag is set and passed to the error handler. For the routine \( \text{GE\_DIAG\_SCALE} \), if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( m \), an error flag is set and passed to the error handler. For the C bindings of \( \text{GE\_DIAG\_SCALE} \), if \( \text{order} = \text{blas\_rowmajor} \) and if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler; if \( \text{order} = \text{blas\_colmajor} \) and if \( \text{lda} \) is less than one or \( \text{lma} \) is less than \( m \), an error flag is set and passed to the error handler. For the routine \( \text{GB\_DIAG\_SCALE} \), if \( \text{lma} \) is less than \( k_l \) plus \( k_u \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  General:
  ```fortran
  SUBROUTINE ge_diag_scale( d, a [, side ] )
  General Band:
  SUBROUTINE gb_diag_scale( d, a, m, k [l [, side ] ]
  all:
  <type>(<wp>), INTENT (IN) :: d(:)
  <type>(<wp>), INTENT (INOUT) :: a(:,:)
  INTEGER, INTENT (IN) :: m, k_l
  TYPE (blas\_side\_type), INTENT (IN), OPTIONAL :: side
  where
  a has shape (m,n) for general matrix
  (1,n) for general banded matrix (l > k_l)
  d has shape (p) where p = m if side = blas\_left\_side
  p = n if side = blas\_right\_side
  ```

- Fortran 77 binding:

  General:
  ```fortran
  SUBROUTINE BLAS\_xGE\_DIAG\_SCALE( SIDE, M, N, D, INCD, A, LDA )
  General Band:
  SUBROUTINE BLAS\_xGB\_DIAG\_SCALE( SIDE, M, N, KL, KU, D, INCD, A, LDA )
  all:
  INTEGER INCD, KL, KU, LDA, M, N, SIDE
  <type> A( LDA, * ), D( * )
  ```

- C binding:

  General:
  ```c
  void BLAS\_xge\_diag\_scale( enum blas\_order\_type order,
                              enum blas\_side\_type side, int m, int n,
                              const ARRAY d, int incd, ARRAY a, int lda );
  General Band:
  ```
void BLAS_xgb_diag_scale( enum blas_order_type order,
    enum blas_side_type side, int m, int n, int kl,
    int ku, const ARRAY d, int incd, ARRAY a, int lda );

{GE,GB}_LRSCALE (Two-sided diagonal scaling) \[ A \leftarrow D_L A D_R \]

These routines scale a general (or banded) matrix \( A \) on the left side by a diagonal matrix \( D_L \)
and on the right side by a diagonal matrix \( D_R \). This routine returns immediately if \( m \) or \( n \) or \( kl \) or
\( ku \) (for band matrices) is less than or equal to zero. As described in section 2.5.3, the value incd or
incr less than zero is permitted. However, if either incd or incr is equal to zero, an error flag is set
and passed to the error handler. For the routine GE_LRS scale, if \( lda \) is less than one or \( lda \) is less
than \( m \), an error flag is set and passed to the error handler. For the C bindings of GE_LRS SCALE,
if order = blas_rowmajor and if \( lda \) is less than one or \( lda \) is less than \( n \), an error flag is set and
passed to the error handler; if order = blas_colmajor and if \( lda \) is less than one or \( lda \) is less than
\( m \), an error flag is set and passed to the error handler. For the routine GB_LRS SCALE, if \( lda \) is less
than \( kl \) plus \( ku \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  General:
  
  SUBROUTINE ge_lrscale( dl, dr, a )

  General Band:
  
  SUBROUTINE gb_lrscale( dl, dr, a, m, kl )

  all:
  
  <type>(<wp>), INTENT (IN) :: dl(:,), dr(:)
  <type>(<wp>), INTENT (INOUT) :: a(:,:),
  INTEGER, INTENT (IN) :: m, kl

  where
  
  a has shape (m,n) for general matrix
  (1,n) for general banded matrix (l > kl)
  dl has shape (m)
  dr has shape (n)

- Fortran 77 binding:

  General:
  
  SUBROUTINE BLAS_xGE_LRS SCALE( M, N, DL, INCDL, DR, INCDDR, A, LDA )

  General Band:
  
  SUBROUTINE BLAS_xGB_LRS SCALE( M, N, KL, KU, DL, INCDL, DR, INCDDR,
  $ A, LDA )

  all:
  
  INTEGER INCDL, INCDDR, KL, KU, LDA, M, N
  <type> A( LDA, * ), DL( * ), DR( * )

- C binding:
CHAPTER 2. DENSE AND BANDED BLAS

General:
void BLAS_xge_lrscale( enum blas_order_type order, int m, int n,
const ARRAY dl, int incdl, const ARRAY dr,
int incdr, ARRAY a, int lda );

General Band:
void BLAS_xgb_lrscale( enum blas_order_type order, int m, int n, int kl,
int ku, const ARRAY dl, int incdl, const ARRAY dr,
int incdr, ARRAY a, int lda );

\{SY,SB,SP\}_LRSCALE (Two-sided diagonal scaling of a symmetric matrix)

\[ A \leftarrow DAD \text{ with } A = A^T \]

These routines perform a two-sided scaling of a symmetric (or symmetric banded or symmetric packed) matrix \( A \) by a diagonal matrix \( D \). This routine returns immediately if \( n \) or \( k \) (for symmetric band matrices) is less than or equal to zero. As described in section 2.5.3, the value incd less than zero is permitted. However, if incd is equal to zero, an error flag is set and passed to the error handler. For the routines \SY_LRSCALE and SP_LRSCALE, if lda is less than one or lda is less than \( n \), an error flag is set and passed to the error handler. For the routine \SB_LRSCALE, if lda is less than \( k \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

Symmetric:
SUBROUTINE sy_lrscale( d, a [, uplo] )
Symmetric Band:
SUBROUTINE sb_lrscale( d, a [, uplo] )
Symmetric Packed:
SUBROUTINE sp_lrscale( d, ap [, uplo] )
all:
<type>(<wp>), INTENT (IN) :: d(:)
<type>(<wp>), INTENT (INOUT) :: a(:,:) | ap(:)
TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
where
a has shape (n,n) for symmetric
(k+1,n) for symmetric banded (k=band width)
ap has shape (n*(n+1)/2).
d has shape (n)

- Fortran 77 binding:

Symmetric:
SUBROUTINE BLAS_xSY_LRSCALE( UPLO, N, D, INCD, A, LDA )
Symmetric Band:
SUBROUTINE BLAS_xSB_LRSCALE( UPLO, N, K, D, INCD, A, LDA )
Symmetric Packed:
SUBROUTINE BLAS_xSP_LRSCALE( UPLO, N, D, INCD, AP )
all:

\[
\begin{align*}
\text{INTEGER} & \quad \text{INCD, K, LDA, N, UPLO} \\
\text{<type>} & \quad A( \text{LDA, *}) \text{ or AP( *), D( *)}
\end{align*}
\]

- C binding:

```
Symmetric:
void BLAS_xsy_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                       int n, const ARRAY d, int incd, ARRAY a, int lda );
```

```
Symmetric Band:
void BLAS_xsb_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                       int n, int k, const ARRAY d, int incd, ARRAY a,
                       int lda );
```

```
Symmetric Packed:
void BLAS_xsp_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                       int n, const ARRAY d, int incd, ARRAY ap );
```

\{HE,HB,HP\}_LRSCALE (Two-sided diagonal scaling of a Hermitian matrix)

\[
A \leftarrow DAD^H \text{ with } A = A^H
\]

These routines perform a two-sided scaling of a Hermitian (or Hermitian banded or Hermitian packed) matrix \( A \) by a diagonal matrix \( D \). This routine returns immediately if \( n \) or \( k \) (for Hermitian band matrices) is less than or equal to zero. As described in section 2.5.3, the value incd less than zero is permitted. However, if incd is equal to zero, an error flag is set and passed to the error handler. For the routines \texttt{HE}_LRSCALE, if lda is less than one or lda is less than n, an error flag is set and passed to the error handler. For the routine \texttt{HB}_LRSCALE, if lda is less than k plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```
Hermitian:
  SUBROUTINE he_lrscale( d, a [, uplo] )
```

```
Hermitian Band:
  SUBROUTINE hb_lrscale( d, a [, uplo] )
```

```
Hermitian Packed:
  SUBROUTINE hp_lrscale( d, ap [, uplo] )
```

all:

```
COMPLEX(<wp>), INTENT (IN) :: d(:)
COMPLEX(<wp>), INTENT (INOUT) :: a(:,:), ap(:)
TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
```

where

- a has shape \((n,n)\) for Hermitian
- \((k+1,n)\) for Hermitian banded \((k=\text{band width})\)
- ap has shape \((n*(n+1)/2)\).
- d has shape \((n)\)

- Fortran 77 binding:
Hermitian:
  SUBROUTINE BLAS_xHE_LRScale( UPLO, N, D, INCD, A, LDA )
Hermitian Band:
  SUBROUTINE BLAS_xHB_LRScale( UPLO, N, K, D, INCD, A, LDA )
Hermitian Packed:
  SUBROUTINE BLAS_xHP_LRScale( UPLO, N, D, INCD, AP )
all:
  INTEGER INCD, K, LDA, N, UPLO
  <ctype> A( LDA, * ) or AP( * ), D( * )
• C binding:

Hermitian:
  void BLAS_xhe_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, const ARRAY d, int incd, ARRAY a, int lda );
Hermitian Band:
  void BLAS_xhb_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, int k, const ARRAY d, int incd, ARRAY a,
                        int lda );
Hermitian Packed:
  void BLAS_xhp_lrscale( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, const ARRAY d, int incd, ARRAY ap );

{GE,GB}_DIAG_SCALE_ACC (Diagonal scaling and accumulation)  \[ A \leftarrow A + BD \]

These routines perform the diagonal scaling of a general (or banded) matrix \( B \) and accumulate
the result in the matrix \( A \). This routine returns immediately if \( m \) or \( n \) or \( k l \) or \( k u \) (for band
matrices) is less than or equal to zero. As described in section 2.5.3, the value incd less than zero
is permitted. However, if incd is equal to zero, an error flag is set and passed to the error handler.
For the routine GE\_DIAG\_SCALE\_ACC, if lda or ldb is less than one or lda or ldb is less than \( m \), an
error flag is set and passed to the error handler. For the C bindings of GE\_DIAG\_SCALE\_ACC, if
order = blas_rowmajor and if lda or ldb is less than one or lda or ldb is less than \( n \), an error flag
is set and passed to the error handler; if order = blas_colmajor and if lda or ldb is less than one
or lda or ldb is less than \( m \), an error flag is set and passed to the error handler. For the routine
GB\_DIAG\_SCALE\_ACC, if lda is less than kl plus ku plus one, an error flag is set and passed to the
error handler.

• Fortran 95 binding:

General:
  SUBROUTINE ge_diag_scale_acc( b, d, a )
Band:
  SUBROUTINE gb_diag_scale_acc( b, m, kl, d, a )
all:
  <type>(<wp>), INTENT (IN) :: b(:,,:), d(:)
  <type>(<wp>), INTENT (INOUT) :: a(:,:)
  INTEGER, INTENT (IN) :: m, kl
where
a has shape (m,n)
b has shape (m,n) for general matrix
   (1,n) for general banded matrix (l > k)
d has shape (n)

• Fortran 77 binding:

General:
SUBROUTINE BLAS_xGE_DIAG_SCALE_ACC( M, N, B, LDB, D, INCD, A, $)
                     LDA )

Band:
SUBROUTINE BLAS_xGB_DIAG_SCALE_ACC( M, N, KL, KU, B, LDB, D, INCD, $)
                     A, LDA )

all:
INTEGER INCD, KL, KU, LDB, LDB, M, N
<type> A( LDA, * ), B( LDB, * ), D( * )

• C binding:

General:
void BLAS_xge_diag_scale_acc( enum blas_order_type order, int m, int n,
   const ARRAY b, int ldb, const ARRAY d,
   int incd, ARRAY a, int lda );

General Band:
void BLAS_xgb_diag_scale_acc( enum blas_order_type order, int m, int n,
   int kl, int ku, const ARRAY b, int ldb,
   const ARRAY d, int incd, ARRAY a, int lda );

\{GE,SY,SB,SP\}_ACC (Matrix accumulation and scale) \quad B \leftarrow \alpha A + \beta B, B \leftarrow \alpha A^T + \beta B

These routines scale a matrix \(A\) (or its transpose) and scale a matrix \(B\) and accumulate the result in the matrix \(B\). Matrices \(A\) and \(B\) have the same storage format. These routines return immediately if \(\text{alpha}\) is equal to zero and \(\text{beta}\) is equal to one, or if \(m\) (for nonsymmetric matrices) or \(n\) or \(k\) (for symmetric band matrices) is less than or equal to zero. As described in section 2.5.3, for the routine GE_ACC, if \(lda\) or \(ldb\) is less than one or \(lda\) or \(ldb\) is less than \(m\), an error flag is set and passed to the error handler. For the C bindings for GE_ACC, if \text{order} = \text{blas_rowmajor} and if \(lda\) or \(ldb\) is less than one or \(lda\) or \(ldb\) is less than \(n\), an error flag is set and passed to the error handler; if \text{order} = \text{blas_colmajor} and if \(lda\) or \(ldb\) is less than one or \(lda\) or \(ldb\) is less than \(m\), an error flag is set and passed to the error handler. For the routine SY_ACC, if \(lda\) or \(ldb\) is less than one or \(lda\) or \(ldb\) is less than \(n\), an error flag is set and passed to the error handler. For the routine SB_ACC, if \(lda\) or \(ldb\) is less than \(k\) plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:

General:
SUBROUTINE ge_acc( a, b [, trans] [, alpha] [, beta] )
Symmetric:

```fortran
SUBROUTINE sy_acc( a, b [, uplo] [, trans] [, alpha] [, beta] )
```

Symmetric Band:

```fortran
SUBROUTINE sb_acc( a, b [, uplo] [, trans] [, alpha] [, beta] )
```

Symmetric Packed:

```fortran
```

**all:**

```fortran
[type] (<type>), INTENT(IN) :: a(:,:), b(:,:), ap(:)
[type] (<type>), INTENT(INOUT) :: b(:,:), bp(:)
```

* TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
* TYPE (blas_trans_type), INTENT (IN), OPTIONAL :: trans
* TYPE (<type>), INTENT(IN), OPTIONAL :: alpha, beta

The default value for $\beta$ is 1.0 or (1.0,0,0).

- **Fortran 77 binding:**

  **General:**
  ```fortran
  SUBROUTINE BLAS_xGE_ACC( TRANS, M, N, ALPHA, A, LDA, BETA, B,
  $  
  LDB )
  ```
  ```fortran
  Symmetric:
  ```fortran
  SUBROUTINE BLAS_xSY_ACC( UPLO, TRANS, N, ALPHA, A, LDA, BETA, B,
  $  
  LDB )
  ```
  ```fortran
  Symmetric Band:
  ```fortran
  SUBROUTINE BLAS_xSB_ACC( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA,
  $  
  B, LDB )
  ```
  ```fortran
  Symmetric Packed:
  ```fortran
  SUBROUTINE BLAS_xSP_ACC( UPLO, TRANS, N, ALPHA, AP, BETA, BP )
  ```

  **all:**

  ```fortran
  INTEGER K, LDA, LDB, M, N, TRANS, UPLO
  <type> ALPHA, BETA
  <type> A( LDA, * ) or AP( * ), B( LDB, * ) or BP( * )
  ```

- **C binding:**

  **General:**
  ```c
  void BLAS_xge_acc( enum blas_order_type order, enum blas_trans_type trans,
  int m, int n, SCALAR_IN alpha, const ARRAY a, int lda,
  SCALAR_IN beta, ARRAY b, int ldb );
  ```
  ```c
  Symmetric:
  ```c
  void BLAS_xsy_acc( enum blas_order_type order, enum blas_uplo_type uplo,
  enum blas_trans_type trans, int n, SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta, ARRAY b, int ldb );
  ```
  ```c
  Symmetric Band:
  ```c
  void BLAS_xsb_acc( enum blas_order_type order, enum blas_uplo_type uplo,
  enum blas_trans_type trans, int n, int k, SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta, ARRAY b, int ldb );
  ```
  ```c
  Symmetric Packed:
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void BLAS_xsp_acc( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, int n, SCALAR_IN alpha,
    const ARRAY ap, SCALAR_IN beta, ARRAY bp );

{GB,TR,TB,TP}_ACC (Matrix accumulation and scale) \( B \leftarrow \alpha A + \beta B \)

These routines scale matrices \( A \) and \( B \) and accumulate the result in the matrix \( B \). Matrices \( A \) and \( B \) have the same storage format. These routines return immediately if alpha is equal to zero and beta is equal to one, or if \( m \) or \( k \) or \( k_\text{u} \) (for general band matrices) or \( n \) or \( k \) (for triangular band matrices) is less than or equal to zero. For the routine GB_ACC, if \( \text{lda} \) is less than \( k_\text{l} \) plus \( k_\text{u} \) plus one, an error flag is set and passed to the error handler. For the routines TR_ACC and TP_ACC, if \( \text{lda} \) is less than one or \( \text{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the routine TB_ACC, if \( \text{lda} \) is less than \( k \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  General Band:
  ```fortran
  SUBROUTINE gb_acc( a, m, k1, b [, alpha] [, beta] )
  ```

  Triangular:
  ```fortran
  SUBROUTINE tr_acc( a, b [, uplo] [, diag] [, alpha] [, beta] )
  ```

  Triangular Band:
  ```fortran
  SUBROUTINE tb_acc( a, b [, uplo] [, diag] [, alpha] [, beta] )
  ```

  Triangular Packed:
  ```fortran
  ```

  all:
  ```fortran
  <type>(<wp>), INTENT(IN) :: a(:,:), ap(:)
  INTEGER, INTENT (IN) :: m, k1
  <type>(<wp>), INTENT(INOUT) :: b(:,:), bp(:)
  TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
  TYPE (blas_diag_type), INTENT (IN), OPTIONAL :: diag
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  ```

  The default value for \( \beta \) is 1.0 or (1,0,0).

- Fortran 77 binding:

  General Band:
  ```fortran
  SUBROUTINE BLAS_xGB_ACC( M, N, KL, KU, ALPHA, A, LDA, BETA, B,
    LDB )
  ```

  Triangular:
  ```fortran
  SUBROUTINE BLAS_xTR_ACC( UPLO, DIAG, N, ALPHA, A, LDA, BETA, B,
    LDB )
  ```

  Triangular Band:
  ```fortran
  SUBROUTINE BLAS_xTB_ACC( UPLO, DIAG, N, K, ALPHA, A, LDA, BETA, B,
    LDB )
  ```

  Triangular Packed:
  ```fortran
  SUBROUTINE BLAS_xTP_ACC( UPLO, DIAG, N, ALPHA, AP, BETA, BP )
  ```
all:
  INTEGER    DIAG, K, KL, KU, LDA, LDB, M, N, UPLO
  <type>     ALPHA, BETA
  <type>     A( LDA, * ) or AP( * ), B( LDB, * ) or BP( * )

- C binding:

  General Band:
  void BLAS_xgb_acc( enum blas_order_type order, int m, int n, int kl, int ku,
                     SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta,
                     ARRAY b, int ldb);

  Triangular:
  void BLAS_xtr_acc( enum blas_order_type order, enum blas_uplo_type uplo,
                     enum blas_diag_type diag, int n, SCALAR_IN alpha,
                     const ARRAY a, int lda, SCALAR_IN beta, ARRAY b, int ldb);

  Triangular Band:
  void BLAS_xtb_acc( enum blas_order_type order, enum blas_uplo_type uplo,
                     enum blas_diag_type diag, int n, int k, SCALAR_IN alpha,
                     const ARRAY a, int lda, SCALAR_IN beta, ARRAY b, int ldb);

  Triangular Packed:
  void BLAS_xtp_acc( enum blas_order_type order, enum blas_uplo_type uplo,
                     enum blas_diag_type diag, int n, SCALAR_IN alpha,
                     const ARRAY ap, SCALAR_IN beta, ARRAY bp );

\{GE,GB,SY,SP,TR,TB,TP\}_ADD (Matrix add and scale) \[ C \leftarrow \alpha A + \beta B \]

This routine scales two matrices \( A \) and \( B \) and stores their sum in a matrix \( C \). Matrices \( A \), \( B \),
and \( C \) have the same storage format. This routine returns immediately if \( m \) or \( kl \) or \( ku \) (for
general band matrices) or \( n \) or \( k \) (for symmetric or triangular band matrices) is less than or equal to zero.
For the routine \( \text{GE}_\text{ADD} \), if \( \text{lda} \) or \( \text{ldb} \) is less than one or less than \( m \), an error flag is set and passed to the
error handler. For the \( C \) bindings for \( \text{GE}_\text{ADD} \), if \( \text{order} = \text{blas_rowmajor} \) and if \( \text{lda} \) or \( \text{ldb} \) is less than one or \( \text{lda} \) or \( \text{ldb} \) is less than \( n \), an error flag is set and passed to the error handler; if \( \text{order} = \text{blas_colmajor} \) and if \( \text{lda} \) or \( \text{ldb} \) is less than one or \( \text{lda} \) or \( \text{ldb} \) is less than \( m \), an error flag is set and passed to the error handler. For the routine \( \text{GB}_\text{ADD} \), if \( \text{lda} \) or \( \text{ldb} \) is less than \( kl \) plus \( k \) plus one, an error flag is set and passed to the error handler. For the routines \( \text{SY}_\text{ADD}, \text{TR}_\text{ADD}, \text{SP}_\text{ADD}, \text{TB}_\text{ADD} \), if \( \text{lda} \) or \( \text{ldb} \) is less than one or \( \text{lda} \) or \( \text{ldb} \) is less than \( n \), an error flag is set and passed to the error handler. For the routines \( \text{SB}_\text{ADD} \) and \( \text{TB}_\text{ADD} \), if \( \text{lda} \) or \( \text{ldb} \) is less than \( k \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  General:
  SUBROUTINE ge_add( a, b, c [, alpha] [, beta] )

  General Band:
  SUBROUTINE gb_add( a, m, kl, b, c [, alpha] [, beta] )

  Symmetric:
  SUBROUTINE sy_add( a, b, c [, uplo] [, alpha] [, beta] )
Symmetric Band:
   SUBROUTINE sb_add( a, b, c [, uplo] [, alpha] [, beta] )
Symmetric Packed:
   SUBROUTINE sp_add( ap, bp, cp [, uplo] [, alpha] [, beta] )
Triangular:
   SUBROUTINE tr_add( a, b, c [, uplo] [, diag] [, alpha] [, beta] )
Triangular Band:
   SUBROUTINE tb_add( a, b, c [, uplo] [, diag] [, alpha] [, beta] )
Triangular Packed:
all:
   <type>(<wp>), INTENT(IN) :: a(:,:) | ap(:)
   INTEGER, INTENT (IN) :: m, kl
   <type>(<wp>), INTENT(IN) :: b(:,:) | bp(:)
   <type>(<wp>), INTENT(OUT) :: c(:,:) | cp(:)
   TYPE (blas_uplo_type), INTENT (IN), OPTIONAL :: uplo
   TYPE (blas_diag_type), INTENT (IN), OPTIONAL :: diag
   <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
assuming A, B and C all the same (general, banded or packed) with
the same size.
   a, b and c have shape (m,n) for general matrix
   (l,n) for general banded matrix (l > kl)
   (n,n) for symmetric or triangular
   (k+1,n) for symmetric banded or triangular
   banded (k=band width)
   ap, bp and cp have shape (n*(n+1)/2).

The default value for β is 1.0 or (1.0,0.0).

- Fortran 77 binding:

General:
   SUBROUTINE BLAS_xGE_ADD( M, N, ALPHA, A, LDA, BETA, B, LDB, C,
   $                   LDC )
General Band:
   SUBROUTINE BLAS_xGB_ADD( M, N, KL, KU, ALPHA, A, LDA, BETA, B,
   $                   LDB, C, LDC )
Symmetric:
   SUBROUTINE BLAS_xSY_ADD( UPLO, N, ALPHA, A, LDA, BETA, B, LDB,
   $                   C, LDC )
Symmetric Band:
   SUBROUTINE BLAS_xSB_ADD( UPLO, N, K, ALPHA, A, LDA, BETA, B, LDB,
   $                   C, LDC )
Symmetric Packed:
   SUBROUTINE BLAS_xSP_ADD( UPLO, N, ALPHA, AP, BETA, BP, CP )
Triangular:
   SUBROUTINE BLAS_xTR_ADD( UPLO, DIAG, N, ALPHA, A, LDA, BETA, B,
Triangular Band:

SUBROUTINE BLAS_xTB_ADD( UPLO, DIAG, N, K, ALPHA, A, LDA, BETA,
             B, LDB, C, LDC )

Triangular Packed:

SUBROUTINE BLAS_xTP_ADD( UPLO, DIAG, N, ALPHA, AP, BETA, BP, CP )

all:

INTEGER        DIAG, K, KL, KU, LDA, LDB, M, N, TRANS, UPLO
<type>         ALPHA, BETA
<type>         A( LDA, * ) or AP( * ), B( LDB, * ) or BP( * ),
<type>         C( LDC, * ) or CP( * )

• C binding:

General:
void BLAS_xge_add( enum blas_order_type order, int m, int n, SCALAR_IN alpha,
                   const ARRAY a, int lda, SCALAR_IN beta, const ARRAY b,
                   int ldb, ARRAY c, int ldc );

General Band:
void BLAS_xgb_add( enum blas_order_type order, int m, int n, int kl, int ku,
                   SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta,
                   const ARRAY b, int ldb, ARRAY c, int ldc );

Symmetric:
void BLAS_xsy_add( enum blas_order_type order, enum blas_uplo_type uplo, int n,
                   SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta,
                   const ARRAY b, int ldb, ARRAY c, int ldc );

Symmetric Band:
void BLAS_xsb_add( enum blas_order_type order, enum blas_uplo_type uplo,
                   int n, int k, SCALAR_IN alpha, const ARRAY a, int lda,
                   SCALAR_IN beta, const ARRAY b, int ldb, ARRAY c, int ldc );

Symmetric Packed:
void BLAS_xsp_add( enum blas_order_type order, enum blas_uplo_type uplo,
                   int n, SCALAR_IN alpha, const ARRAY ap, SCALAR_IN beta,
                   const ARRAY bp, ARRAY cp );

Triangular:
void BLAS_xtr_add( enum blas_order_type order, enum blas_uplo_type uplo,
                   enum blas_diag_type diag, int n, SCALAR_IN alpha,
                   const ARRAY a, int lda, SCALAR_IN beta, const ARRAY b,
                   int ldb, ARRAY c, int ldc );

Triangular Band:
void BLAS_xtb_add( enum blas_order_type order, enum blas_uplo_type uplo,
                   enum blas_diag_type diag, int n, int k, SCALAR_IN alpha,
                   const ARRAY a, int lda, SCALAR_IN beta, const ARRAY b,
                   int ldb, ARRAY c, int ldc );

Triangular Packed:
void BLAS_xtp_add( enum blas_order_type order, enum blas_uplo_type uplo,
                   enum blas_diag_type diag, int n, SCALAR_IN alpha,
                   const ARRAY ap, SCALAR_IN beta, const ARRAY bp,
2.8. LANGUAGE BINDINGS

ARRAY cp);

2.8.8 Matrix-Matrix Operations

In the following section, \( op(X) \) denotes \( X \), or \( X^T \) or \( X^H \) where \( X \) is a matrix.

GEMM (General Matrix Matrix Product)

\[
C \leftarrow \alpha op(A) op(B) + \beta C
\]

The routine performs a general matrix matrix multiply \( C \leftarrow \alpha op(A) op(B) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, and \( A, B, \) and \( C \) are general matrices. This routine returns immediately if \( \alpha \) is equal to zero and \( \beta \) is equal to one, or if \( m \) or \( n \) or \( k \) is less than or equal to zero. If \( \text{lda} \) is less than one, or \( \text{transa} = \text{blas_no_trans} \) and \( \text{lda} \) is less than \( m \), or \( \text{transa} \neq \text{blas_no_trans} \) and \( \text{lda} \) is less than \( k \), or \( \text{ldb} \) is less than one, or \( \text{transb} = \text{blas_no_trans} \) and \( \text{ldb} \) is less than \( k \), or \( \text{transb} \neq \text{blas_no_trans} \) and \( \text{ldb} \) is less than \( n \), or \( \text{ldc} \) is less than one or less than \( m \), an error flag is set and passed to the error handler.

This interface encompasses the Legacy BLAS routine \text{xGEMM}.

- Fortran 95 binding:

```
SUBROUTINE gemm( a, b, c [, transa] [, transb] [, alpha] [, beta] )
  <type>(<wp>), INTENT(IN) :: <aa>, <bb>
  <type>(<wp>), INTENT(INOUT) :: <cc>
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transa, transb
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
  <aa> :: a(:, :) or a(:)
  <bb> :: b(:, :) or b(:)
  <cc> :: c(:, :) or c(:)
and
  c, rank 2, has shape \((m,n)\)
    a has shape \((m,k)\) if \( \text{transa} = \text{blas_no_trans} \) (the default)
    \((k,m)\) if \( \text{transa} /= \text{blas_no_trans} \)
    \((m)\) if rank 1
  b has shape \((k,n)\) if \( \text{transb} = \text{blas_no_trans} \) (the default)
    \((n,k)\) if \( \text{transb} /= \text{blas_no_trans} \)
    \((n)\) if rank 1
  c, rank 1, has shape \((m)\)
    a has shape \((m,n)\) if \( \text{transa} = \text{blas_no_trans} \) (the default)
    \((n,m)\) if \( \text{transa} /= \text{blas_no_trans} \)
    b has shape \((n)\)
```
The functionality of xGEMV, xGER, xGERU, and xGERC are also covered by this generic procedure.

- Fortran 77 binding:

```
SUBROUTINE BLAS_xGEMM( TRANSA, TRANSB, M, N, K, ALPHA, A, LDA,
          $                     B, LDB, BETA, C, LDC )
INTEGER K, LDA, LDB, LDC, M, N, TRANSA, TRANSB,
<type>     ALPHA, BETA
<type>     A( LDA, * ), B( LDB, * ), C( LDC, * )
```

- C binding:

```
void BLAS_xgemm( enum blas_order_type order, enum blas_trans_type transa,
    enum blas_trans_type transb, int m, int n, int k,
    SCALAR_IN alpha, const ARRAY a, int lda, const ARRAY b,
    int ldb, SCALAR_IN beta, ARRAY c, int ldc );
```

SYMM (Symmetric Matrix Matrix Product) \( C \leftarrow \alpha AB + \beta C \) or \( C \leftarrow \alpha BA + \beta C \)

This routine performs one of the symmetric matrix matrix operations \( C \leftarrow \alpha AB + \beta C \) or \( C \leftarrow \alpha BA + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( A \) is a symmetric matrix, and \( B \) and \( C \) are general matrices. This routine returns immediately if alpha is equal to zero and beta is equal to one, or if \( m \) or \( n \) is less than or equal to zero. For side equal to blas_left_side, and if \( \text{lda} \) is less than one or less than \( m \), or if \( \text{ldb} \) is less than one or less than \( m \), or if \( \text{ldc} \) is less than one or less than \( m \), an error flag is set and passed to the error handler. For side equal to blas_right_side, and if \( \text{lda} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( n \), or if \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler.

The interfaces encompass the Legacy BLAS routine xSYMM with added functionality for complex symmetric matrices.

- Fortran 95 binding:
2.8. LANGUAGE BINDINGS

SUBROUTINE symm( a, b, c [, side] [, uplo] [, alpha] [, beta] )
  <type>(<wp>), INTENT(IN) :: a(:,:), bb
  <type>(<wp>), INTENT(INOUT) :: cc
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
  <bb> ::= b(:,:) or b(:)
  <cc> ::= c(:,:) or c(:)

and
  c, rank 2, has shape (m,n), b same shape as c
  SY  a has shape (m,m) if side = blas_left_side (the default)
  a has shape (n,n) if side /= blas_left_side
  c, rank 1, has shape (m), b same shape as c
  SY  a has shape (m,m)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>Rank c</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>L</td>
<td>C &lt;- alpha A B + beta C</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>R</td>
<td>C &lt;- alpha B A + beta C</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>c &lt;- alpha A b + beta c</td>
</tr>
</tbody>
</table>

The functionality of xSYMV is covered by symm.

- Fortran 77 binding:

  SUBROUTINE BLAS_xSYM( SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB,
                          $ INTEGER LDA, LDB, LDC, M, N, SIDE, UPLO
                          <type> ALPHA, BETA
                          <type> A( LDA, * ), B( LDB, * ), C( LDC, * )

- C binding:

  void BLAS_xsymm( enum blas_order_type order, enum blas_side_type side,
                   enum blas_uplo_type uplo, int m, int n, SCALAR_IN alpha,
                   const ARRAY a, int lda, const ARRAY b, int ldb,
                   SCALAR_IN beta, ARRAY c, int ldc );

HEMM (Hermitian Matrix Matrix Product)  
  C <- alpha A B + beta C or C <- alpha B A + beta C

This routine performs one of the Hermitian matrix matrix operations
  C <- alpha A B + beta C or
  C <- alpha B A + beta C where alpha and beta are scalars, A is a Hermitian matrix,
  and B and C are general matrices. This routine returns immediately if alpha
  is equal to zero and beta is equal to one, or if
  m or n is less than or equal to zero. For side equal to blas_left_side, and if lda
  is less than one or less than m, or if ldb is less than one or less than m, or if ldc
  is less than one or less than m, an error flag is set and passed to the error
  handler. For side equal to blas_right_side, and if lda is less than
  one or less than n, or if ldb is less than one or less than n, or if ldc
  is less than one or less than n, an error flag is set and passed to the error
  handler.

The interfaces encompass the Legacy BLAS routine xHEMM.
• Fortran 95 binding:

Hermitian:

```fortran
SUBROUTINE hemm( a, b, c [, side] [, uplo] [, alpha] [, beta] )
  COMPLEX(<wp>), INTENT(IN) :: a(:,:), bb
  COMPLEX(<wp>), INTENT(INOUT) :: <cc>
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
  <bb> ::= b(:,:) or b(:)
  <cc> ::= c(:,:) or c(:)
and
  c, rank 2, has shape (m,n), b same shape as c
  HE a has shape (m,m) if "side" = blas_left_side (the default)
    a has shape (n,n) if "side" /= blas_left_side
  c, rank 1, has shape (m), b same shape as c
  HE a has shape (m,m)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>Rank c</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>L</td>
<td>(C \leftarrow \alpha A B + \beta C)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>R</td>
<td>(C \leftarrow \alpha B A + \beta C)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>(c \leftarrow \alpha A b + \beta c)</td>
</tr>
</tbody>
</table>
```

The functionality of xHEMV is covered by hemm.

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xHEMM( SIDE, UPLO, M, N, ALPHA, A, LDA, B, LDB,
$                               \quad BETA, C, LDC )
INTEGER                     LDA, LDB, LDC, M, N, SIDE, UPLO
<ctype>                      ALPHA, BETA
<ctype>                      A( LDA, * ), B( LDB, * ), C( LDC, * )
```

• C binding:

```c
void BLAS_xhemm( enum blas_order_type order, enum blas_side_type side,
    enum blas_uplo_type uplo, int m, int n, CSCALAR_IN alpha,
    const CARRAY a, int lda, const CARRAY b, int ldb,
    CSCALAR_IN beta, CARRAY c, int ldc );
```

TRMM (Triangular Matrix Matrix Multiply)  \(B \leftarrow \alpha o p(T) B\) or \(B \leftarrow \alpha B o p(T)\)

These routines perform one of the matrix-matrix operations \(B \leftarrow \alpha o p(T) B\) or \(B \leftarrow \alpha B o p(T)\)
where \(\alpha\) is a scalar, \(B\) is a general matrix, and \(T\) is a unit, or non-unit, upper or lower triangular
(or triangular band) matrix. This routine returns immediately if \(m, n, \) or \(k\) (for triangular band
matrices), is less than or equal to zero. For side equal to blas_left_side, and if ldt is less than one
or less than \( m \), or if \( \text{ldb} \) is less than one or less than \( m \), an error flag is set and passed to the error handler. For \( \text{side} \) equal to \( \text{blas\_right\_side} \), and if \( \text{ldt} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( m \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine \( \text{xTRMM} \).

- Fortran 95 binding:

```fortran
  <type>(<wp>), INTENT(IN) :: t(:, :)
  <type>(<wp>), INTENT(INOUT) :: b
  TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
where
  b \cdot \text{bb} \cdot b(:, :) \text{ or b( : )}
and
  \( b \), rank 2, has shape \(( m, n)\)
  TR \( t \) has shape \(( m, m)\) if \( \text{side} = \text{blas\_left\_side}\) (the default)
  \( t \) has shape \(( n, n)\) if \( \text{side} /= \text{blas\_left\_side}\)
  \( b \), rank 1, has shape \(( m)\)
  TR \( t \) has shape \(( m, m)\)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>transt</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>L</td>
<td>( B \leftarrow \alpha TB )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>L</td>
<td>( B \leftarrow \alpha T^T B )</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>L</td>
<td>( B \leftarrow \alpha T^H B )</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>R</td>
<td>( B \leftarrow \alpha BT )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>R</td>
<td>( B \leftarrow \alpha BT^T )</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>R</td>
<td>( B \leftarrow \alpha BT^H )</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>-</td>
<td>( b \leftarrow \alpha Tb )</td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>-</td>
<td>( b \leftarrow \alpha T^Tb )</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>-</td>
<td>( b \leftarrow \alpha T^Hb )</td>
</tr>
</tbody>
</table>
```

The functionality of \( \text{xTRMV} \) is covered by \( \text{trmm} \).

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_XTRMM( SIDE, UPLO, TRANST, DIAG, M, N, ALPHA, T,
  $ \quad \text{LDT, B, LDB} )$
  INTEGER            DIAG, LDB, LDT, M, N, SIDE, TRANST, UPLO
  <type>              ALPHA
  <type>              T( LDT, * ), B( LDB, * )
```

- C binding:
void BLAS_xtrmm( enum blas_order_type order, enum blas_side_type side,
    enum blas_uplo_type uplo, enum blas_trans_type trans,
    enum blas_diag_type diag, int m, int n, SCALAR_IN alpha,
    const ARRAY t, int ldt, ARRAY b, int ldb );

TRSM (Triangular Solve)

This routine solves one of the matrix equations \( B \leftarrow \alpha op(T^{-1})B \) or \( B \leftarrow \alpha B op(T^{-1}) \) where \( \alpha \) is a scalar, \( B \) is a general matrix, and \( T \) is a unit, or non-unit, upper or lower triangular matrix. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For side equal to blas_left_side, and if ldt is less than one or less than \( m \), or if ldb is less than one or less than \( m \), an error flag is set and passed to the error handler. For side equal to blas_right_side, and if ldt is less than one or less than \( n \), or if ldb is less than one or less than \( m \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xTRSM.

*Advice to implementors.* Note that no check for singularity, or near singularity is specified for these triangular equation-solving routines. The requirements for such a test depend on the application, and so we felt that this should not be included, but should instead be performed before calling the triangular solver. (*End of advice to implementors.*)

- Fortran 95 binding:

```fortran
  <type>(<wp>), INTENT(IN) :: t(:,:)
  <type>(<wp>), INTENT(INOUT) :: <bb>
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
  TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha

where

<bb> :: b(:,:) or b(:)

and

b, rank 2, has shape \((m,n)\)

TR \( t \) has shape \((m,m)\) if side = blas_left_side (the default)

\( t \) has shape \((n,n)\) if side \(!=\) blas_left_side

b, rank 1, has shape \((m)\)

TR \( t \) has shape \((m,m)\)
```

<table>
<thead>
<tr>
<th>Rank b</th>
<th>transt</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>L</td>
<td>( B \leftarrow \alpha T^{-1}B )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>L</td>
<td>( B \leftarrow \alpha T^T B )</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>L</td>
<td>( B \leftarrow \alpha T^{-H}B )</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>R</td>
<td>( B \leftarrow \alpha BT^{-1} )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>R</td>
<td>( B \leftarrow \alpha BT^{-T} )</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>R</td>
<td>( B \leftarrow \alpha BT^{-H} )</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>-</td>
<td>( b \leftarrow \alpha T^{-1}b )</td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>-</td>
<td>( b \leftarrow \alpha T^{-T}b )</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>-</td>
<td>( b \leftarrow \alpha T^{-H}b )</td>
</tr>
</tbody>
</table>
The functionality of xTRSV is covered by trsm.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xTRSM( SIDE, UPLO, TRANS, DIAG, M, N, ALPHA,
                        $ T, LDT, B, LDB )
INTEGER          DIAG, LDB, LDT, M, N, SIDE, TRANS, UPLO
<type>            ALPHA
<type>            T( LDT, * ), B( LDB, * )
```

- C binding:

```c
void BLAS_xtrsm( enum blas_order_type order, enum blas_side_type side,
    enum blas_uplo_type uplo, enum blas_trans_type trans,
    enum blas_diag_type diag, int m, int n, SCALAR_IN alpha,
    const ARRAY t, int ldt, ARRAY b, int ldb );
```

SYRK (Symmetric Rank K update) \( C \leftarrow \alpha A A^T + \beta C \), \( C \leftarrow \alpha A^T A + \beta C \)

This routine performs one of the symmetric rank k operations \( C \leftarrow \alpha A A^T + \beta C \) or \( C \leftarrow \alpha A^T A + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a symmetric matrix, and \( A \) is a general matrix. This routine returns immediately if alpha is equal to zero and beta is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \text{trans} equal to \text{blas_no_trans}, and if \( \text{lda} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \text{trans} equal to \text{blas_trans}, and if \( \text{lda} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xSYRK with added functionality for complex symmetric matrices.

- Fortran 95 binding:

```fortran
SUBROUTINE syrk( a, c [, uplo] [, trans] [, alpha] [, beta] )
<t>(<wp>), INTENT(IN) :: <aa>
<t>(<wp>), INTENT(INOUT) :: c(:,:)
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
<t>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
```

where

\(<\text{aa}> ::= \text{a}(:,:) \) or \( \text{a}(:,) \)

and

\(<\text{c}> \text{ has shape } (n,n)\)
\(<\text{a}> \text{ has shape } (n,k) \text{ if } \text{trans} = \text{blas_no_trans} \text{ (the default)}\)
\(<(k,n) \text{ if } \text{trans} \neq \text{blas_no_trans}\)
\(<(n) \text{ if rank 1}\>

<table>
<thead>
<tr>
<th>Rank a</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha A A^T + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^T A + \beta C )</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>( C \leftarrow \alpha a a^T + \beta C )</td>
</tr>
</tbody>
</table>
The functionality of xSYR is covered by syrk.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYRK( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA, 
$ C, LDC )
INTEGER K, LDA, LDC, N, TRANS, UPLO
<type> ALPHA, BETA
<type> A( LDA, * ), C( LDC, * )
```

- C binding:

```c
void BLAS_xsyrk( enum blas_order_type order, enum blas_uplo_type uplo, 
enum blas_trans_type trans, int n, int k, SCALAR_IN alpha, 
const ARRAY a, int lda, SCALAR_IN beta, ARRAY c, int ldc );
```

**HERK** (Hermitian Rank K update) $C \leftarrow \alpha AA^H + \beta C, C \leftarrow \alpha A^H A + \beta C$

This routine performs one of the Hermitian rank k operations $C \leftarrow \alpha AA^H + \beta C$ or $C \leftarrow \alpha A^H A + \beta C$ where $\alpha$ and $\beta$ are scalars, $C$ is a Hermitian matrix, and $A$ is a general matrix. This routine returns immediately if $\alpha$ is equal to zero and $\beta$ is equal to one, or if n or k is less than or equal to zero. If ldc is less than one or less than n, an error flag is set and passed to the error handler. For trans equal to blas_no_trans, and if lda is less than one or less than n, an error flag is set and passed to the error handler. For trans equal to blas_trans, and if lda is less than one or less than k, an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xHERK.

- Fortran 95 binding:

```fortran
SUBROUTINE herk( a, c [, uplo] [, trans] [, alpha] [, beta] )
COMPLEX(<wp>), INTENT(IN) :: <aa>
COMPLEX(<wp>), INTENT(INOUT) :: c(:,:)
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
REAL(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
```

where

```fortran
<aa> ::= a(:,:) or a(:)
```

and

```fortran
c has shape (n,n)
a has shape (n,k) if trans = blas_no_trans (the default)
(k,n) if trans /= blas_no_trans
(n) if rank 1
```

<table>
<thead>
<tr>
<th>Rank a</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>$C \leftarrow \alpha AA^H + \beta C$</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>$C \leftarrow \alpha A^H A + \beta C$</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>$C \leftarrow \alpha a a^H + \beta C$</td>
</tr>
</tbody>
</table>
The functionality of xHER is covered by herk.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xHERK( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA, C, $
                      LDC )
  INTEGER K, LDA, LDC, N, TRANS, UPLO
  <type> ALPHA, BETA
  <ctype> A( LDA, * ), C( LDC, * )
```

- C binding:

```c
void BLAS_xherk( enum blas_order_type order, enum blas_uplo_type uplo,
                 enum blas_trans_type trans, int n, int k, RSCALAR_IN alpha,
                 const CARRAY a, int lda, RSCALAR_IN beta, CARRAY c, int ldc );
```

---

**SY_TRIDIAG_RK** (Symmetric Rank K update with symmetric tridiagonal matrix)

\[
C \leftarrow \alpha A A^T + \beta C, \quad C \leftarrow \alpha A^T J A + \beta C
\]

This routine performs one of the symmetric rank k operations \( C \leftarrow \alpha A A^T + \beta C \) or \( C \leftarrow \alpha A^T J A + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a symmetric matrix, \( A \) is a general matrix, and \( J \) is a symmetric tridiagonal matrix. This routine returns immediately if alpha is equal to zero and beta is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_no_trans} \), and if \( \text{lda} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_trans} \), and if \( \text{lda} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE sy_tridiag_rk( a, d, e, c [, uplo] [, trans] [, alpha] &
                         [, beta] )
  <type>(<wp>), INTENT(IN) :: a(:,:),
  <type>(<wp>), INTENT(IN) :: d(:,:), e(:)
  <type>(<wp>), INTENT(INOUT) :: c(:,:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
  c has shape (n,n)
  if trans = blas_no_trans (the default)
    a has shape (n,k)
    d has shape (k)
    e has shape (k-1)
  if trans /= blas_no_trans
    a has shape (k,n)
    d has shape (n)
    e has shape (n-1)
```
<table>
<thead>
<tr>
<th>Rank a</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha A A^T + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^T J A + \beta C )</td>
</tr>
</tbody>
</table>

- Fortran 77 binding:

```
SUBROUTINE BLAS_xSY_TRIANG_RK( UPLO, TRANS, N, K, ALPHA, A, LDA, D, $  E, BETA, C, LDC )
INTEGER K, LDA, LDC, N, TRANS, UPLO
<type> ALPHA, BETA
<type> A( LDA, * ), C( LDC, * ), D( * ), E( * )
```

- C binding:

```
void BLAS_xsy_tridiag_rk( enum blas_order_type order, enum blas_uplo_type uplo, 
  enum blas_trans_type trans, int n, int k, 
  SCALAR_IN alpha, const ARRAY a, int lda, 
  const ARRAY d, const ARRAY e, SCALAR_IN beta, 
  ARRAY c, int ldc );
```

**HE_TRIDIAG_RK** (Hermitian Rank K update with symmetric tridiagonal matrix)

\[
C \leftarrow \alpha A A^H + \beta C, C \leftarrow \alpha A^H J A + \beta C
\]

This routine performs one of the Hermitian rank k operations \( C \leftarrow \alpha A A^H + \beta C \) or \( C \leftarrow \alpha A^H J A + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a Hermitian matrix, \( A \) is a general matrix, and \( J \) is a symmetric tridiagonal matrix. This routine returns immediately if alpha is equal to zero and beta is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( ldc \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For trans equal to blas_no_trans, and if \( lda \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For trans equal to blas_trans, and if \( lda \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```
SUBROUTINE he_tridiag_rk( a, d, e, c [, uplo] [, trans] [, alpha] &  
  [, beta] )
COMPLEX(<wp>), INTENT(IN) :: a(:,:)
COMPLEX(<wp>), INTENT(IN) :: d(:,), e(:)
COMPLEX(<wp>), INTENT(INOUT) :: c(:,:),
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_trans_type), INTENT(IN), OPTIONAL :: trans
REAL(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
  c has shape (n,n)
if trans = blas_no_trans (the default)
  a has shape (n,k)
  d has shape (k)
```
e has shape (k-1)
if trans /= blas_no_trans
  a has shape (k,n)
  d has shape (n)
e has shape (n-1)

<table>
<thead>
<tr>
<th>Rank</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha A A^H + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^H A + \beta C )</td>
</tr>
</tbody>
</table>

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xHE_TRIDIAG_RK( UPLO, TRANS, N, K, ALPHA, A, LDA,
                                D, E, BETA, C, LDC )
  INTEGER K, LDA, LDC, N, TRANS, UPLO
  <rtype> ALPHA, BETA
  <ctype> A( LDA, * ), C( LDC, * ), D( * ), E( * )
```

- C binding:

```c
void BLAS_xhe_tridiag_rk( enum blas_order_type order, enum blas_uplo_type uplo,
                          enum blas_trans_type trans, int n, int k,
                          RSCALAR_IN alpha, const CARRAY a, int lda,
                          const CARRAY d, const CARRAY e, RSCALAR_IN beta,
                          CARRAY c, int ldc );
```

SYR2K (Symmetric rank 2k update)

\[
C \leftarrow (\alpha A)B^T + B(\alpha A)^T + \beta C
\]

\[
C \leftarrow (\alpha A)^T B + B^T(\alpha A) + \beta C
\]

These routines perform the symmetric rank 2k operation \( C \leftarrow (\alpha A)B^T + B(\alpha A)^T + \beta C \) or \( C \leftarrow (\alpha A)^T B + B^T(\alpha A) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a symmetric matrix, and \( A \) and \( B \) are general matrices. This routine returns immediately if \( \alpha \) is equal to zero and \( \beta \) is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_no_trans} \), and if \( \text{lda} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_trans} \), and if \( \text{lda} \) is less than one or less than \( k \), or if \( \text{ldb} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xSYR2K with added functionality for complex symmetric matrices.

- Fortran 95 binding:

```fortran
SUBROUTINE syr2k( a, b, c [, uplo] [, trans] [, alpha] [, beta] )
  <type>(<wp>), INTENT(IN) :: <aa>, <bb>
  <type>(<wp>), INTENT(INOUT) :: c(:,:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
```
CHAPTER 2. DENSE AND BANDED BLAS

\[
\text{TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans} \\
<\text{type>(<up>), INTENT(IN), OPTIONAL :: alpha, beta}
\]

\[\text{where}
\]
\[\text{<aa> ::= a(;++), or a(;;)}
\]
\[\text{<bb> ::= b(;++), or b(;;)}
\]

\[\text{and}
\]
\[\text{c has shape (n,n)}
\]
\[\text{if trans = blas_no_trans (the default)}
\]
\[\text{a has shape (n,k)}
\]
\[\text{b has shape (n,k)}
\]
\[\text{if trans /= blas_no_trans}
\]
\[\text{a has shape (k,n)}
\]
\[\text{b has shape (k,n)}
\]

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Rank a} & \text{Rank b} & \text{trans} & \text{Operation} \\
\hline
2 & 2 & N & C \leftarrow \alpha A B^T + \alpha B A^T + \beta C \\
2 & 2 & T & C \leftarrow \alpha A^T B + \alpha B^T A + \beta C \\
1 & 1 & - & C \leftarrow \alpha A B^T + \alpha A^T B + \beta C \\
\hline
\end{array}
\]

The functionality of xSYR2 is covered by syr2k.

- Fortran 77 binding:

\[
\text{SUBROUTINE BLAS_xSYR2K( UPLO, TRANS, N, K, ALPHA, A, LDA, B, LDB,}
\text{ BETA, C, LDC )}
\]

\[
\text{INTEGER}
\]
\[
\text{K, LDA, LDB, LDC, N, TRANS, UPLO}
\]

\[
\text{<type>}
\]
\[
\text{<type>}
\]

\[
\text{A( LDA, * ), B( LDB, * ), C( LDC, * )}
\]

- C binding:

\[
\text{void BLAS_xsyr2k( enum blas_order_type order, enum blas_uplo_type uplo,}
\text{ enum blas_trans_type trans, int n, int k, SCALAR_IN alpha,}
\text{ const ARRAY a, int lda, const ARRAY b, int ldb,}
\text{ SCALAR_IN beta, ARRAY c, int ldc );}
\]

HER2K (Hermitian rank 2k update)

\[
C \leftarrow (\alpha A) B^H + B (\alpha A)^H + \beta C
\]

\[
C \leftarrow (\alpha A)^H B + B^H (\alpha A) + \beta C
\]

These routines perform the Hermitian rank 2k operation \( C \leftarrow (\alpha A) B^H + B (\alpha A)^H + \beta C \) or \( C \leftarrow (\alpha A)^H B + B^H (\alpha A) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a Hermitian matrix, and \( A \) and \( B \) are general matrices. This routine returns immediately if \( \alpha \) is equal to zero and \( \beta \) is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_no_trans} \), and if \( \text{lda} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_trans} \), and if \( \text{lda} \) is less than one or less than \( k \), or if \( \text{ldb} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xHER2K.
2.8. LANGUAGE BINDINGS

- Fortran 95 binding:

```fortran
SUBROUTINE her2k( a, b, c [, uplo] [, trans] [, alpha] [, beta] )
COMPLEX(<wp>), INTENT(IN) :: <aa>, <bb>
COMPLEX(<wp>), INTENT(INOUT) :: c(:,:)
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha
REAL(<wp>), INTENT(IN), OPTIONAL :: beta
where
<aa> ::= a(:,:) or a(:)
<bb> ::= b(:,:) or b(:)
and
c has shape (n,n)
a and b have shape (n,k) if trans = blas_no_trans (the default)
(k,n) if trans /= blas_no_trans
(n) if rank 1

<table>
<thead>
<tr>
<th>Rank a</th>
<th>Rank b</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha A B^H + \alpha B A^H + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^H B + \alpha B^H A + \beta C )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>( C \leftarrow aab^H + \alpha ba^H + \beta C )</td>
</tr>
</tbody>
</table>
```

The functionality of xHER2 is covered by her2k.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xHER2K( UPLO, TRANS, N, K, ALPHA, A, LDA, B, LDB,
                        \$                                BETA, C, LDC )
INTEGER K, LDA, LDB, LDC, N, TRANS, UPLO
<ctype>       ALPHA
<rtype>       BETA
<ctype>       A( LDA, * ), B( LDB, * ), C( LDC, * )
```

- C binding:

```c
void BLAS_xher2k( enum blas_order_type order, enum blas_uplo_type uplo,
                  enum blas_trans_type trans, int n, int k, CSCALAR_IN alpha,
                  const CARRAY A, int lda, const CARRAY b, int ldb,
                  RSCALAR_IN beta, CARRAY c, int ldc );
```

SY_TRIDIAG_R2K (Symmetric rank 2k update with symmetric tridiagonal matrix)

\[
C \leftarrow (\alpha A J) B^T + B (\alpha A J)^T + \beta C
\]

\[
C \leftarrow (\alpha A J)^T B + B^T (\alpha A J) + \beta C
\]
These routines perform the symmetric rank 2k operation $C \leftarrow (\alpha A J) B^T + B (\alpha A J)^T + \beta C$ or $C \leftarrow (\alpha A J)^T B + B^T (\alpha A J) + \beta C$ where $\alpha$ and $\beta$ are scalars, $C$ is a symmetric matrix, $A$ and $B$ are general matrices, and $J$ is a symmetric tridiagonal matrix. This routine returns immediately if $\alpha$ is equal to zero and $\beta$ is equal to one, or if $n$ or $k$ is less than or equal to zero. If $\text{ldc}$ is less than one or less than $n$, an error flag is set and passed to the error handler. For $\text{trans}$ equal to $\text{blas_no_trans}$, and if $\text{lda}$ is less than one or less than $n$, or if $\text{ldb}$ is less than one or less than $n$, an error flag is set and passed to the error handler. For $\text{trans}$ equal to $\text{blas_trans}$, and if $\text{lda}$ is less than one or less than $k$, or if $\text{ldb}$ is less than one or less than $k$, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE sy_tridiag_r2k( a, d, e, b, c [, uplo] [, trans] &
                          [, alpha] [, beta] )
  <type>(<wp>), INTENT(IN) :: a(:,,:), b(:,:)
  <type>(<wp>), INTENT(IN) :: d(:,:), e(:)
  <type>(<wp>), INTENT(INOUT) :: c(:,:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha
  <type>(<wp>), INTENT(IN), OPTIONAL :: beta

where

c has shape (n,n)
if trans = blas_no_trans (the default)
  a and b have shape (n,k)
  d has shape (k)
  e has shape (k-1)
if trans /= blas_no_trans
  a and b have shape (k,n)
  d has shape (n)
  e has shape (n-1)
```

<table>
<thead>
<tr>
<th>Rank a</th>
<th>Rank b</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>N</td>
<td>$C \leftarrow (\alpha A J) B^T + B (\alpha A J)^T + \beta C$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>T</td>
<td>$C \leftarrow (\alpha A J)^T B + B^T (\alpha A J) + \beta C$</td>
</tr>
</tbody>
</table>

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_XSY_TRIDIAG_R2K( UPLO, TRANS, N, K, ALPHA, A, LDA,
                                 D, E, B, LDB, BETAN, C, LDC )
  INTEGER K, LDA, LDB, LDC, N, TRANS, UPLO
  <type> ALPHA, BETAN
  <type> A( LDA, * ), B( LDB, * ), C( LDC, * ),
  D( * ), E( * )
```

- C binding:
2.8. LANGUAGE BINDINGS

void BLAS_xsy_tridiag_r2k( enum blas_order_type order,
    enum blas_uplo_type uplo,
    enum blas_trans_type trans, int n, int k,
    SCALAR_IN alpha, const ARRAY a, int lda,
    const ARRAY d, const ARRAY e, const ARRAY b,
    int ldb, SCALAR_IN beta, ARRAY c, int ldc );

HE_TRIDIAG_R2K (Hermitian rank 2k update with symmetric tridiagonal matrix)

\[
C \leftarrow (\alpha A)B^H + B(\alpha A)^H + \beta C \\
C \leftarrow (\alpha A)^H B + B^H(\alpha A) + \beta C
\]

These routines perform the symmetric rank 2k operation \( C \leftarrow (\alpha A)B^H + B(\alpha A)^H + \beta C \) or \( C \leftarrow (\alpha A)^H B + B^H(\alpha A) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a Hermitian matrix, \( A \) and \( B \) are general matrices, and \( J \) is a symmetric tridiagonal matrix. This routine returns immediately if \( \text{alpha} \) is equal to zero and \( \text{beta} \) is equal to one, or if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_no_trans} \), and if \( \text{lda} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_trans} \), and if \( \text{lda} \) is less than one or less than \( k \), or if \( \text{ldb} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```
SUBROUTINE he_tridiag_r2k( a, d, e, b, c [, uplo] [, trans] &
    [, alpha] [, beta] )
    COMPLEX (<wp>), INTENT(IN) :: a(:,:), b(:,:)
    COMPLEX (<wp>), INTENT(IN) :: d(:,), e(:)
    COMPLEX (<wp>), INTENT(INOUT) :: c(:,:)
    TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
    TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
    COMPLEX (<wp>), INTENT(IN), OPTIONAL :: alpha
    REAL(<wp>), INTENT(IN), OPTIONAL :: beta

where

c has shape (n,n)
if "trans" = blas_no_trans (the default)
a and b have shape (n,k)
d has shape (k)
e has shape (k-1)
if "trans" /= blas_no_trans
a and b have shape (k,n)
d has shape (n)
e has shape (n-1)
```

<table>
<thead>
<tr>
<th>Rank a</th>
<th>Rank b</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>N</td>
<td>( C \leftarrow (\alpha A)B^H + B(\alpha A)^H + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>T</td>
<td>( C \leftarrow (\alpha A)^H B + B^H(\alpha A) + \beta C )</td>
</tr>
</tbody>
</table>
• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xHE_TRIDIAG_R2K( UPLO, TRANS, N, K, ALPHA, A, LDA,
   $                      D, E, B, LDB, BETA, C, LDC )
INTEGER K, LDA, LDB, LDC, N, TRANS, UPLO
<ctype>      ALPHA
<rtype>      BETA
<ctype>      A( LDA, * ), B( LDB, * ), C( LDC, * ),
$                      D( * ), E( * )
```

• C binding:

```c
void BLAS_xhe_tridiag_r2k( enum blas_order_type order,
   enum blas_uplo_type uplo,
   enum blas_trans_type trans, int n, int k,
   CSCALAR_IN alpha, const CARRAY a, int lda,
   const CARRAY d, const CARRAY e, const CARRAY b,
   int ldb, RSCALAR_IN beta, CARRAY c, int ldc );
```

### 2.8.9 Data Movement with Matrices

{GE,GB,SY,SB,SP,TR,TB,TP}.COPY (Matrix copy) \( B \leftarrow A, B \leftarrow A^T, B \leftarrow A^H \)

This routine copies a matrix (or its transpose or conjugate-transpose) \( A \) and stores the result in a matrix \( B \). Matrices \( A \) and \( B \) have the same storage format. This routine returns immediately if \( m \) (for nonsymmetric matrices), \( n, k \) (for symmetric band matrices), or \( kl \) or \( ku \) (for general band matrices), is less than or equal to zero. For the routine GE_COPY, if trans equal to blas_no_trans, and if lda is less than one or less than m, or if ldb is less than one or less than m, an error flag is set and passed to the error handler. For the routine GE_COPY, if trans equal to blas_trans or blas_conj_trans, and if lda is less than one or less than m, or if ldb is less than one or less than n, an error flag is set and passed to the error handler. For the routine GB_COPY, if lda is less than kl plus ku plus one, or if ldb is less than kl plus ku plus one, an error flag is set and passed to the error handler. For the routines SY_COPY and TR_COPY, if lda is less than one or less than n, or if ldb is less than one or less than n, an error flag is set and passed to the error handler. For the routines SB_COPY and TB_COPY, if lda is less than k plus one, or if ldb is less than k plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:

```fortran
General:
   SUBROUTINE ge_copy( a, b [, trans] )
General Band:
   SUBROUTINE gb_copy( a, b, m, kl [, trans] )
Symmetric:
   SUBROUTINE sy_copy( a, b [, uplo] )
Symmetric Band:
```

2.8. LANGUAGE BINDINGS

SUBROUTINE sb_copy( a, b [, uplo] )
Symmetric Packed:
   SUBROUTINE sp_copy( ap, bp [, uplo] )
Triangular:
   SUBROUTINE tr_copy( a, b [, uplo] [,trans] [, diag] )
   Triangular Band:
   SUBROUTINE tb_copy( a, b [, uplo] [,trans] [, diag] )
   Triangular Packed:
   SUBROUTINE tp_copy( ap, bp [, uplo] [,trans] [, diag] )
all:
   <type>(<wp>), INTENT(IN) :: a(:,:) or ap(:)
   <type>(<wp>), INTENT(OUT) :: b(:,:) or bp(:)
   INTEGER, INTENT(IN) :: m, kl
   TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
   TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
   TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
   where
   a and b have shape (n,n) for symmetric or triangular
   (k+1,n) for symmetric banded or triangular
   banded (k=band width)
   ap and bp have shape (n*(n+1)/2).
For a general or general banded matrix:
   If trans = blas_no_trans (the default)
   a, b have shape (m,n) for general matrix
   (1,n) for general banded matrix (l > kl)
   If trans \= blas_no_trans
   a has shape (m,n) and b has shape (n,m) for general matrix
   (1,n) and b has shape (1,m) for general banded matrix (1 > kl)

- Fortran 77 binding:

General:
   SUBROUTINE BLAS_xGE_COPY( TRANS, M, N, A, LDA, B, LDB )
General Band:
   SUBROUTINE BLAS_xGB_COPY( TRANS, M, N, KL, KU, A, LDA, B, LDB )
Symmetric:
   SUBROUTINE BLAS_xSY_COPY( UPLO, N, A, LDA, B, LDB )
Symmetric Band:
   SUBROUTINE BLAS_xSB_COPY( UPLO, N, K, A, LDA, B, LDB )
Symmetric Packed:
   SUBROUTINE BLAS_xSP_COPY( UPLO, N, AP, BP )
Triangular:
   SUBROUTINE BLAS_xTR_Copy( UPLO, TRANS, DIAG, N, A, LDA, B, LDB )
Triangular Band:
   SUBROUTINE BLAS_xtB_Copy( UPLO, TRANS, DIAG, N, K, A, LDA, B,
   $ LDB )
Triangular Packed:
   SUBROUTINE BLAS_xtP_Copy( UPLO, TRANS, DIAG, N, AP, BP )
all:

```
INTEGER       DIAG, LDA, LDB, N, K, KL, KU, TRANS, UPLO
<type>        A( LDA, * ) or AP( * ), B( LDB, * ) or BP( * )
```

• C binding:

General:
```
void BLAS_xge_copy( enum blas_order_type order, enum blas_trans_type trans,
                    int m, int n, const ARRAY a, int lda, ARRAY b, int ldb );
```

General Band:
```
void BLAS_xgb_copy( enum blas_order_type order, enum blas_trans_type trans,
                    int m, int n, int kl, int ku, const ARRAY a, int lda,
                    ARRAY b, int ldb );
```

Symmetric:
```
void BLAS_xsy_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    int n, const ARRAY a, int lda, ARRAY b, int ldb );
```

Symmetric Band:
```
void BLAS_xsb_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    int n, int k, const ARRAY a, int lda, ARRAY b, int ldb );
```

Symmetric Packed:
```
void BLAS_xsp_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    int n, const ARRAY ap, ARRAY bp );
```

Triangular:
```
void BLAS_xtr_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    enum blas_trans_type trans, enum blas_diag_type diag,
                    int n, const ARRAY a, int lda, ARRAY b, int ldb );
```

Triangular Band:
```
void BLAS_xtb_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    enum blas_trans_type trans, enum blas_diag_type diag,
                    int n, int k, const ARRAY a, int lda, ARRAY b, int ldb );
```

Triangular Packed:
```
void BLAS_xtp_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                    enum blas_trans_type trans, enum blas_diag_type diag,
                    int n, const ARRAY ap, ARRAY bp );
```

{HE,HB,HP}_COPY (Matrix copy)

This routine copies a Hermitian matrix $A$ and stores the result in a matrix $B$. This routine returns immediately if $n$ or $k$ is less than or equal to zero. For the routine HE_COPY, if lda is less than one or less than $n$, or if ldb is less than one or less than $n$, an error flag is set and passed to the error handler. For the routine HB_COPY, if lda is less than $k$ plus one, or if ldb is less than $k$ plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:

Hermitian:
```
SUBROUTINE he_copy( a, b [, uplo] )
```
2.8. LANGUAGE BINDINGS

Hermitian Band:
    SUBROUTINE hb_copy( a, b [, uplo] )
Hermitian Packed:
    SUBROUTINE hp_copy( ap, bp [, uplo] )
all:
    COMPLEX(<wp>), INTENT(IN) :: a(:, :) or ap(:)
    COMPLEX(<wp>), INTENT(OUT) :: b(:, :) or bp(:)
    TYPE(blash_uplo_type), INTENT(IN), OPTIONAL :: uplo
where
    a and b have shape (n,n)
    (k+1,n) for banded (k=band width)
    ap and bp have shape (n*(n+1)/2).

• Fortran 77 binding:
    Hermitian:
    SUBROUTINE BLAS_xHE_COPY( UPLO, N, A, LDA, B, LDB )
    Hermitian Band:
    SUBROUTINE BLAS_xHB_COPY( UPLO, N, K, A, LDA, B, LDB )
    Hermitian Packed:
    SUBROUTINE BLAS_xHP_COPY( UPLO, N, AP, BP )
all:
    INTEGER K, LDA, LDB, N, UPLO
    <ctype>    A( LDA, * ) or AP( * ), B( LDB, * ) or BP( * )

• C binding:
    Hermitian:
    void BLAS_xhe_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, const CARRAY a, int lda, CARRAY b, int ldb );
    Hermitian Band:
    void BLAS_xhb_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, int k, const CARRAY a, int lda, CARRAY b, int ldb );
    Hermitian Packed:
    void BLAS_xhp_copy( enum blas_order_type order, enum blas_uplo_type uplo,
                        int n, const CARRAY ap, CARRAY bp );

GE_TRANS (Matrix transposition) \[ A \leftarrow A^T, A \leftarrow A^H \]

This routine performs the matrix transposition or conjugate-transposition of a square matrix
A, overwriting the matrix A. This routine returns immediately if n is less than or equal to zero. If
lda is less than one or less than n, an error flag is set and passed to the error handler.

• Fortran 95 binding:
    SUBROUTINE ge_trans( a [, conj] )
    <ctype>(<wp>), INTENT(INOUT) :: a(:, :)

TYPE (blas_conj_type), INTENT(IN), OPTIONAL :: conj
where
a has shape (n,n)

- Fortran 77 binding:

  SUBROUTINE BLAS_xGE_TRANS( CONJ, N, A, LDA )
  INTEGER CONJ, LDA, N
  <type> A( LDA, * )

- C binding:

  void BLAS_xge_trans( enum blas_order_type order, enum blas_conj_type conj,
                       int n, ARRAY a, int lda );

GE_PERMUTE (Permute matrix) \( A \leftarrow PA, \text{ or } A \leftarrow AP \)

This routine permutes the rows or columns of a matrix \((A \leftarrow PA \text{ or } A \leftarrow AP)\) by the permutation matrix \(P\). The representation of the permutation vector \(p\) is described in section 2.2.6. This routine returns immediately if \(m\) or \(n\) is less than or equal to zero. As described in section 2.5.3, the value incp less than zero is permitted. However, if incp is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or less than \(m\), an error flag is set and passed to the error handler. For the C bindings, if order = blas_rowmajor and if lda is less than one or lda is less than \(n\), an error flag is set and passed to the error handler; if order = blas_colmajor and if lda is less than one or lda is less than \(m\), an error flag is set and passed to the error handler.

- Fortran 95 binding:

  SUBROUTINE ge_permute( p, a [, side] )
  INTEGER, INTENT(IN) :: p(:,)
  <type>(<wp>), INTENT(INOUT) :: a(:,;)
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
where
a has shape (m,n)
d has shape (p) where p = m if side = blas_left_side
  p = n if side = blas_right_side

- Fortran 77 binding:

  SUBROUTINE BLAS_xGE_PERMUTE( SIDE, M, N, P, INCP, A, LDA )
  INTEGER INCP, LDA, M, N, SIDE
  INTEGER P( * )
  <type> A( LDA, * )

The value of INCP may be positive or negative. A negative value of INCP applies the permutation in the opposite direction.
2.8. LANGUAGE BINDINGS

- C binding:

```c
void BLAS_xge_permute( enum blas_order_type order, enum blas_side_type side,
                      int m, int n, const int *p, int incp, ARRAY a,
                      int lda );
```

The value of `incp` may be positive or negative. A negative value of `incp` applies the permutation in the opposite direction.

2.8.10 Environmental Enquiry

FPINFO (Environmental enquiry)

This routine queries for machine-specific floating point characteristics. Refer to section 1.6 for a list of all possible return values of this routine, and sections A.4, A.5, and A.6, for their respective language dependent representations in Fortran 95, Fortran 77, and C.

- Fortran 95 binding:

```fortran
REAL(<wp>) FUNCTION fpinfo( cmach, prec )
  TYPE (blas_cmach_type), INTENT(IN) :: cmach
  REAL (<wp>), INTENT(IN) :: prec
```

- Fortran 77 binding:

```fortran
<rtype> FUNCTION BLAS_xFPINFO( CMACH )
  INTEGER CMACH
```

- C binding:

```c
<rtype> BLAS_xfpinfo( enum blas_cmach_type cmach );
```
Chapter 3

Sparse BLAS

3.1 Overview

A matrix which contains many zero entries is often referred to as being sparse. Many problems arising from engineering and scientific computing give rise to large, sparse matrices, hence their importance in numerical linear algebra. Sparsity provides an opportunity to conserve storage and reduce computational requirements by storing only the significant (typically, nonzero) entries.

The Sparse BLAS interface addresses computational routines for unstructured sparse matrices. These are matrices that do not possess a special sparsity pattern (such as banded or triangular covered in the previous chapter on Dense/Banded specifications). Two fundamental differences between the Sparse BLAS and other chapters are

- **Functionality**: Only a small subset of the BLAS functionality is specified for sparse matrices – essentially only matrix multiply and triangular solve, along with sparse vector update, dot product and gather/scatter. These are among the basic operations used in solving large sparse linear equations using iterative techniques. Not included are general operations for direct solvers, functions for explicit matrix reordering, or operations in which both operands are sparse (e.g. the product of two sparse matrices).

- **Generic interface**: There is no single “best” method to represent a sparse matrix. The selection of the possible storage format is dependent on the algorithm being used, the original sparsity pattern of the matrix, the underlying computer architecture, together with other considerations such as in what format the data already exists, and so on. Because of this, sparse matrix arguments to the Level 2 and 3 Sparse BLAS routines are not the actual data components but rather a placeholder, or *handle*, which refers to an abstract representation of a matrix. (For portability, this handle is an integer variable.) Unlike the dense BLAS, there are many storage representations for sparse matrices, and this handle-based scheme allows one to write numerical algorithms using the Sparse BLAS independently of the matrix storage scheme.

Several routines are provided to create Sparse BLAS matrices, but the internal representation is implementation dependent. This provides BLAS library developers the best opportunity for optimizing and fine-tuning their kernels for specific situations.

Matrices in the Sparse BLAS can be constructed piece-by-piece, directly from common formats. The result is a matrix handle that can be passed as a parameter to Sparse BLAS computational kernels. Routines are also provided to extract information on a matrix identified by its handle and
3.2. NAMING CONVENTIONS

to release any resources related to the handle when computations with the matrix are completed. Thus, typical use of the Sparse BLAS consists of three phases:

1. create an internal sparse matrix representation and return its handle
   (Sections 3.8.6, 3.8.7, and 3.8.8).

2. use this handle as a parameter in computational Sparse BLAS routines (Sections 3.8.2, 3.8.3, and 3.8.4).

3. when the matrix is no longer needed, call a cleanup routine to free resources associated with the handle (Section 3.8.10).

Note that the releasing a matrix handle does not affect any of the user's data, but only internal BLAS resources (housekeeping data structures and internal copies of matrix data) that are not visible to the user. Thus, program resources available to the user after releasing a matrix handle should be the same as before creating that handle.

In Section 3.3 we describe the functionality of the Level 1, 2 and 3 Sparse BLAS. Section 3.4 provides an overview of the data structures used to express the sparsity of the sparse vectors and matrices, including a discussion of index bases in Section 3.4.2 and repeated indices in Section 3.4.3. Section 3.5.1 illustrates how to initialize Sparse BLAS matrices and Section 3.5.2 how to specify properties of the matrices. Sections 3.6.1–3.6.3 discuss interface issues. Section 3.7 briefly discusses numerical accuracy and environmental enquiry. Finally, in Section 3.8, we present the interfaces for the kernels, giving details for each specific language binding for Fortran 95, Fortran 77, and C programming languages.

3.2 Naming Conventions

Because this standard addresses multiple language bindings and various precisions, the BLAS routines are typically referred to in the text by their root names. Sparse BLAS root names use the two-letter identifier US, for Unstructured Sparse, e.g. as in USMV, or USDOT. These names are a compact way to represent the various instantiations. For example, the root for matrix-vector multiplication, USMV, is the general form of routines such as BLAS_dasmv (the C version for double-precision), or BLAS_CUSMV (the Fortran 77 version of single-precision complex). Functions listed in the Language Bindings Section 3.8 appear under their root name, followed by their detailed language-specific bindings.

Where an x appears in the name of a subroutine or function binding, it should be replaced in the call by one of the letters S, D, C, Z to indicate whether the floating-point data types are real single precision, real double precision, complex single precision, or complex double precision, respectively. Notice that, for some calls, this letter and substitution does not appear since the data type is not referenced explicitly and is only accessed through the matrix handle. In the F95 language, generic calls enable the use of this letter to be avoided in all cases except the creation routines.

3.3 Functionality

This section describes the Level 1, 2, and 3 routines defined for sparse vectors and matrices. In all cases only one of the basic operands is sparse, that is there are no sparse-sparse operations. For the sake of compactness, the case involving complex operators is usually omitted. For matrices, whenever a transpose operation is described, the conjugate transpose is implied for the complex case.
3.3.1 Scalar and Vector Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>USDOT</td>
<td>sparse dot product</td>
<td>( r \leftarrow x^T y, ) \  ( r \leftarrow x^H y )</td>
</tr>
<tr>
<td>USAIPY</td>
<td>sparse vector update</td>
<td>( y \leftarrow \alpha x + y )</td>
</tr>
<tr>
<td>USGA</td>
<td>sparse gather</td>
<td>( x \leftarrow y</td>
</tr>
<tr>
<td>USGZ</td>
<td>sparse gather and zero</td>
<td>( x \leftarrow y</td>
</tr>
<tr>
<td>USSC</td>
<td>sparse scatter</td>
<td>( y</td>
</tr>
</tbody>
</table>

Table 3.1: Sparse Vector Operations

This subsection lists the operations corresponding to the Level 1 Sparse BLAS. Table 3.1 lists the scalar and vector operations. The following notation is used: \( r \) and \( \alpha \) are scalars, \( x \) is a compressed sparse vector, \( y \) is a dense vector, and \( y|_x \) refers to the entries of \( y \) that have common indices with the sparse vector \( x \). Details of the sparse vector storage format are given in Section 3.4.1.

3.3.2 Matrix-Vector Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>USMV</td>
<td>sparse matrix/vector multiply</td>
<td>( y \leftarrow \alpha Ax + y ) \  ( y \leftarrow \alpha A^T x + y ) \  ( y \leftarrow \alpha A^H x + y )</td>
</tr>
<tr>
<td>USSV</td>
<td>sparse triangular solve</td>
<td>( x \leftarrow \alpha T^{-1} x ) \  ( x \leftarrow \alpha T^{-T} x ) \  ( x \leftarrow \alpha T^{-H} x )</td>
</tr>
</tbody>
</table>

Table 3.2: Sparse Matrix-Vector Operations

Table 3.2 lists matrix/vector (Level 2) operations. The notation \( A \) represents a sparse matrix and \( T \) denotes a sparse triangular matrix. \( x \) and \( y \) are dense vectors, \( \alpha \) is a scalar.

3.3.3 Matrix-Matrix Operations

<table>
<thead>
<tr>
<th>Operation</th>
<th>Description</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>USMM</td>
<td>sparse matrix/matrix multiply</td>
<td>( C \leftarrow \alpha AB + C ) \  ( C \leftarrow \alpha A^T B + C ) \  ( C \leftarrow \alpha A^H B + C )</td>
</tr>
<tr>
<td>USSM</td>
<td>sparse triangular solve</td>
<td>( B \leftarrow \alpha T^{-1} B ) \  ( B \leftarrow \alpha T^{-T} B ) \  ( B \leftarrow \alpha T^{-H} B )</td>
</tr>
</tbody>
</table>

Table 3.3: Sparse Matrix-Matrix Operations

Table 3.3 lists matrix/matrix (Level 3) operations, using the following notation: \( \alpha \) is a scalar, \( A \) denotes a general sparse matrix, \( T \) denotes a sparse triangular matrix, \( B \) and \( C \) are dense matrices.
3.4. Describing sparsity

3.4.1 Sparse Vectors

Sparse vectors are represented by a pair of conventional vectors, one denoting the nonzero values and the other denoting their indices. That is, if \( x \) is a vector that we wish to represent in sparse format, then it is represented by a one-dimensional array, \( X \), of the entries of \( x \), and an integer vector of equal length to \( X \) whose values indicate the location in \( x \) of the corresponding floating-point values in \( X \). The index values may follow the Fortran convention (where the first element has an index of 1) or the C/C++ convention (where the first element has an index of 0). These are referred to as 1-based and 0-based indexing, respectively, and the Sparse BLAS specification usually handles both (see Section 3.4.2). For example, using 1-based (Fortran) indexing, the vector

\[
\begin{bmatrix}
11.0 & 0.0 & 13.0 & 14.0 & 0.0
\end{bmatrix}
\]

can be represented by two vectors as

\[
\begin{align*}
X &= \begin{bmatrix}
11.0 & 13.0 & 14.0
\end{bmatrix} \\
\text{INDX} &= \begin{bmatrix}
1 & 3 & 4
\end{bmatrix}
\end{align*}
\]

although the permutation

\[
\begin{align*}
X &= \begin{bmatrix}
14.0 & 13.0 & 11.0
\end{bmatrix} \\
\text{INDX} &= \begin{bmatrix}
4 & 3 & 1
\end{bmatrix}
\end{align*}
\]

or any other such permutation is equally valid.

We illustrate the use of this structure, through the Fortran 77 routine for a double precision real sparse dot product:

\[
W = \text{BLAS DUSDOT}( \text{CONJ}, \text{NZ}, X, \text{INDX}, Y, \text{INCY} )
\]

where \( NZ \) is the number of nonzero entries in the sparse vector \( x \), the argument \( X \) is the double precision vector containing the entries of \( x \), \( \text{INDX} \) is the index vector for \( x \), \( Y \) is a dense vector with \( \text{INCY} \) defining the stride between consecutive components, and \( \text{CONJ} \) is a flag specifying if \( x \) or \( x^{\text{T}} \) is used (although this has no effect in the case of real arguments). This call computes

\[
w = \sum_{I=1}^{NZ} X(I) \ast Y(\text{INDX}(I))
\]

3.4.2 Index bases

The Fortran and C programming languages utilize different conventions to index entries of a vector. Fortran uses a 1-based convention, (that is \( x(1) \) is the first entry of vector \( x \)); C assumes 0-based index values (that is \( x[0] \) is the first entry of the vector \( x \)).

For dense array operations, this difference can often be dealt with by adjustments to the array parameters in function and subroutine calls. For sparse data structures, however, the index information is part of the semantics of the data structure, so this must be dealt with explicitly.

The Fortran interface for the Sparse BLAS defaults to a 1-based indexing, while the C interface defaults to 0-base indexing. Both interfaces, however, can explicitly override this default with only one exception: the Fortran interfaces to the Level 1 sparse routines. In the following sections, we use 1-based conventions in examples and discussions, unless otherwise stated.

For Level 2 and Level 3 operations, the index base may be specified by the \texttt{blas_one_base} or \texttt{blas_zero_base} property, which can be set when constructing BLAS matrices (see Section 3.5.2).
3.4.3 Repeated Indices

In general, having the same matrix or vector entry specified multiple times in a sparse representation can lead to ambiguities. There are some cases, however, where it is useful to define the result as the sum of all entries with a common index. For example, the sparse data structure

\[
N = 5 \\
X = \begin{pmatrix} 11.0 & 13.0 & 14.0 & 22.0 \end{pmatrix} \\
\text{INDX} = \begin{pmatrix} 1 & 3 & 4 & 3 \end{pmatrix}
\]

may be interpreted as a representation of the vector

\[
x = \begin{pmatrix} 11.0 & 0.0 & 35.0 & 14.0 & 0.0 \end{pmatrix}
\]

Analogously, a similar convention can be adopted for sparse matrices: whenever an \((i,j)\) index is specified multiple times, the result is that its corresponding nonzero values are added together. (This is useful, for example, in the assembling of elemental matrices from finite-element formulations as in Section 3.5.6).

Because of possible ambiguities and inefficiencies, the use of repeated indices is not supported in the Level 1 BLAS operations. That is, for those routines the sparse vector parameter must have unique indices, otherwise the computational results are undefined.

3.5 Sparse BLAS Matrices

3.5.1 Creation Routines

A Sparse BLAS matrix and its associated handle are created by a sequence of calls to the routines listed in Sections 3.8.6, 3.8.7, and 3.8.8. A call must first be made to a routine to begin the matrix construction. This can be of three forms depending on whether the input matrix has entries which are scalars or are dense matrices. The calls for the scalar or single entries case have the form

\[
\text{CALL DUSCR\_BEGIN( m, n, A, istat ) (Fortran 95)} \\
\text{CALL BLAS\_DUSCR\_BEGIN( M, N, A, ISTAT ) (Fortran 77)} \\
A = \text{BLAS\_duscr\_begin( m, n ); (C)}
\]

where \(m\) and \(n\) are the matrix dimensions and \(A\) is the matrix handle.

When initializing Sparse BLAS matrices from a block-structured format, two variants of the creation routines may be used. For fixed size \(k \times l\) blocks, the declaration

\[
\text{CALL DUSCR\_BLOCK\_BEGIN( mb, nb, k, l, A, istat ) (Fortran 95)} \\
\text{CALL BLAS\_DUSCR\_BLOCK\_BEGIN( MB, NB, K, L, A, ISTAT ) (Fortran 77)} \\
A = \text{BLAS\_duscr\_block\_begin( Mb, Nb, k, l ); (C)}
\]

signifies that the input matrix contains \(Mb \times Nb\) blocks, each of size \(k \times l\), that is the total dimensions of the matrix are \((Mb \times k) \times (Nb \times l)\).

Likewise, for variable block matrices, the declaration

\[
\text{CALL DUSCR\_VARIABLE\_BLOCK\_BEGIN( mb, nb, K, L, A, istat ) (Fortran 95)} \\
\text{CALL BLAS\_DUSCR\_VARIABLE\_BLOCK\_BEGIN( MB, NB, K, L, A, ISTAT ) (Fortran 77)} \\
A = \text{BLAS\_duscr\_variable\_block\_begin(Mb, Nb, K, L ); (C)}
\]

denotes that the input matrix has a variable block structure denoted by the integer vectors \(K\) and \(L\).
3.5. SPARSE BLAS MATRICES

<table>
<thead>
<tr>
<th>blas_non_unit_diag</th>
<th>nonzero diagonal entries are stored (Default)</th>
</tr>
</thead>
<tbody>
<tr>
<td>blas_unit_diag</td>
<td>diagonal entries are not stored and assumed to be 1.0</td>
</tr>
<tr>
<td>blas_no_repeated_indices</td>
<td>indices are unique (Default)</td>
</tr>
<tr>
<td>blas_repeated_indices</td>
<td>nonzero values of repeated indices are summed</td>
</tr>
<tr>
<td>blas_lower_symmetric</td>
<td>only lower half of symmetric matrix is specified by user.</td>
</tr>
<tr>
<td>blas_upper_symmetric</td>
<td>only upper half of symmetric matrix is specified by user.</td>
</tr>
<tr>
<td>blas_lower_hermitian</td>
<td>only lower half of Hermitian matrix is specified by user.</td>
</tr>
<tr>
<td>blas_upper_hermitian</td>
<td>only upper half of Hermitian matrix is specified by user.</td>
</tr>
<tr>
<td>blas_lower_triangular</td>
<td>sparse matrix is lower triangular</td>
</tr>
<tr>
<td>blas_upper_triangular</td>
<td>sparse matrix is upper triangular</td>
</tr>
<tr>
<td>blas_zero_base</td>
<td>indices of inserted items are 0-based (Default for C)</td>
</tr>
<tr>
<td>blas_one_base</td>
<td>indices of inserted items are 1-based (Default for Fortran)</td>
</tr>
<tr>
<td>blas_rowmajor</td>
<td>Applicable for block entries only</td>
</tr>
<tr>
<td>blas_colmajor</td>
<td>dense block stored row major order (Default for C)</td>
</tr>
<tr>
<td>blas_irregular</td>
<td>general unstructured matrix</td>
</tr>
<tr>
<td>blas_regular</td>
<td>structured matrix</td>
</tr>
<tr>
<td>blas_block_irregular</td>
<td>unstructured matrix best represented by blocks</td>
</tr>
<tr>
<td>blas_block_regular</td>
<td>structured matrix best represented by blocks</td>
</tr>
<tr>
<td>blas_unassembled</td>
<td>matrix is best represented by cliques</td>
</tr>
</tbody>
</table>

Table 3.4: Matrix properties (can be set by USSP).

3.5.2 Specifying matrix properties

The creation routines allow one to specify various properties about the matrix and optionally provide hints to the underlying BLAS implementation about how the matrix will be used in subsequent BLAS calls, so that possible optimization may take place.

When creating a matrix to a BLAS sparse matrix, one or more of the properties in Table 3.4 may be specified with the use of the USSP (set property) routine (See Section 3.8.9). For example,

```
USSP( A, blas_lower_triangular );
USSP( A, blas_unit_diag );
```

denotes a lower triangular matrix, with an implicit unit diagonal.

The input properties (Table 3.4), are mutually exclusive for each category and may be specified only once. The result is undefined if incompatible properties are requested.

An optional description of the sparsity pattern of the matrix may be specified at construction time. These properties are listed as the last group in Table 3.4 and their use may assist the underlying implementation in choosing the most efficient internal data structure for subsequent computation. Note that each description is mutually exclusive. The specification of these properties is optional and does not effect the correctness of the program.

3.5.3 Sparse Matrices: Inserting a Single Entry

The basic insertion routine USR_INSERT_ENTRY allows one to build a sparse matrix, one scalar entry at a time, by specifying its row and column index together with its numeric value. Although there are other insertion routines for special structures (see below) this version is the simplest and most universal, as it allows one to build a BLAS Sparse Matrix from any given format.
<table>
<thead>
<tr>
<th>Property</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>blas_num_rows</td>
<td>returns the number of rows of matrix</td>
</tr>
<tr>
<td>blas_num_cols</td>
<td>returns the number of columns of matrix</td>
</tr>
<tr>
<td>blas_num_nonzeros</td>
<td>returns the number of stored entries</td>
</tr>
<tr>
<td>blas_complex</td>
<td>matrix values are complex</td>
</tr>
<tr>
<td>blas_real</td>
<td>matrix values are real</td>
</tr>
<tr>
<td>blas_integer</td>
<td>matrix values are integer</td>
</tr>
<tr>
<td>blas_double_precision</td>
<td>matrix values are single precision</td>
</tr>
<tr>
<td>blas_single_precision</td>
<td>matrix values are double precision</td>
</tr>
<tr>
<td>blas_general</td>
<td>neither symmetric nor Hermitian (Default)</td>
</tr>
<tr>
<td>blas_symmetric</td>
<td>sparse matrix is symmetric</td>
</tr>
<tr>
<td>blas_hermitian</td>
<td>(complex) sparse matrix is Hermitian</td>
</tr>
<tr>
<td>blas_lower_triangular</td>
<td>sparse matrix is lower triangular</td>
</tr>
<tr>
<td>blas_upper_triangular</td>
<td>sparse matrix is upper triangular</td>
</tr>
<tr>
<td>blas_zero_base</td>
<td>indices of inserted items are 0-based (Default for C)</td>
</tr>
<tr>
<td>blas_one_base</td>
<td>indices of inserted items are 1-based (Default for Fortran)</td>
</tr>
<tr>
<td>blas_rowmajor</td>
<td>Applicable for block entries only</td>
</tr>
<tr>
<td>blas_colmajor</td>
<td>dense block stored row major order (Default for C)</td>
</tr>
<tr>
<td>blas_void_handle</td>
<td>handle not currently in use</td>
</tr>
<tr>
<td>blas_new_handle</td>
<td>handle created but no entries inserted so far</td>
</tr>
<tr>
<td>blas_open_handle</td>
<td>an entry has been inserted but creation not yet finished</td>
</tr>
<tr>
<td>blas_valid_handle</td>
<td>creation completed (USCR_END has been called)</td>
</tr>
</tbody>
</table>

Table 3.5: Matrix properties (can be read by USGP).

3.5.4 Sparse Matrices: Inserting List of Entries

The insertion routine USCR_INSERT_ENTRIES allows us to pass a list of entries with arbitrary row and column indices. We describe this list with a similar set of data structures as used for sparse vectors, but now need two integer vectors, one containing the row indices (called INDX) and another containing the column indices (called JNDX).

To illustrate this, consider the following matrix:

\[
A = \begin{pmatrix}
1.1 & 0 & 0 & 0 \\
0 & 2.2 & 0 & 2.4 \\
0 & 0 & 3.3 & 0 \\
4.1 & 0 & 0 & 4.4 \\
\end{pmatrix}.
\]  

(3.1)

We can pass in all entries (following a call to one of the BEGIN routines) by defining NZ = 6 and setting

\[
\begin{align*}
\text{VAL} &= ( 1.1 \ 2.2 \ 2.4 \ 3.3 \ 4.1 \ 4.4 ) \\
\text{INDX} &= ( 1 \ 2 \ 2 \ 3 \ 4 \ 4 ) \\
\text{JNDX} &= ( 1 \ 2 \ 4 \ 3 \ 1 \ 4 ).
\end{align*}
\]

Note that calls to the C interface would default to using 0-based indices (see Section 3.4.2). The ordering of the entries is arbitrary.
3.6. INTERFACE ISSUES

3.5.5 Sparse Matrices: Inserting Row and Column Vectors

The insertion routines USCR_INSERT_COL and USCR_INSERT_ROW allow us to pass a list of entries that all belong to the same column or row of a matrix. The data structures used to pass the information are identical to those used to describe a sparse vector in Section 3.4.1.

3.5.6 Sparse Matrices: Inserting Cliques

A clique is a two-dimensional array of values with integer row and column vectors that describe how the values will be scattered into the sparse matrix. Such data structures are common in finite element computations. Consider the matrix $A$ in Section 3.5.4. We can pass in the (2,2), (2,4), (4,2) and (4,4) entries as a clique by defining a two-dimensional array

$$\text{VAL} = \begin{pmatrix} 2.2 & 2.4 \\ 0.0 & 4.4 \end{pmatrix}$$

(3.2)

and its associated row and column scattering vectors as

$$\text{INDX} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$\text{JNDX} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}.$$  

Note that the structure allows cliques to be other than principal submatrices (in which case arrays INDX and JNDX could differ) and indeed allows the clique matrices to be rectangular.

3.6 Interface Issues

3.6.1 Interface Issues for Fortran 95

- Predefined constants for the Sparse BLAS are included in the module "blas_sparse_namedconstants". These include the sparse matrix properties constants defined in Tables 3.4 and 3.5. A module "blas_sparse_proto" of explicit interfaces to all routines is also provided.
- Sparse matrix/vector indices are assumed to begin at 1 (that is they are 1-based), but can be overridden by specifying blas_zero_base at the time of creation.
- The values of the named constants are as specified in Section A.4.
- Error handling is as defined in Section 2.4.6.

The interface example below illustrates multiplying a sparse $4 \times 4$ matrix

$$A = \begin{pmatrix} 1.1 & 0 & 0 & 0 \\ 0 & 2.2 & 0 & 2.4 \\ 0 & 0 & 3.3 & 0 \\ 4.1 & 0 & 0 & 4.4 \end{pmatrix}$$

(3.3)

with the vector $x = \{1.0, 1.0, 1.0, 1.0\}$ performing the operation $y \leftarrow Ax$. In this example, the sparse matrix is input by point (rather than block) entries.
! Fortran 95 example: sparse matrix-vector multiplication

PROGRAM F95_EX
USE blas_sparse

IMPLICIT NONE
INTEGER NMAX, NNZ
PARAMETER (NMAX = 4, NNZ = 6)
INTEGER i, n, a, istat
INTEGER, DIMENSION(:,), ALLOCATABLE:: indx, jindx
DOUBLE PRECISION, DIMENSION(:,), ALLOCATABLE:: val, x, y

ALLOCATE(val(NNZ),x(NMAX),y(NMAX),indx(NNZ),jindx(NNZ))

indx(=1,2,3,4,4/)
jindx(=1,2,4,3,1,4/)
val(=1.1,2.2,2.4,3.3,4.1,4.4/)

N = NMAX
!
! -----------------------------------------------
! Step 1: Create Sparse BLAS Handle
! -----------------------------------------------
!
CALL duscr_begin(n,n,a,istat)
!
! -----------------------------------------------
! Step 2: Insert entries one-by-one
! -----------------------------------------------
!
DO i=1, nnz
   CALL uscr_insert_entry(A, val(i), indx(i), jindx(i), istat)
END DO
!
! -----------------------------------------------
! Step 3: Complete construction of sparse matrix
! -----------------------------------------------
!
CALL uscr_end(a, istat)
!
! -----------------------------------------------
! Step 4: Compute Matrix vector product y = A*x
! -----------------------------------------------
!
CALL usmv(a, x, y, istat)
!
! -----------------------------------------------
! Step 5: Release Matrix Handle
!
3.6. INTERFACE ISSUES

!  -----------------------------
!  CALL usds(a, istat)
END

3.6.2 Interface Issues for Fortran 77

Although Fortran 77 is no longer a standard, Fortran 77 compilers are still heavily used and there are many Fortran applications that, even if compiled with a Fortran 95 compiler, use a subset of the language that is very close to Fortran 77. In addition, we have seen in the C interface to the legacy BLAS (see Chapter B) that a Fortran 77 library can provide the vast majority of functionality required by a higher level interface and greatly reduce the overall amount of work required to develop and support multiple language bindings. For these reasons we provide a Fortran 77 interface to the sparse BLAS.

- Predefined constants for the Sparse BLAS are included in the header file “bls_namedconstants.h”. These include the sparse matrix properties constants defined in Tables 3.4 and 3.5.

- Sparse matrix/vector indices are assumed to begin at 1 (that is they are 1-based), but can be overridden by specifying blas_zero_base at the time of creation.

- The values of the named constants are as specified in Section A.5.

- Error handling is as defined in Section 2.5.6.

The following program illustrates the use of Fortran 77 codes on the matrix 3.3.

C  Fortran 77 example: sparse matrix-vector multiplication

PROGRAM F77_EX
IMPLICIT NONE
INCLUDE "bls_namedconstants.h"
INTEGER NMAX, NNZ
PARAMETER (NMAX = 4, NNZ = 6)
INTEGER I, N, ISTAT, A
INTEGER INDX(NNZ), JNDX(NNZ)
DOUBLE PRECISION VAL(NNZ), X(NMAX), Y(NMAX)

C  -----------------------------
C  Define Matrix, LHS and RHS in Coordinate format
C  -----------------------------

C DATA VAL / 1.1, 2.2, 2.4, 3.3, 4.1, 4.4/
DATA INDX / 1, 2, 2, 3, 4, 4/
DATA JNDX / 1, 2, 4, 3, 1, 4/
C DATA X / 1., 1., 1., 1./
DATA Y / 0., 0., 0., 0./
C
N = NMAX

-------------------------
Step 1: Create Sparse BLAS Handle
-------------------------

CALL BLAS_DUSCR_BEGIN( N, N, A, ISTAT)

-------------------------
Step 2: Insert entries one-by-one
-------------------------

DO 10 I=1, NNZ
    CALL BLAS_DUSCR_INSERT_ENTRY(A, VAL(I), INDX(I), JNDX(I), ISTAT)
10 CONTINUE

-------------------------
Step 3: Complete construction of sparse matrix
-------------------------

CALL BLAS_USCR_END(A, ISTAT)

-------------------------
Step 4: Compute Matrix vector product y = A*x
-------------------------

CALL BLAS_DUSMV( BLAS_NO_TRANS, 1.0, A, X, 1, Y, 1, ISTAT )

-------------------------
Step 5: Release Matrix Handle
-------------------------

CALL BLAS_USDS(A, ISTAT)

END

3.6.3 Interface Issues for C

- Predefined constants for the Sparse BLAS are included in the header file “blas_enum.h”. These include the sparse matrix properties constants defined in Tables 3.4 and 3.5.
- Sparse matrix/vector indices are assumed to begin at 0 (that is they are 0-based), but can be overridden by specifying blas_one_base at the time of creation.
- Sparse matrix handles are integers, but are typedef to blas_sparse_matrix for clarity.
- The values of the enumerated types are as specified in Section A.6.
- Error handling is as defined in Section 2.6.9.
The following program illustrates the use of C codes on the matrix 3.3.

/* C example: sparse matrix/vector multiplication */

#include "blas_sparse.h"

int main()
{
    const int N = 4;
    const int nz = 6;
    double val[] = { 1.1, 2.2, 2.4, 3.3, 4.1, 4.4 };
    int indx[] = { 0, 1, 1, 2, 3, 3};
    int jndx[] = { 0, 1, 3, 2, 0, 3};
    double x[] = { 1.0, 1.0, 1.0, 1.0 };
    double y[] = { 0.0, 0.0, 0.0, 0.0 };

    blas_sparse_matrix A;
    int i;
    double alpha = 1.0;

    /*************************************************************************/
    /* Step 1: Create Sparse BLAS Handle */
    /*************************************************************************/
    A = BLAS_duscr_begin( N, N );

    /*************************************************************************/
    /* Step 2: insert entries */
    /*************************************************************************/
    for (i=0; i<nz; i++)
        BLAS_duscr_insert_entry(A, val[i], indx[i], jndx[i]);

    /*************************************************************************/
    /* Step 3: Complete construction of sparse matrix */
    /*************************************************************************/
    BLAS_uscr_end(A);

    /*************************************************************************/
    /* Step 4: Compute Matrix vector product y = A*x */
    /*************************************************************************/
    BLAS_dusmv( blas_no_trans, alpha, A, x, 1, y, 1 );

    /*************************************************************************/
    /* Step 5: Release Matrix Handle */
    /*************************************************************************/
3.7 Numerical Accuracy and Environmental Enquiry

All the comments on the accuracy of numerical methods made in Sections 1.6 and 2.7 apply here. In particular, subroutine FPINFO described in Section 2.7 should be used to get floating-point parameters needed for error bounds.

3.8 Language Bindings

3.8.1 Overview

This sections lists BLAS routines by their root name (see Section 3.2) together with their specific bindings for Fortran 95, Fortran 77, and C.

- Level 1 computational routines (Section 3.8.2)
  - USDOT sparse dot product
  - USAXPY sparse vector update
  - USGA sparse gather
  - USGZ sparse gather and zero
  - USSC sparse scatter

- Level 2 computational routines (Section 3.8.3)
  - USMV matrix/vector multiply
  - USMV matrix/vector triangular solve

- Level 3 computational routines (Section 3.8.4)
  - USMM matrix/matrix multiply
  - USMM matrix/matrix triangular solve

- Handle Management routines (Level 2/3) (Section 3.8.5)
  - Creation routine (Section 3.8.6)
    * USCR-BEGIN begin construction
    * USCR_BLOCK-BEGIN begin block-entry construction
    * USCR_VARIABLE_BLOCK-BEGIN begin variable block-entry construction
  - Insertion routines (Section 3.8.7)
    * USCR_INSERT_ENTRY add point-entry to construction
    * USCR_INSERT_ENTRIES add list of point-entries to construction
    * USCR_INSERT_COL add a compressed column to construction
3.8. LANGUAGE BINDINGS

* USCR_INSERT_ROW add a compressed row to construction
* USCR_INSERT_CLIQUE add a dense matrix clique to construction
* USCR_INSERT_BLOCK add a block entry at block coordinate (bi, bj)

- Completion of construction routine (Section 3.8.8)
  * USCR_END entries completed; build internal representation

- Matrix property routines (Section 3.8.9)
  * USGP get/test for matrix property
  * USSP set matrix property

- Destruction routine (Section 3.8.10)
  * USDS release matrix handle

3.8.2 Level 1 Computational Routines

General conventions: in all Level 1 routines, the following common arguments are used:

- $x$: a sparse vector $x$, with $nz$ nonzeros
- $\text{indx}$: an (integer) index vector corresponding to $x$,
- $y$: a dense vector
- $\text{index\_base}$: (C bindings only.) By convention, the Fortran 77 and Fortran 95 bindings assume that all offsets begin at 1 (that is $x(1)$ is the first entry). For the C language bindings, offsets can start at 0 (the default for C arrays) or 1 (for Fortran compatibility).

Note that, as stated in Section 3.4.3, the result of a Level 1 BLAS operation called with repeated indices in array $\text{indx}$ will be undefined. The actual return will be dependent on the implementation.

**USDOT** (Sparse dot product)

$$ r := x^T y $$

The function **USDOT** computes the dot product of sparse vector $x$ with dense vector $y$. The routine returns a real zero if the length of arrays $x$ and $\text{indx}$ are less than or equal to zero. When $x$ and $y$ are complex vectors, the vector components $x_i$ are used unconjugated or conjugated as specified by the operator argument $\text{conj}$. If $x$ and $y$ are real vectors, the operator argument $\text{conj}$ has no effect. For the C binding, the lack of a complex data type forces us to return the result in the parameter $r$.

- Fortran 95 binding:

  ```fortran
  <type>(<wp> FUNCTION usdot( x, indx, y [, conj] )
  INTEGER, INTENT(IN) :: indx(:)
  <type>(<wp>), INTENT(IN) :: x(:,), y(:,
  TYPE blas_conj_type), INTENT(IN), OPTIONAL :: conj
  ```

- Fortran 77 binding:
<type> FUNCTION BLAS_xUSDOT( CONJ, NZ, X, INDX, Y, INCY )
<type> X( * ), Y( * )
INTEGER NZ, INDX( * ), INCY
INTEGER CONJ

- C binding:

void BLAS_xusdot( enum blas_conj_type conj, int nz, const ARRAY x,
      const int *indx, const ARRAY y, int incy,
      SCALAR_INOUT r, enum blas_base_type index_base );

USAXPY (Sparse vector update) $y \leftarrow \alpha x + y$

The routine USAXPY scales the sparse vector $x$ by $\alpha$ and adds the result to the dense vector $y$. If the length of arrays $x$ and $indx$ are less than or equal to zero or if $\alpha$ is equal to zero, this routine returns without modifying $y$. Note that we do not allow a scaling on the vector $y$ (that is, we do not implement a USAXPY) as this would change the complexity of our routine because scaling a dense vector requires $n$ operations while the sparse operations are only $O(nz)$. If the dense vector $y$ is to be scaled, the appropriate Level 1 dense BLAS kernel should be used.

- Fortran 95 binding:

   SUBROUTINE usaxpy( x, indx, y [ , alpha] )
   <type>(<wp>), INTENT(IN) :: x(:)
   <type>(<wp>), INTENT(INOUT) :: y(:)
   INTEGER, INTENT(IN) :: indx(:)
   <type>(<wp>), INTENT(IN), OPTIONAL :: alpha

   The default value for $\alpha$ is 1.0.

- Fortran 77 binding:

   SUBROUTINE BLAS_xUSAXPY( NZ, ALPHA, X, INDX, Y, INCY )
   <type> ALPHA
   <type> X( * ), Y( * )
   INTEGER NZ, INDX( * ), INCY

- C binding:

void BLAS_xusaxpy( int nz, SCALAR_IN alpha, const ARRAY x, const int *indx,
      ARRAY y, int incy, enum blas_base_type index_base );

USGA (Sparse gather into compressed form) $x \leftarrow y|_x$

Using indx to denote the list of indices of the sparse vector $x$, for each component $i$ in this list, the routine USGA assigns $x(i) = y(\text{indx}(i))$. For example, if $x$ is a sparse vector with nonzeros $\{3.1, 4.9\}$ and indices $\{1, 4\}$ (using 1-based offsets), and $y$ is the dense vector $\{12.7, 68.1, 38.1, 54.0\}$, then the USGA routine changes $x$ to $\{12.7, 54.0\}$. If the length of $x$ and indx is non-positive, this routine returns without any modification to its parameters.
3.8. LANGUAGE BINDINGS

- Fortran 95 binding:

```fortran
SUBROUTINE usga( y, x, indx )
  <type>(<wp>), INTENT(IN) :: y(:)
  <type>(<wp>), INTENT(OUT) :: x(:)
  INTEGER, INTENT(IN) :: indx(:)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSGA( NZ, Y, INCY, X, INDX )
  INTEGER               NZ, INDEX( * ), INCY
  <type>                Y( * ), X( * )
```

- C binding:

```c
void BLAS_xusga( int nz, const ARRAY y, int incy, ARRAY x, const int *indx,
                 enum blas_base_type index_base );
```

USGZ (Sparse gather and zero) \( x \leftarrow y|_x, y|_x \leftarrow 0 \)

This routine combines two operations: (1) a sparse gather of \( y \) into \( x \). (see USGA above), followed by (2) setting the corresponding values of \( y \) (\( y(\text{indx}()) \)) to zero. For example, if \( x \) is a sparse vector with nonzeros \( \{3.1, 4.9\} \) and indices \( \{1, 4\} \) (using 1-based offsets), and \( y \) is the dense vector \( \{12.7, 68.1, 38.1, 54.0\} \), then the USGA routine changes the nonzero values of \( x \) to \( \{12.7, 54.0\} \) and changes \( y \) to \( \{0.0, 68.1, 38.1, 0.0\} \).

- Fortran 95 binding:

```fortran
SUBROUTINE usgz( y, x, indx )
  <type>(<wp>), INTENT(INOUT) :: y(:)
  <type>(<wp>), INTENT(OUT) :: x(:)
  INTEGER, INTENT(IN) :: indx(:)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSGZ( NZ, Y, INCY, X, INDX )
  INTEGER               NZ, INDEX( * ), INCY
  <type>                Y( * ), X( * )
```

- C binding:

```c
void BLAS_xusgz( int nz, ARRAY y, int incy, ARRAY x, const int *indx,
                 enum blas_base_type index_base );
```

USSC (Sparse scatter) \( y|_x \leftarrow x \)
This routine copies the nonzero values of \( x \) into the corresponding locations in the dense vector \( y \). For example, if \( x \) is a sparse vector with nonzeros \{3.1, 4.9\} and indices \{1, 4\} (using 1-based offsets), and \( y \) is the dense vector \{12.7, 68.1, 38.1, 54.0\}, then the USSC routine changes \( y \) to \{3.1, 68.1, 38.1, 4.9\}. If the length of arrays \( x \) and \( \text{indx} \) are less than or equal to zero, this routine returns without any modification to its parameters.

- Fortran 95 binding:

```fortran
SUBROUTINE ussc( x, y, indx )
   <type>(<wp>), INTENT(IN) :: x(:)
   <type>(<wp>), INTENT(INOUT) :: y(:)
   INTEGER, INTENT(IN) :: indx(:)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSSC( NZ, X, Y, INCY, INDX )
   INTEGER NZ, INDX(*), INCY
   <type> X(*), Y(*)
```

- C binding:

```c
void BLAS_xussc( int nz, const ARRAY x, ARRAY y, int incy, const int *indx,
    enum blas_base_type index_base );
```

### 3.8.3 Level 2 Computational Routines

**USMV (Sparse Matrix/Vector Multiply)**

\[
y \leftarrow \alpha A x + y \\
y \leftarrow \alpha A^T x + y
\]

This routine multiplies a dense vector \( x \) by a sparse matrix \( A \) (or its transpose), and adds it to the vector operand \( y \). The matrix handle \( A \) must be valid, i.e. USGP(\( A \), blas_valid_handle) must be true, and the precision type of the sparse matrix represented by the handle \( A \) must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE usmv( a, x, y, istat [, transa [, alpha] ] )
   INTEGER, INTENT(IN) :: a
   <type>(<wp>), INTENT(IN) :: x(:)
   <type>(<wp>), INTENT(INOUT) :: y(:)
   INTEGER, INTENT(OUT) :: istat
   TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transa
   <type>(<wp>), INTENT(IN), OPTIONAL :: alpha
```

Default values for \( \text{transa} \) and \( \alpha \) are blas_no_trans and 1.0, respectively.
3.8. LANGUAGE BINDINGS

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSMV( TRANS, ALPHA, A, X, INCX, Y, INCY, ISTAT )
INTEGER INCX, INCY, A, TRANS, ISTAT
<type> ALPHA
<type> X( * ), Y( * )
```

- C binding:

```c
int BLAS_xusmv( enum blas_trans_type transa, SCALAR_IN alpha,
    blas_sparse_matrix A, const ARRAY x, int incx, ARRAY y, int incy );
```

**USSV (Sparse Triangular Solve)**

\[ x \leftarrow \alpha T^{-1} x \]
\[ x \leftarrow \alpha T^{-T} x \]

This routine solves one of the systems of equations \( x \leftarrow \alpha T^{-1} x \) or \( x \leftarrow \alpha T^{-T} x \), where \( x \) is a dense vector and the matrix \( T \) is a triangular sparse matrix. The matrix handle \( T \) must be valid, i.e. USGP(T, blas_valid_handle) is true, must represent a valid triangular matrix, i.e. either USGP(T, blas_lower_triangular) or USGP(T, blas_upper_triangular) must be true, and the precision type of the sparse matrix represented by the handle \( T \) must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE ussv( t, x, istat, [, transt] [, alpha] )
INTEGER, INTENT(IN) :: t
<type>(<wp>), INTENT(INOUT) :: x(:)
INTEGER, INTENT(INOUT) :: istat
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
<type>(<wp>), INTENT(IN), OPTIONAL :: alpha
```

Default values for transt and \( \alpha \) are .TRUE. and 1.0 respectively.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSSV( TRANST, ALPHA, T, X, INCX, ISTAT )
INTEGER T, INCX, TRANST, ISTAT
<type> ALPHA
<type> X( * )
```

- C binding:

```c
int BLAS_xussv( enum blas_trans_type transt, SCALAR_IN alpha,
    blas_sparse_matrix T, ARRAY x, int incx );
```
3.8.4 Level 3 Computational Routines

USMM (Sparse Matrix Multiply)

\[
C \leftarrow \alpha AB + C \\
C \leftarrow \alpha A^T B + C
\]

This routine multiplies a dense matrix \( B \) by a sparse matrix \( A \) (or its transpose), and adds it to a dense matrix operand \( C \). \( A \) is of size \( m \) by \( n \), \( B \) is of size \( n \) by \( \text{nrhs} \), and \( C \) is of size \( m \) by \( \text{nrhs} \). The input argument \( \text{nrhs} \) must be greater than zero, and the matrix handle \( A \) must be valid, i.e. \( \text{USGP}(A, \text{blas_valid_handle}) \) must be true, and the precision type of the sparse matrix represented by the handle \( A \) must match the remaining floating-point arguments. \( \text{istat} \) is used as an error flag and will be zero if the routine executes successfully. The C binding returns \( \text{istat} \) as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE usmm( a, b, c, istat, [, transa] [, alpha] )
  INTEGER, INTENT(IN) :: a
  <type>(<wp>), INTENT(IN) :: b(:,:,)
  <type>(<wp>), INTENT(INOUT) :: c(:,:,)
  INTEGER, INTENT(OUT) :: istat
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transa
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha

Default values for \( \text{transa} \) and \( \alpha \) are \text{.TRUE.} and 1.0, respectively.
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSMM( TRANSA, NRHS, ALPHA, A, B, LDB, C, LDC,

$ 
  INTEGER NRHS, A, LDB, LDC, TRANSA, ISTAT
  <type> ALPHA
  <type> B( LDB, * ), C( LDC, * )
```

- C binding:

```c
int BLAS_xusmm( enum blas_order_type order, enum blas_trans_type transa,
  int nrhs, SCALAR_IN alpha, blas_sparse_matrix A,
  const ARRAY B, int ldb, ARRAY C, int ldc );
```

USSM (Sparse Triangular Solve)

\[
B \leftarrow \alpha T^{-1} B \\
B \leftarrow \alpha T^{-T} B
\]

This routine solves one of the systems of equations \( B \leftarrow \alpha T^{-1} B \) or \( B \leftarrow \alpha T^{-T} B \), where \( B \) is a dense matrix and \( T \) is a triangular sparse matrix. \( T \) is of size \( n \) by \( n \), \( B \) is of size \( n \) by \( \text{nrhs} \), and \( C \) is of size \( n \) by \( \text{nrhs} \). The input argument \( \text{nrhs} \) must be greater than zero, and the matrix handle \( T \) must be valid, i.e. \( \text{USGP}(T, \text{blas_valid_handle}) \) must be true, and represent a valid triangular matrix, i.e. either \( \text{USGP}(T, \text{blas_lower_triangular}) \) or \( \text{USGP}(T, \text{blas_upper_triangular}) \) must be true. The precision type of the sparse matrix represented by the handle \( T \) must match the remaining floating-point arguments. \( \text{istat} \) is used as an error flag and will be zero if the routine executes successfully. The C binding returns \( \text{istat} \) as the function return value.
3.8. LANGUAGE BINDINGS

- Fortran 95 binding:

```fortran
SUBROUTINE ussm( t, b, istat [, transt] [, alpha] )
  INTEGER, INTENT(IN) :: t
  <type>(<wp>), INTENT(INOUT) :: b(:, :)
  INTEGER, INTENT(OUT) :: istat
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha

Default values for transt and α are .TRUE. and 1.0 respectively.
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSSM( TRANS, NRHS, ALPHA, T, B, LDB, ISTAT )
  INTEGER NRHS, T, LDB, TRANS, ISTAT
  <type> ALPHA
  <type> B( LDB, * )
```

- C binding:

```c
int BLAS_xussm( enum blas_order_type order, enum blas_trans_type transt,
               int nrhs, SCALAR_IN alpha, blas_sparse_matrix T, ARRAY B, int ldb
             );
```

3.8.5 Handle Management

The Handle Management routines can be divided into five sets; the creation routines (Section 3.8.6),
the insertion routines (Section 3.8.7), the completion routine (Section 3.8.8), matrix property rou
tines (Section 3.8.9), and the destruction routine (Section 3.8.10). A brief discussion of these rou
tines was given in Section 3.5.1.

3.8.6 Creation Routines

USCR_BEGIN (begin point-entry construction) \( A \leftarrow (\ldots) \)

USCR_BEGIN is used to create a sparse matrix handle where the matrix is held in normal point-
wise form (by single scalar entries). \( m \) and \( n \) must be greater than zero. The \( x \) prefix in the binding
names specifies the scalar type and precision of the matrix, as described in 3.2. istat is used as an
error flag and will be zero if the routine executes successfully. The C binding returns a new handle
as its function return value; this handle is void, i.e. USGP(return_value, blas_void_handle) is
true, if the routine did not execute successfully.

- Fortran 95 binding:

```fortran
SUBROUTINE xuscr_begin( m, n, a, istat )
  INTEGER, INTENT(IN) :: m, n
  INTEGER, INTENT(OUT) :: a, istat
```
• Fortran 77 binding:

\[
\text{SUBROUTINE BLAS_xUSCR\_BEGIN( M, N, A, ISTAT )}
\]

\[
\text{INTEGER M, N, A, ISTAT}
\]

• C binding:

\[
\text{blas\_sparse\_matrix BLAS\_xuscr\_begin( int m, int n );}
\]

\[
\text{USCR\_BLOCK\_BEGIN (begin constant block-entry construction) } A \leftarrow (\ldots)
\]

\text{USCR\_BLOCK\_BEGIN is used to create a sparse matrix handle referring to a block-entry matrix where the blocksize of all entries is constant, that is block entries are } k \times l. \text{ Mb, Nb, k and l must all be greater than zero. The x prefix in the binding names specifies the scalar type and precision of the matrix, as described in 3.2. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns a new handle as its function return value; this handle is void, i.e. USGP(\text{return\_value, blas\_void\_handle}) is true, if the routine did not execute successfully.}

• Fortran 95 binding:

\[
\text{SUBROUTINE xuscr\_block\_begin( Mb, Nb, k, l, a, istat )}
\]

\[
\text{INTEGER, INTENT(IN) :: Mb, Nb, k, l}
\]

\[
\text{INTEGER, INTENT(OUT) :: a, istat}
\]

• Fortran 77 binding:

\[
\text{SUBROUTINE BLAS\_xUSCR\_BLOCK\_BEGIN( MB, NB, K, L, A, ISTAT )}
\]

\[
\text{INTEGER MB, NB, K, L, A, ISTAT}
\]

• C binding:

\[
\text{blas\_sparse\_matrix BLAS\_xuscr\_block\_begin( int Mb, int Nb, int k, int l );}
\]

\[
\text{USCR\_VARIABLE\_BLOCK\_BEGIN (begin variable block-entry construction) } A \leftarrow (\ldots)
\]

\text{USCR\_VARIABLE\_BLOCK\_BEGIN is used to create a sparse matrix handle referring to a block-entry matrix whose entries may have variable block sizes. The blocksizes are given by the integer arrays K and L such that the dimension of the (i, j) block entry is } K(i) \times L(j). \text{ Mb, Nb, and all elements of K and L must be greater than zero. The x prefix in the binding names specifies the scalar type and precision of the matrix, as described in 3.2. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns a new handle as its function return value; this handle is void, i.e. USGP(\text{return\_value, blas\_void\_handle}) is true, if the routine did not execute successfully.}

• Fortran 95 binding:
3.8. LANGUAGE BINDINGS

SUBROUTINE xuscr_variable_block_begin( Mb, Nb, k, l, a, istat )
  INTEGER, INTENT(IN) :: Mb, Nb, k(:), l(:)
  INTEGER, INTENT(OUT) :: a, istat

- Fortran 77 binding:

  SUBROUTINE BLAS_xUSCR_VARIABLE_BLOCK_BEGIN( MB, NB, K, L, A, ISTAT )
  INTEGER MB, NB, A, ISTAT
  INTEGER K(*), L(*)

- C binding:

  blas_sparse_matrix BLAS_xuscr_variable_block_begin( int Mb, int Nb,
                                                    const int *k,
                                                    const int *l );

3.8.7 Insertion routines

USCR_INSERT_ENTRY (insert single value at coordinate (i, j))

USCR_INSERT_ENTRY is used to build a sparse matrix, passing in one scalar entry at a time.
This routine may only be called on a matrix handle that was opened via the USCR_BEGIN routine
and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be
modified after any insertions, so this call must follow all settings made to the matrix via the USSP
routine. The matrix handle must be in a new state (i.e USPG(A, blas_new_handle) is true) upon
the first call to this routine. Upon successful completion, the matrix handle is an open state (i.e.
USGP(A, blas_open_handle) is true) and subsequent calls to this routine will keep the matrix in
this state, until a call to USCR_END is issued. The precision type of the sparse matrix represented
by the handle A must match the remaining floating-point arguments. istat is used as an error flag
and will be zero if the routine executes successfully. The C binding returns istat as the function
return value.

- Fortran 95 binding:

  SUBROUTINE uscr_insert_entry( a, val, i, j, istat )
  INTEGER, INTENT(IN) :: a, i, j
  <type>(<wp>), INTENT(IN) :: val
  INTEGER, INTENT(OUT) :: istat

- Fortran 77 binding:

  SUBROUTINE BLAS_xUSCR_INSERT_ENTRY ( A, VAL, I, J, ISTAT )
  INTEGER A, I, J, ISTAT
  <type> VAL

- C binding:
int BLAS_xuscr_insert( blas_sparse_matrix A, SCALAR val, int i, int j );

USCR_INSERT_ENTRIES (insert a list of values in coordinate form (val, i, j))  
\[ A \leftarrow (val, i, j) \]

USCR_INSERT_ENTRIES is used to build a sparse matrix, passing in a list of point entries. This routine may only be called on a matrix handle that was opened via the USCR_BEGIN routine and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be modified after any insertions, so this call must follow all settings made to the matrix via the USSP routine. The matrix handle must be in a new state (i.e. USGP(A, blas_new_handle) is true) upon the first call to this routine. Upon successful completion, the matrix handle is an open state (i.e. USGP(A, blas_open_handle) is true) and subsequent calls to this routine will keep the matrix in this state, until a call to USCR_END is issued. The precision type of the sparse matrix represented by the handle A must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

  SUBROUTINE uscr_insert_entries( a, val, indx, jndx, istat )
  INTEGER, INTENT(IN) :: a, indx(:), jndx(:)
  <type>(<wp>), INTENT(IN) :: val (:)
  INTEGER, INTENT(IN) :: istat

- Fortran 77 binding:

  SUBROUTINE BLAS_xUSCR_INSERT_ENTRIES( A, NZ, VAL, INDX, JNDX, $
  INTEGER A, NZ, INDX( * ), JNDX( * ), ISTAT
  <type> VAL( * )

- C binding:

  int BLAS_xuscr_insert_entries( blas_sparse_matrix A, int nz,
  const ARRAY val,
  const int *indx, const int *jndx );

USCR_INSERT_COL (insert a compressed column)  
\[ A \leftarrow (\ldots) \]

USCR_INSERT_COL is used to build a sparse matrix, passing in one column at a time. This routine may only be called on a matrix handle that was opened via the USCR_BEGIN routine and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be modified after any insertions, so this call must follow all settings made to the matrix via the USSP routine. The matrix handle must be in a new state (i.e. USGP(A, blas_new_handle) is true) upon the first call to this routine. Upon successful completion, the matrix handle is an open state (i.e. USGP(A, blas_open_handle) is true) and subsequent calls to this routine will keep the matrix in this state, until a call to USCR_END is issued. The precision type of the sparse matrix represented by the handle A must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.
3.8. LANGUAGE BINDINGS

- Fortran 95 binding:

```fortran
SUBROUTINE uscr_insert_col( a, j, val, indx, istat )
    INTEGER, INTENT(IN) :: a, j, indx(:)
    <type>(<wp>), INTENT(IN) :: val(:)
    INTEGER, INTENT(OUT) :: istat
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSCR_INSERT_COL( A, J, NZ, VAL, INDX, ISTAT )
    INTEGER A, J, NZ, INDX( * ), ISTAT
    <type> VAL( * )
```

- C binding:

```c
int BLAS_xuscr_insert_col( blas_sparse_matrix A, int j, int nz,
    const ARRAY val, const int *indx );
```

USCR_INSERT_ROW (insert a compressed row) \( A \leftarrow (\ldots) \)

USCR_INSERT_ROW is used to build a sparse matrix, passing in one row at a time. This routine may only be called on a matrix handle that was opened via the USCR_BEGIN routine and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be modified after any insertions, so this call must follow all settings made to the matrix via the USSP routine. The matrix handle must be in a new state (i.e. USPG(A, blas_new_handle) is true) upon the first call to this routine. Upon successful completion, the matrix handle is an open state (i.e. USGP(A, blas_open_handle) is true) and subsequent calls to this routine will keep the matrix in this state, until a call to USCR_END is issued. The precision type of the sparse matrix represented by the handle A must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE uscr_insert_row( a, i, val, indx, istat )
    INTEGER, INTENT(IN) :: a, i, indx(:)
    <type>(<wp>), INTENT(IN) :: val(:)
    INTEGER, INTENT(OUT) :: istat
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSCR_INSERT_ROW( A, I, NZ, VAL, INDX, ISTAT )
    INTEGER A, I, NZ, INDX( * ), ISTAT
    <type> VAL( * )
```

- C binding:
int BLAS_xuscr_insert_row( blas_sparse_matrix A, int i, int nz,
const ARRAY val, const int *indx );

USCR_INSERT_CLIQUE (insert a dense matrix clique) \( A \leftarrow (val, i, j) \)

**USCR_INSERT_CLIQUE** is used to build a sparse matrix, passing in a dense matrix \( val \) of dimension \( k \times l \) and corresponding integer arrays containing the list of \( (i, j) \) indices describing the clique. This routine may only be called on a matrix handle that was opened via the USCR_BEGIN routine and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be modified after any insertions, so the call must follow all settings made to the matrix via the USSP routine. The matrix handle must be in a new state (i.e. USPG (A, blas_new_handle) is true) upon the first call to this routine. Upon successful completion, the matrix handle is an open state (i.e. USGP (A, blas_open_handle) is true) and subsequent calls to this routine will keep the matrix in this state, until a call to USCR_END is issued. The precision type of the sparse matrix represented by the handle A must match the remaining floating-point arguments. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE uscr_insert_clique( a, val, indx, jndx, istat )
  INTEGER, INTENT(IN) :: a, indx(:), jndx(:)
  <type>(<wp>), INTENT(IN) :: val(:,:)
  INTEGER, INTENT(OUT) :: istat
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xUSCR_INSERT_CLIQUE( A, K, L, VAL, LDV, INDX,
$                JNDX, ISTAT )
  INTEGER A, K, L, LDV, INDX(*), JNDX(*), ISTAT
  <type>    VAL( LDV, * )
```

- C binding:

```c
int BLAS_xuscr_insert_clique( blas_sparse_matrix A, const int k,
const int l, const ARRAY val,
const int row_stride, const int col_stride,
const int *indx,
const int *jndx );
```

USCR_INSERT_BLOCK (insert a block entry at block coordinate \((bi, bj)\)) \( A \leftarrow (val, bi, bj) \)

**USCR_INSERT_BLOCK** is used to insert a block entry into a block-entry matrix. This routine may only be called on a matrix handle that was opened with one of the block creation routines (USCR_BLOCK_BEGIN or USCR_VARIABLE_BLOCK_BEGIN) and has not yet been closed via the USCR_END routine. Furthermore, matrix properties cannot be modified after any insertions, so
this call must follow all settings made to the matrix via the USSP routine. The matrix handle must
be in a new state (i.e. USGP(A, blas_new_handle) is true) upon the first call to this routine. Upon
successful completion, the matrix handle is an open state (i.e. USGP(A, blas_open_handle) is
true) and subsequent calls to this routine will keep the matrix in this state, until a call to USCR_END
is issued. The dimensions of the block entry are determined from the blocksize information passed to
USCR_BLOCK_BEGIN or USCR_VARIABLE_BLOCK_BEGIN. In the Fortran 77 binding, LDV denotes
the leading dimension of the dense array VAL. The precision type of the sparse matrix represented
by the handle A must match the remaining floating-point arguments. istat is used as an error flag
and will be zero if the routine executes successfully. The C binding returns istat as the function
return value.

- Fortran 95 binding:

```fortran
SUBROUTINE uscr_insert_block(a, val, bi, bj, istat)
  INTEGER, INTENT(IN) :: a, bi, bj
  INTEGER, INTENT(OUT) :: istat
  <type>(<wp>), INTENT(IN) :: val(:, :)
END SUBROUTINE uscr_insert_block
```

- Fortran 77 binding:

```fortran
SUBROUTINE F_xUSCR_INSERT_BLOCK(A, VAL, LDV, BI, BJ, ISTAT)
  INTEGER A, LDV, BI, BJ, ISTAT
  <type> VAL(LDV, *)
END SUBROUTINE F_xUSCR_INSERT_BLOCK
```

- C binding:

```c
int BLAS_xuscr_insert_block(int a, const ARRAY val, int row_stride,
int col_stride, int bi, int bj);
```

### 3.8.8 Completion of construction routine

USCR_END (entries completed; build valid matrix handle) \( A \leftarrow (...) \)

USCR_END is used to complete the construction phase and build a valid sparse matrix han-
dle. This routine may be called only with a sparse matrix handle that was previously cre-
ated via the routines USCR_BEGIN, USCR_BLOCK_BEGIN or USCR_VARIABLE_BLOCK_BEGIN.
The matrix handle must be in an open or new state, i.e. either USGP(A, blas_open_handle)
or USGP(A, blas_new_handle) is true. istat is used as an error flag and will be zero if the routine
executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:

```fortran
SUBROUTINE uscr_end(a, istat)
  INTEGER, INTENT(IN) :: a
  INTEGER, INTENT(OUT) :: istat
END SUBROUTINE uscr_end
```

- Fortran 77 binding:
3.8.9 Matrix property routines

**USGP (get/test matrix property)**

For a given sparse matrix $A$, the routine USGP returns the value of the given property name. The first argument is the matrix handle and the second argument is one of the properties listed in Table 3.5. Each grouping denotes a subset of mutually exclusive properties. The properties `blas_num_rows`, `blas_num_cols`, and `blas_num_nonzeros` return integer values, all other properties return 1 if true, and 0 otherwise. If the matrix handle is corrupt, i.e. `USGP(A, blas_void_handle)` is true, all other Boolean properties are false, and integer valued properties (`blas_num_rows`, `blas_num_cols`, and `blas_num_nonzeros`) return 0.

- **Fortran 95 binding:**
  ```fortran
  SUBROUTINE usgp( a, pname, m )
  INTEGER, INTENT(IN) :: a
  INTEGER, INTENT(IN) :: pname
  INTEGER, INTENT(OUT) :: m
  ```

- **Fortran 77 binding:**
  ```fortran
  SUBROUTINE BLAS_USGP( A, PNAME, M )
  INTEGER A, PNAME, M
  ```

- **C binding:**
  ```c
  int BLAS_usgp( blas_sparse_matrix A, int pname );
  ```

**USSP (set matrix property)**

For a given valid sparse matrix handle $A$, the routine USSP sets the value of the given matrix property. This routine must be called after the handle has been created, and before any of the INSERT routines have been called. That is, the matrix handle must be in a new state, i.e. `USGP(A, blas_new_handle)` is true. `istat` is used as an error flag and will be zero if the routine executes successfully and is set to -1 if the handle is corrupt, i.e. if `USGP(A, blas_void_handle)` is true. The C binding returns `istat` as the function return value.

The first argument is the matrix handle; the second argument is one of the properties listed in Table 3.4. Each grouping denotes a subset of mutually exclusive properties.

If two incompatible properties from the same group are set, the results are undefined. For example, the sequence
3.8. LANGUAGE BINDINGS

```c
BLAS_ussp(A, blas_zero_base);
BLAS_ussp(A, blas_one_base);
```

leads to an ambiguity and the resulting handle is void (i.e. USGP(A, blas_void_handle) is true). It is possible to guard against this by testing the properties first.

- Fortran 95 binding:
  ```fortran
  SUBROUTINE ussp( a, pname, istat )
  INTEGER, INTENT(INOUT) :: a
  INTEGER, INTENT(IN) :: pname
  INTEGER, INTENT(OUT) :: istat
  ```

- Fortran 77 binding:
  ```fortran
  SUBROUTINE BLAS_USSP( A, PNAME, ISTAT )
  INTEGER A, PNAME, ISTAT
  ```

- C binding:
  ```c
  int BLAS_ussp( blas_sparse_matrix A, int pname );
  ```

3.8.10  Destruction routine

USDS (release matrix handle)

```fortran
(...) ← A
```

The routine USDS releases any memory internally used by the sparse matrix handle A. The handle must have been previously closed by the USCR_END routine, i.e. USGP(A, blas_valid_handle) must be true. It turns this into a handle that is no longer in use, i.e. USGP(A, blas_void_handle) is true. istat is used as an error flag and will be zero if the routine executes successfully. The C binding returns istat as the function return value.

- Fortran 95 binding:
  ```fortran
  SUBROUTINE usds( a, istat )
  INTEGER, INTENT(IN) :: a
  INTEGER, INTENT(OUT) :: istat
  ```

- Fortran 77 binding:
  ```fortran
  SUBROUTINE BLAS_USDS( A, ISTAT )
  INTEGER A, ISTAT
  ```

- C binding:
  ```c
  int BLAS_usds( blas_sparse_matrix A );
  ```
Chapter 4

Extended and Mixed Precision BLAS

4.1 Overview

This Chapter describes extended and mixed precision implementations of the BLAS described in other chapters. Extended precision is used only internally to the BLAS; the input and output arguments remain as before. Extended precision permits us to implement some algorithms that may be simpler, more accurate, and sometimes even faster than without it. Mixed precision refers to having some input/output parameters that are both single precision and double precision, or both real and complex. Mixed precision similarly permits us to write simpler or faster algorithms. But given the complexity that could result by allowing too many combinations of types and precisions, we must choose a parsimonious subset that is both useful and reasonable to implement.

The rest of this chapter is organized as follows. Section 4.2 summarizes the designs goals and decisions that guide our design, with details left to [42]. Section 4.3 summarizes the functions supported in extended and mixed precision. This includes a discussion of the error bounds that routines must satisfy. Section 4.4 summarizes the issues in our design of language bindings for Fortran 95, Fortran 77 and C. Section 4.5 contains the detailed calling sequences for the subroutines in the three languages. A complete justification of our design appears in [42].

4.2 Design Goals and Summary

Our proposal to have extended and mixed precision in the BLAS is motivated by the following facts:

- A number of important linear algebra algorithms can become simpler, more accurate and sometimes faster if internal computations carry more precision (and sometimes more range) than is used for the input and output arguments. These include linear system solving, least squares problems, and eigenvalue problems. Often the benefits of wider arithmetic cost only a small fractional addition to the total work.

- For single precision input, the computer’s native double precision is a way to achieve these benefits easily on all commercially significant computers, at least when only a few extra-precision operations are needed. (Crays and their emulators implement 64-bit single in hardware and much slower 128-bit double in software, so if a great many double precision operations are needed, these machines will slow down significantly.)

- Intel and similar processors are designed to run fastest performing arithmetic to the full 80-bit width, wider than double precision, of their internal registers. These computers confer
some benefits of wider arithmetic at little or no performance penalty. Some BLAS on these
computers already perform wider arithmetic internally but, without knowing this for sure,
programmers cannot exploit it.

• All computers can simulate quadruple precision or something like it in software at the cost of
arithmetic slower than double precision by at worst an order of magnitude. Less slowdown
is incurred for a rough double-double precision on machines (IBM RS/6000, PowerPC/Mac,
SGI/MIPS R8000, HP PA RISC 2.0) with special fused multiply-accumulate instructions.
Since some algorithms require very little extra precise arithmetic to get a large benefit, the
slowdown is practically negligible.

Given the variety of implementation techniques hinted at above, we need to carefully examine
the costs and benefits of exploiting various arithmetic features beyond the most basic ones, and
choose a parsimonious subset that

**Goal 1:** is reasonable to implement,

**Goal 2:** supports some if not all important application examples,

**Goal 3:** is easy to use,

**Goal 4:** encourages the writing of portable code, and

**Goal 5:** accommodates growth as we learn about new algorithms exploiting our arithmetic fea-
tures.

Here is an outline of our design decisions. These are discussed and justified in detail in [42].

1. We will not require that the user explicitly declare or use any new extended precision data
types, i.e. beyond the standard single and double precisions, since these are not supported
in a standard way by every language and compiler. Thus the only extended precision that
we mandate will be hidden inside the BLAS, and so can be implemented in any convenient
machine dependent way. This supports Goals 1, 3 and 4 above.

2. This internal extended precision will support most of the application examples listed in [42],
supporting Goal 2.

3. Since we cannot predict all the future applications of extended or mixed precision, we will
accommodate growth by making our proposal as orthogonal as possible to the rest of this
proposal, showing how to take any BLAS routine, determine whether extra precision is worth
using (since sometimes it is not), and define the extended precision version if it is. This
supports Goal 5.

4. Since the number of possible routines with mixed precision inputs is very large, we will specify
a small subset of mixed precision routines which seems to cover most foreseeable needs. This
supports Goals 1 and 2.

5. In order to easily estimate error bounds in code by running with different internal precisions
and then comparing the answers, (see Example 8 in [42]), we need to be able to specify
the extended precision at runtime; we will do this with a variable we will call PREC. This
supports Goal 2.
6. Since different machines may best support extended precision in different ways, PREC could potentially take on many machine-dependent values. Instead we have chosen a parsimonious subset that will be available on all machines, permitting the implementor to support others if desired. This supports all the Goals 1 and 4 above.

7. Since the precision specified by one value of PREC can still have different implementations and so different error bounds on different machines, we have specified environmental enquiries for the user to be able to discover the actual machine precision (or over/underflow thresholds) used at runtime. This lets the user pick appropriate stopping criteria for iterations, etc. This supports Goals 3 and 4.

4.3 Functionality

This section describes the functionality of extended and mixed precision BLAS in a language independent way. Section 4.3.1 describes how extra precision is specified via the PREC argument. Section 4.3.2 describes in general what kind of mixed precision operations will be supported, Section 4.3.3 describes the error bounds that BLAS operations must satisfy; this is where the semantics of “extra precision” are precisely specified. Finally, section 4.3.4 lists the functions that will be supported in extra and/or mixed precision.

4.3.1 Specifying Extra Precision

The internal precision to be used by an extended precision routine will be specified by an argument called PREC. It is not entirely straightforward to describe PREC because even on a single machine there may be multiple ways of implementing wider-than-double-precision arithmetic (see [42]).

To encourage portability, we specify names for precisions that may map to different formats and techniques on different machines. As discussed in section 1.6, historically the words “single” and “double” have referred to very different formats on different architectures. Nonetheless, we all agree on single precision as a word with a certain meaning, and double precision too, meaning twice or more precision than single. The definitions below add two more precisions, whose implementation details are discussed in [42].

PREC = Single . This means single precision, whatever single means on the particular machine, language and compiler.

PREC = Double . This means double precision, again whatever that means on a particular machine, language and compiler.

PREC = Indigenous . This means the widest hardware-supported format available. Its intention is to let the machine run close to top speed, while being as accurate as possible. On some machines this would be a 64-bit format (whether it is called single or double), but on Intel machines and ones like them it means the 80-bit IEEE format of the floating point registers.

PREC = Extra . This means anything at least 1.5 times as accurate than double, and in particular wider than 80-bits (see section 4.3.3 for details). An existing quadruple precision format could be used to implement this, but it can probably be implemented more efficiently using native double (or indigenous) operations in a technique called “double-double”, described in [42, 46, 47]. It is possible to write a portable and reasonably efficient reference implementation of all proposed routines using these techniques [42].
The actual names for PREC values are specified in section A.3. Here are the rules for using PREC:

1. The internal precision used must always be at least as high as the most precise input or output. So if the user requests less internal precision than in the most precise input or output, then the implementor must use more than requested.

2. The implementor may always use a higher precision than the one requested in the subroutine call, if this is convenient or faster.

3. The precision actually used is available to the user via the environmental enquiry in section 4.3.3.

4. PREC may take on other machine dependent values provided by the implementor, provided these are documented via the environmental enquiry routine.

Advice to implementors: While it appears that as many as seven new implementations of each routine are needed (four when the arguments are single, and three when the arguments are double), in fact fewer are needed: Two exist already as the standard BLAS (single input/output with PREC = Single, and double input/output when PREC = Double), Indigenous = Double or Indigenous = Single on many machines, and wider precision than requested may be used. Thus the only really new implementations may be single input/output with Double or Extra internal precision, and double input/output with Extra internal precision. Of these, only Extra internal precision may need arithmetic not already native to the machine. A reference implementation is described in [42].

4.3.2 Mixed Precision

Suppose a subroutine has several floating point arguments, some scalars and some arrays. Mixed precision refers to permitting these arguments to have different mathematical types, meaning real and complex, or different precisions, meaning single and double. Some BLAS in Chapter 2 are naturally defined with arguments of mixed mathematical type (e.g. HERK), but most have a single mathematical type; all are defined with the same precision for all arguments.

The permitted combinations of mathematical types and precisions are defined as follows. There are two cases:

1. The mathematical types of the input/output floating point arguments are identical to the BLAS as defined in Chapter 1. All scalar arguments and the output argument (scalar or array) are double precision. At least one array argument must be single precision.

For example, suppose the function being implemented is matrix-matrix multiplication $C = \alpha \cdot A \cdot B + \beta \cdot C$, where $\alpha$ and $\beta$ are scalars and $A$, $B$ and $C$ are arrays. Then the allowed types are as follows ($S = \text{Single real}, D = \text{Double real}, C = \text{Single complex}, Z = \text{Double complex}$).

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$A$</th>
<th>$B$</th>
<th>$\beta$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>S</td>
<td>S</td>
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</tr>
<tr>
<td>Z</td>
<td>C</td>
<td>C</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>C</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>C</td>
<td>Z</td>
<td>Z</td>
</tr>
</tbody>
</table>
2. The precision of all floating point arguments must be single, or all must be double. All scalar arguments and the output argument (scalar or array) are complex (unless a scalar argument must be real for mathematical reasons, like $\alpha$ and $\beta$ in HERK). At least one input array argument must be real.

For example, suppose the function being implemented is matrix-matrix multiplication as before. Then the allowed types are as follows:

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$A$</th>
<th>$B$</th>
<th>$\beta$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>S</td>
<td>S</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>S</td>
<td>C</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>S</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Z</td>
<td>D</td>
<td>D</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>D</td>
<td>Z</td>
<td>Z</td>
<td>Z</td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>D</td>
<td>Z</td>
<td>Z</td>
</tr>
</tbody>
</table>

Note that we specify only 16 versions of matrix-matrix multiplication (the 12 mixed ones above, and 4 unmixed), in contrast to the maximum possible $4^5 = 1024$.

4.3.3 Numerical Accuracy and Environmental Enquiries

The machine dependent interpretations of PREC require us to have a more complicated environmental enquiry routine to describe the numerical behavior of the routine in this chapter than the simpler FPINFO routine described in sections 1.6 and 2.7. While FPINFO should still be available for the user to call to get basic properties of the single and double precision floating point types, here we will specify an additional routine FPINFO_X that depends on PREC.

The calling sequence of this function is

```
result = FPINFO_X (CMACH, PREC)
```

Both arguments are input arguments, with the requested information returned as the integer value of FPINFO_X. The exact input values depend on the language, and are described in section 4.4. PREC has the same meaning as before. Input argument CMACH may take on the named constant values below, which are a subset of those permitted by function FPINFO as described in section 2.7. Only the first six values of CMACH from section 2.7 are permitted, because 1) they are sufficient to define the remaining parameters by using the formulas in section 1.6, and 2) the values returned by FPINFO_X are representable integer values, whereas the other possible return values, like the overflow and underflow thresholds, may not be representable in any user-declarable format.

<table>
<thead>
<tr>
<th>Floating Point parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BASE</td>
<td>base of the machine</td>
</tr>
<tr>
<td>T</td>
<td>number of (BASE) digits in the mantissa</td>
</tr>
<tr>
<td>RND</td>
<td>1 when &quot;proper rounding&quot; occurs in addition, 0 otherwise</td>
</tr>
<tr>
<td>IEEE</td>
<td>1 when rounding in addition occurs in &quot;IEEE style&quot;, 0 otherwise</td>
</tr>
<tr>
<td>EMIN</td>
<td>minimum exponent before (gradual) underflow</td>
</tr>
<tr>
<td>EMAX</td>
<td>largest exponent before overflow</td>
</tr>
</tbody>
</table>
4.3. FUNCTIONALITY

We will use the following notation to describe machine parameters derivable from the values returned by FPINFO_X using the formulas in section 1.6:

- \( \epsilon_{\text{prec}} \) is relative machine precision or machine epsilon of the internal precision specified by PREC,
- \( \epsilon_o \) is the machine epsilon for the output precision,
- \( \text{OV}_{\text{PREC}} \) and \( \text{UN}_{\text{PREC}} \) are the overflow and underflow thresholds for internal precision PREC, and
- \( \text{OV}_o \) and \( \text{UN}_o \) are the overflow and underflow thresholds for the output precision.

Here are the error bounds satisfied by the extra precision routines, and how they depend on \( \epsilon \).

There are two cases of interest.

1. Suppose each component of the computed result is of the form

\[
r_{\text{true}} = \alpha \cdot \left( \sum_{i=1}^{n} a_i \cdot b_i \right) + \beta \cdot c,
\]

where all quantities are scalars. This covers the dot product, scaled vector addition and scaled vector accumulation, all variants of matrix-vector and matrix-matrix products, and low-rank updates (sometimes with \( \alpha \) and \( \beta \) taking on special values like zero and one). In this case the error bound, in the absence of over/underflow of any intermediate or output quantities, should satisfy

\[
|r_{\text{computed}} - r_{\text{true}}| \leq \gamma (n + 2) \cdot \epsilon_{\text{prec}} (|\alpha| \cdot \sum_{i=1}^{n} |a_i \cdot b_i| + |\beta \cdot c|) + \epsilon_o \cdot |r_{\text{true}}|.
\]

where \( \gamma = 1 \) if all data is real and \( \gamma = 2\sqrt{2} \) if any data is complex.

Rationale: This accommodates all reasonable, non-Strassen based implementations, with real or complex scalars (and conventional multiplication of complex scalars), that perform all intermediate floating point operations with machine epsilon \( \epsilon_{\text{prec}} \), with or without a guard digit, before rounding the final result to precision \( \epsilon_o \). Underflow is guaranteed to be absent if no intermediate quantity stored in precision PREC is less than \( \text{UN}_{\text{PREC}} \) in magnitude (unless its exact value is zero) and \( |r_{\text{computed}}| \) is not less than \( \text{OV}_o \) (unless its exact value is zero). Similarly, overflow is guaranteed to be absent if no intermediate quantity in precision PREC exceeds \( \text{OV}_{\text{PREC}} \) in magnitude, and \( |r_{\text{computed}}| \) does not exceed \( \text{OV}_o \). We avoid specifying what happens with underflow, because the implementor may reasonably choose to compute \( r \) using \( \alpha \cdot (\sum a_i \cdot b_i) \), \( \sum (\alpha \cdot a_i) \cdot b_i \) or \( \sum a_i \cdot (\alpha \cdot b_i) \) depending on dimensions, and the error bounds in the presence of underflow can differ significantly in these three cases. See [42] for implementation recommendations and detailed error bounds in the presence of underflow.

2. Suppose the computed solution consists of one or more vectors \( x \) satisfying an \( n \)-by-\( n \) triangular system of equations

\[
T x = \alpha b
\]

where \( \alpha \) is a scalar, \( b \) is a vector (or vectors), and \( T \) is a triangular matrix. In this case the computed solution, in the absence of over/underflow in intermediate or output quantities, satisfies

\[
(T + E)(x_{\text{computed}} + e) = \alpha(b + f)
\]
where $|E_{ij}| \leq \rho \varepsilon_{\text{REC}} |T_{ij}|$, $|e_i| \leq \varepsilon_d |x_{\text{computed},i}|$, $|f_i| \leq \rho \varepsilon_{\text{REC}} |b_i|$, $\rho = 1$ if all data is real, and $\rho = 6 + 4\sqrt{2}$ if any data is complex.

**Rationale:** This accommodates all reasonable, substitution-based methods of solution, with summations evaluated in any order, with all intermediate floating point operations done with machine epsilon $\varepsilon_{\text{REC}}$, and with all intermediate quantities stored to the same precision. In particular, this means that the entries of $x_{\text{computed}}$ must be temporarily stored with precision $\varepsilon_{\text{REC}}$ before being rounded to the output precision at the end. Overflow and underflow are defined and treated as before. See [42] for implementation recommendations and detailed error bounds in the presence of underflow.

The values of $\varepsilon_{\text{REC}}$ must satisfy the following inequalities:

$$
\varepsilon_{\text{DOUBLE}} \leq \varepsilon_{\text{SINGLE}}^2
$$

$$
\varepsilon_{\text{INDIGENOUS}} \leq \varepsilon_{\text{SINGLE}}
$$

$$
\varepsilon_{\text{EXTRA}} \leq \varepsilon_{\text{DOUBLE}}^{1.5}
$$

The first inequality says that double precision is at least twice as accurate (has twice as many significant digits) as single precision. The second inequality says that indigenous is at least as accurate as single precision. The third inequality says that extra precision is at least 1.5 times as accurate (has 1.5 times as many significant digits) as double precision.

**Advice to implementors:** This is only a lower bound on the number of significant digits in extra precision; most reasonable implementations can get close to twice as many digits as double precision [42]. The lower bound is intended to exclude the use of the 80-bit IEEE format as Extra precision when Double is the 64-bit IEEE format. It is important that $BASE$, $T$, and $RND$ are chosen so that $EPS$ defined by $EPS = BASE^{1-T}$ if $RND = 0$ and $EPS = .5 \times BASE^{1-T}$ if $RND = 1$ can be used for error analysis. For example in the reference implementation of EXTRA precision in [42], $T = 105$ even though 106 bits are stored. Though we do not require this, the simplest way to achieve the error bounds described above is for floating operations $\odot \in \{+,-,\times,\div\}$ to satisfy the following bounds in the absence of over/underflow: $fl(a \odot b) = (a \odot b)(1 + \delta)$ for some $|\delta| \leq EPS$ when $a$ and $b$ are real, $fl(a \pm b) = (a \pm b)(1 + \delta)$ for some $|\delta| \leq \sqrt{2} \cdot EPS$ when $a$ and $b$ are complex, $fl(a \times b) = (a \times b)(1 + \delta)$ for some $|\delta| \leq 2\sqrt{2} \cdot EPS$ when $a$ and $b$ are complex, and $fl(a/b) = (a/b)(1 + \delta)$ for some $|\delta| \leq (6 + 4\sqrt{2}) \cdot EPS$ when $a$ and $b$ are complex.

The semantics of overflow and underflow are discussed more carefully in [42]; they become more complicated concepts when using implementation techniques like double-double for extra precision. The important properties they should satisfy are

1. In any precision, a quantity greater than $OV$ generates an exception, a $\pm \infty$ symbol, or otherwise somehow indicates its complete loss of precision.

2. In any precision, the error in a floating point operation that might underflow (during some part of the calculation, if for example it is double-double) is described by $fl(a \odot b) = (a \odot b)(1 + \delta) + \eta$, for some $|\delta| \leq EPS$ and $|\eta| \leq UN$ if $a$ and $b$ are real, and for slightly larger $|\delta|$ and $|\eta|$ if $a$ and $b$ are complex.

We choose not to specify the overflow and underflow thresholds in more detail, in order not to eliminate innovative ways of implementing extra precision.
4.3.4 Function Tables

As discussed in [42], not all BLAS routines from Chapter 2 are worth converting to extra or mixed precision, so we only include the subset that is worth converting.

Table 4.1 is a subset of Table 2.1 in Chapter 2, Reduction Operations.
Table 4.2 is a subset of Table 2.3 in Chapter 2, Vector Operations.
Table 4.3 is a subset of Table 2.5 in Chapter 2, Matrix-Vector Operations.
Table 4.4 is a subset of Table 2.7 in Chapter 2, Matrix Matrix Operations.

| Dot product | $r \leftarrow \beta r + \alpha x^T y$ | DOT |
| Sum         | $r \leftarrow \sum_i x_i$          | SUM |

Table 4.1: Extra and Mixed Precision Reduction Operations

| Scaled vector accumulation | $y \leftarrow \alpha x + \beta y$, AXPBY |
| Scaled vector addition    | $w \leftarrow \alpha x + \beta y$, WAXPBY |

Table 4.2: Extra and Mixed Precision Vector Operations

| Matrix vector product | $y \leftarrow \alpha Ax + \beta y$ | GE, GB, SY, SP, SB, MV |
|                       | $y \leftarrow \alpha A^T x + \beta y$ | GE, GB, MV |
|                       | $x \leftarrow \alpha Tx, x \leftarrow \alpha T^T x$ | TR, TB, TP, MV |
| Summed matrix vector multiplies | $y \leftarrow \alpha Ax + \beta Bx$ | GE, SUM, MV |
| Triangular solve       | $x \leftarrow \alpha T^{-1} x, x \leftarrow \alpha T^{-T} x$ | TR, TB, TP, SV |

Table 4.3: Extra and Mixed Precision Matrix Vector Operations

| Matrix matrix product | $C \leftarrow \alpha AB + \beta C, C \leftarrow \alpha A^T B + \beta C, C \leftarrow \alpha A^T B^T + \beta C$ | GE, MM |
| Triangular multiply   | $B \leftarrow \alpha TB, B \leftarrow \alpha BT$ | SY, HE, MM |
|                       | $B \leftarrow \alpha T^T B, B \leftarrow \alpha BT^T$ | TR, MM |
| Triangular solve      | $B \leftarrow \alpha T^{-1} B, B \leftarrow \alpha BT^{-1}$ | TR, SM |
| Symmetric rank $k \& 2k$ updates ($C = C^T$) | $C \leftarrow \alpha A^T + \beta C, C \leftarrow \alpha A^T A + \beta C, C \leftarrow (\alpha A)B^T + B(\alpha A)^T + \beta C$ | SY, HE, RK |

Table 4.4: Extra and Mixed Precision Matrix Matrix Operations

4.4 Interface Issues

This section describes the common issues for our three language bindings: Fortran 95, Fortran 77 and C. Here is a summary of the systematic way we take a subroutine name and its argument list,
and modify them to allow for extra or mixed precision:

1. **Subroutine names and mixed precision inputs.** If the language permits a subroutine argument to have more than one type, because it can dispatch the right routine based on the actual type at compile time (Fortran 95, but not Fortran 77 or C), then the subroutine name does not have to change to accommodate mixed precision. Otherwise, a new subroutine name is required, and will be created from the old one by appending characters indicating the types of the arguments.

2. **Subroutine names and extended precision.** If the language permits PREC to be an optional argument (Fortran 95, but not Fortran 77 or C), then the same subroutine name as for the non-extended precision version can be used without change. If a new name is required, it will be formed by appending \( \_X \) (or \( \_x \)) to the existing name. If the name has already been modified to accommodate mixed precision, \( \_X \) (or \( \_x \)) should be added to the end of the new name.

3. **Location of PREC in the calling sequence.** The new calling sequence will consist of the original calling sequence (for the BLAS routine without extra or mixed precision) with PREC appended at the end.

4. **Type of PREC.** It will be a derived type in Fortran 95, an integer in Fortran 77, and an enumerated type in C. Standard names are listed below.

5. **Environmental enquiry function.** Its output type is an integer. The input PREC is specified as above.

### 4.4.1 Interface Issues for Fortran 95

1. **Subroutine names and mixed precision inputs.** No new subroutine names are needed because we can exploit the optional argument interface of Fortran 95.

2. **Subroutine names and extended precision.** No new subroutine names are needed by letting PREC be an optional argument. The default in the case of no mixed precision is the standard BLAS implementation. The default in the case of mixed precision is at the discretion of the implementor, subject to the constraints of section 4.3.1.

3. **Type of PREC.** PREC is a derived type, as defined in the module blas_operator_arguments (see section A.4).

4. **Environmental enquiry function.** \( \text{fpinfo}\_X(CMACH,PREC) \) returns an integer. PREC is as specified above. CMACH is as defined in sections 1.6, 2.7, 4.3.3, and A.4.

5. **Error Handling.** Error handling is as defined in section 2.4.6.
4.4. INTERFACE ISSUES

4.4.2 Interface Issues for Fortran 77

As described in Chapter 2, this proposal violates the letter of the ANSI Fortran 77 standard by having subroutine and variable names longer than 6 characters and with embedded underscores.

1. **Subroutine names and mixed precision inputs.** The unmodified subroutine name has a character (S, D, C or Z) that specifies the floating point argument types. This will be the type of the output argument. By applying the rules in Section 4.3.2, this also determines the types of the scalar arguments. The possible types of the remaining array arguments are listed in Section 4.3.2. The types of these arguments (written S, D, C or Z) are appended to the unmodified subroutine name, in the order in which they appear in the argument list.

   For example, consider BLAS_ZGEMM(α, A, B, β, C) (only the floating point arguments are shown). The Z in BLAS_ZGEMM means that C, α and β are all double-complex. The possible types of A and B, and the corresponding subroutine names, are:

<table>
<thead>
<tr>
<th>Type of A</th>
<th>Type of B</th>
<th>Modified subroutine name</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C</td>
<td>BLAS_ZGEMM_C_C</td>
</tr>
<tr>
<td>C</td>
<td>Z</td>
<td>BLAS_ZGEMM_C_Z</td>
</tr>
<tr>
<td>Z</td>
<td>C</td>
<td>BLAS_ZGEMM_Z_C</td>
</tr>
<tr>
<td>D</td>
<td>D</td>
<td>BLAS_ZGEMM_D_D</td>
</tr>
<tr>
<td>D</td>
<td>Z</td>
<td>BLAS_ZGEMM_D_Z</td>
</tr>
<tr>
<td>Z</td>
<td>D</td>
<td>BLAS_ZGEMM_Z_D</td>
</tr>
</tbody>
</table>

2. **Subroutine names and extended precision.** To accommodate extended precision, PREC is added as the last argument, and _X is appended to the end of subroutine name (which may already have been modified to accommodate mixed precision).

   For example, double-complex matrix-matrix multiplication implemented with extended precision is named BLAS_ZGEMM_X. Double-complex matrix-matrix multiplication where the A and B arguments are single-complex is named BLAS_ZGEMM_C_C_X.

3. **Type of PREC.** PREC is an integer (named constant), as defined in the include file blas_namedconstants.h (see section A.5).

4. **Environmental enquiry function.** BLAS_FPINFO_X(CMACH, PREC) returns an integer. PREC is as specified above. CMACH is as defined in sections 1.6, 2.7, 4.3.3, and A.5.

5. **Error Handling.** Error handling is as defined in section 2.5.6.

To shorten the subroutine specifications in section 4.5, we will abbreviate the list of possible subroutine names for GEMM to a single one: BLAS_xGEMM{a_b}{_X}. The prefix x may be S (single), D (double), C (complex) or Z (double complex). Also, the subroutine name may optionally be appended with _a_b, where a and b are the types of A and B respectively, and then optionally be appended with _X. At least one of _a_b or _X must appear.

4.4.3 Interface Issues for C

1. **Subroutine names and mixed precision inputs.** The same scheme is used as in Fortran 77, as described above, except that all characters in subroutine names are lower case.
2. **Subroutine names and extended precision.** The same scheme is used as in Fortran 77, as described above, except that all characters in subroutine names are lower case.

3. **Type of PREC.** PREC is an enumerated type, as defined in the include file `blas_enum.h` (see section A.6).

4. **Environmental query function.** `BLAS_fpinf_x(CMACH, PREC)` returns an integer. PREC is as specified above. CMACH is as defined in sections 1.6, 2.7, 4.3.3, and A.6.

5. **Error Handling.** Error handling is as defined in section 2.6.9.

### 4.5 Language Bindings

#### 4.5.1 Overview

As in Chapter 2, each specification of a routine will correspond to an operation outlined in the functionality tables. Operations are organized analogous to the order in which they are presented in the functionality tables. The specification will have the form:

\[
\text{NAME (multi-word description of operation) < mathematical representation >}
\]

- Fortran 95 binding
- Fortran 77 binding
- C binding

Section 4.4 describes abbreviations we use below. For example,

```
SUBROUTINE BLAS_xDOT{_%a_%b}{_X}(N, ALPHA, X, INCX, BETA, Y, INCY, R [_, PREC])
```

means that the subroutine name may optionally be appended with \(_a\_b\), where \(a\) and \(b\) are the types of \(X\) and \(Y\), respectively, and also optionally appended with \(_X\), in which case the parameter PREC must also appear.

The routines specified here are

- **Reduction Operations (section 4.5.2)**
  - DOT (Dot product)
  - SUM (Sum)

- **Vector Operations (section 4.5.3)**
  - AXPBY (Scaled vector accumulation)
  - WAXPBY (Scaled vector addition)

- **Matrix-Vector Operations (section 4.5.4)**
  - \{GE,GB\}MV (Matrix vector product)
  - \{SY,SB,SP\}MV (Symmetric matrix vector product)
4.5. LANGUAGE Bindings

- \{HE,HB,HP\}MV (Hermitian matrix vector product)
- \{TR,TB,TP\}MV (Triangular matrix vector product)
- GE, SUM, MV (Summed matrix vector multiplies)
- \{TR,TB,TP\}SV (Triangular solve)

- Matrix-Matrix Operations (section 4.5.5)
  - GEMM (General Matrix Matrix product)
  - SYMM (Symmetric matrix matrix product)
  - HEMM (Hermitian matrix matrix product)
  - TRMM (Triangular matrix matrix multiply)
  - TRSM (Triangular solve)
  - SYRK (Symmetric rank-k update)
  - HERK (Hermitian rank-k update)
  - SYR2K (Symmetric rank-2k update)
  - HER2K (Hermitian rank-2k update)

4.5.2 Mixed and Extended Precision Reduction Operations

DOT (Dot Product)

\[ x, y \in \mathbb{R}^n, r \leftarrow \beta r + \alpha x^T y = \beta r + \alpha \sum_{i=0}^{n-1} x_i y_i \]

\[ x, y \in \mathbb{C}^n, r \leftarrow \beta r + \alpha x^H y = \beta r + \alpha \sum_{i=0}^{n-1} \bar{x}_i \bar{y}_i \text{ or } \bar{r} \leftarrow \beta \bar{r} + \alpha \bar{x}^H \bar{y} = \beta \bar{r} + \alpha \sum_{i=0}^{n-1} \bar{x}_i \bar{y}_i \]

The routine DOT adds the scaled dot product of two vectors \( x \) and \( y \) into a scaled scalar \( r \). The routine returns immediately if \( n \) is less than zero, or, if beta is equal to one and either alpha or \( n \) is equal to zero. If alpha is equal to zero then \( x \) and \( y \) are not read. Similarly, if beta is equal to zero, \( r \) is not read. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler.

When \( x \) and \( y \) are complex vectors, the vector components \( x_i \) are used unconjugated or conjugated as specified by the operator argument conj. If \( x \) and \( y \) are real vectors, the operator argument conj has no effect.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```fortran
SUBROUTINE dot( x, y, r [, conj] [, alpha] [, beta] [, prec] )
  <type>(<wp>), INTENT (IN) :: x(:)
  <type>(<wp>), INTENT (IN) :: y(:)
  <type>(<wp>), INTENT (INOUT) :: r
  TYPE (blas_conj_type), INTENT(IN), OPTIONAL :: conj
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
  TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
WHERE
  x and y have shape (n)
```
The types of alpha, x, y, beta and r are governed according to the rules of mixed precision
arguments set down in section 4.3: the types of x and y can optionally differ from that of r,
alpha and beta.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xDOT(a, b, x, y, incx, incy, r, [, PREC ])
INTEGER CONJ, INCX, INCY, N [, PREC] <type> ALPHA, BETA, R
<type> X( * )
<type> Y( * )
```

The types of ALPHA, X, Y, BETA and R are governed according to the rules of mixed precision
arguments set down in section 4.3. The prefix x is the floating point type of the arguments,
but if a_b is present then a is the type of x and b is the type of y. The suffix _x is present
if and only if PREC is present. One or both of the suffixes a_b and _x must be present.

- C binding:

```c
void BLAS_xdot(a, b, x, y, incx, incy, r, [, enum blas_prec_type prec ]);
```

The types of alpha, x, y, beta and r are governed according to the rules of mixed precision
arguments set down in section 4.3. The prefix x is the floating point type of the arguments,
but if a_b is present then a is the type of argument x and b is the type of argument y. The
suffix _x is present if and only if prec is present. One or both of the suffixes a_b and _x must
be present.

---

**SUM (Sum)**

\[ r \leftarrow \sum_{i=0}^{n-1} x_i \]

The routine SUM computes the sum of the entries of a vector x. If n is less than or equal to
zero, this routine returns immediately with the output scalar r set to zero. As described in section
2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is
set and passed to the error handler.

Extended precision is permitted, but not mixed precision.

This routine has the same specification as in Chapter 2, except that extended precision is
permitted. Mixed precision is not permitted.

- Fortran 95 binding:

```fortran
<type>(<wp>) FUNCTION sum( x, prec )
<type>(<wp>), INTENT (IN) :: x(:)
TYPE (blas_prec_type), INTENT (IN) :: prec
where
x has shape (n)
```
The types of sum and x are identical.

- Fortran 77 binding:

```
<type> FUNCTION BLAS_xSUM_X( N, X, INCX, PREC )
INTEGER INCX, N, PREC
<type> X( * )
```

The types of BLAS_xSUM_X and argument X are both specified by the prefix x.

- C binding:

```
void BLAS_xsum_x( int n, const ARRAY x, int incx, SCALAR_INOUT sum,
    enum blas_prec_type prec );
```

The types of arguments sum and x are both specified by the prefix x.

## 4.5.3 Mixed and Extended Precision Vector Operations

**AXPY** (Scaled vector accumulation)  
\[ y \leftarrow \alpha x + \beta y \]

The routine AXPBY scales the vector \( x \) by \( \alpha \) and the vector \( y \) by \( \beta \), adds these two vectors to one another and stores the result in the vector \( y \). If \( n \) is less than or equal to zero, or if \( \alpha \) is equal to zero and \( \beta \) is equal to one, this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler.

Extended and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```
SUBROUTINE axpy( x, y [, alpha [, beta [, prec ] ] ] )
    <type>(<wp>), INTENT(IN) :: x(:)
    <type>(<wp>), INTENT(INOUT) :: y(:)
    <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
    TYPE(blas_prec_type), INTENT(IN), OPTIONAL :: prec
where
    x and y have shape (n)
```

The default value for \( \beta \) is 1.0 and (1.0,0.0).

The types of \( x, y, \) alpha, and beta are governed according to the rules of mixed precision arguments set down in section 4.3: the type of \( x \) can optionally differ from that of alpha, beta and \( y \).

- Fortran 77 binding:
SUBROUTINE BLAS_xAXPBY{_a}{_X}( N, ALPHA, X, INCX, BETA, Y, INCY [ , PREC ] )

INTEGER INCX, INCY, N [ , PREC ]
<type> ALPHA, BETA
<type> X( * )
<type> Y( * )

The types of ALPHA, X, Y, and BETA are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix X is the floating point type of the arguments, but if _a is present then _a is the type of X. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a and _X must be present.

• C binding:

void BLAS_xaxpby{_a}{_x}( int n, SCALAR_IN alpha, const ARRAY x, int incx, SCALAR_IN beta, ARRAY y, int incy, [ , enum blas_prec_type prec ] );

The types of alpha, x, y, and beta are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix X is the floating point type of the arguments, but if _a is present then _a is the type of argument x. The suffix _X is present if and only if prec is present. One or both of the suffixes _a and _X must be present.

WAXPBY (Scaled vector addition) \[ w \leftarrow \alpha x + \beta y \]

The routine WAXPBY scales the vector x by \( \alpha \) and the vector y by \( \beta \), adds these two vectors to one another and stores the result in the vector w. If n is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx or incy or incw less than zero is permitted. However, if either incx or incy or incw is equal to zero, an error flag is set and passed to the error handler.

Extended and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

• Fortran 95 binding:

SUBROUTINE waxpby( x, y, w [, alpha] [, beta] [, prec] )
<type>(<wp>), INTENT (IN) :: x(:)
<type>(<wp>), INTENT (IN) :: y(:)
<type>(<wp>), INTENT (OUT) :: w(:)
<type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec

where
x, y and w have shape (n)

The default value for \( \beta \) is 1.0 and (1.0,0,0).

The types of x, y, w, alpha and beta are governed according to the rules of mixed precision arguments set down in section 4.3: the types of x and y can optionally differ from that of w, alpha and beta.
4.5. LANGUAGE BINDINGS

- Fortran 77 binding:

```
SUBROUTINE BLAS_XWAPBY(_a_b){_x}(N, ALPHA, X, INCX, BETA, Y, INCY, W, INCW [, PREC])
INTEGER INCW, INCX, INCY, N [, PREC]
<type> ALPHA, BETA
<type> W(*)
<type> X(*)
<type> Y(*)
```

The types of $X$, $Y$, $W$, $ALPHA$ and $BETA$ are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix $x$ is the floating point type of the arguments, but if $_a_b$ is present then $a$ is the type of $X$ and $b$ is the type of $Y$. The suffix $_x$ is present if and only if $PREC$ is present. One or both of the suffixes $_a_b$ and $_x$ must be present.

- C binding:

```
void BLAS_XWAPBY(_a_b){_x}(int n, SCALAR_IN alpha, const ARRAY x, int incx,
SCALAR_IN beta, const ARRAY y, int incy, ARRAY w, int incw [, enum blas_prec_type prec]);
```

The types of $x$, $y$, $w$, $alpha$ and $beta$ are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix $x$ is the floating point type of the arguments, but if $_a_b$ is present then $a$ is the type of argument $x$ and $b$ is the type of argument $y$. The suffix $_x$ is present if and only if $prec$ is present. One or both of the suffixes $_a_b$ and $_x$ must be present.

#### 4.5.4 Mixed and Extended Precision Matrix-Vector Operations

{GE,GB}MV (Matrix vector product) $y \leftarrow \alpha A x + \beta y$, $y \leftarrow \alpha A^T x + \beta y$ or $y \leftarrow \alpha A^H x + \beta y$

The routines multiply a vector $x$ by a general (or general band) matrix $A$ or its transpose, or its conjugate transpose, scales the resulting vector and adds it to the scaled vector operand $y$. If $m$ or $n$ is less than or equal to zero or if $beta$ is equal to one and $alpha$ is equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler. For the routine GEMV, if lda is less than one or lda is less than $m$, an error flag is set and passed to the error handler. For the routine GBMV, if kl or ku is less than zero, or if lda is less than $k_l$ plus $k_u$ plus one, an error flag is set and passed to the error handler.

Extended and mixed precision are permitted. This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```
SUBROUTINE gbmv( a, m, kl, x, y [, trans] [, alpha] [, beta] [, prec] )
<type>(<wp>), INTENT(IN) :: a(:,,:), x(:)
```
INTEGER, INTENT(IN) :: m, kl
<type>(<wp>), INTENT(INOUT) :: y(:)
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
<type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
TYPE (blas_prec_type), INTENT(IN), OPTIONAL :: prec
where
if trans = blas_no_trans then
   x has shape (n)
   y has shape (m)
else if trans /= blas_no_trans then
   x has shape (m)
   y has shape (n)
end if

The functionality of gemv is covered by gemm.

- Fortran 77 binding:

  General:
  SUBROUTINE BLAS_xGEMV{a_b}{_x}( TRANS, M, N, ALPHA, A, LDA,
                        $X, INCX, BETA, Y, INCY [, PREC] )
  General Band:
  SUBROUTINE BLAS_xGBMV{a_b}{_x}( TRANS, M, N, KL, KU, ALPHA, A,
                        $LDA, X, INCX, BETA, Y, INCY [, PREC] )
  all:
  INTEGER               INCX, INCY, KL, KU, LDA, M, N, [PREC], TRANS
  <type>                ALPHA, BETA
  <type>                A(LDA, *)
  <type>                X(*)
  <type>                Y(*)

  The types of ALPHA, A, X, Y, and BETA are governed according to the rules of mixed precision
  arguments set down in section 4.3. The prefix x is the floating point type of the arguments,
  but if a b is present then a is the type of A and b is the type of X. The suffix _x is present
  if and only if PREC is present. One or both of the suffixes a b and x must be present.

- C binding:

  General:
  void BLAS_xgemv{a_b}{_x}( enum blas_order_type order,
                           enum blas_trans_type trans, int m, int n,
                           SCALAR_IN alpha, const ARRAY a, int lda,
                           const ARRAY x, int incx, SCALAR_IN beta, ARRAY y,
                           int incy [, enum blas_prec_type prec] );

  General Band:
  void BLAS_xgbmv{a_b}{_x}( enum blas_order_type order,
                           enum blas_trans_type trans, int m, int n,
                           int kl, int ku, SCALAR_IN alpha,
                           const ARRAY a, int lda, const ARRAY x, int incx,
4.5. LANGUAGE BINDINGS

SCALAR_IN beta, ARRAY y, int incy
[, enum blas_prec_type prec ]);

The types of alpha, a, x, y and beta are governed according to the rules of mixed precision
arguments set down in section 4.3. The prefix x is the floating point type of the arguments,
but if _a_b is present then _a is the type of argument a and _b is the type of argument x. The
suffix _x is present if and only if prec is present. One or both of the suffixes _a_b and _x must
be present.

{SY,SB,SP}MV (Symmetric matrix vector multiply) y ← αAx + βy with A = A^T

The routines multiply a vector x by a real or complex symmetric matrix A, scales the resulting
vector and adds it to the scaled vector operand y. If n is less than or equal to zero or if beta is
equal to one and alpha is equal to zero, this routine returns immediately. As described in section
2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to
zero, an error flag is set and passed to the error handler. For the routine SYMV, if lda is less than
one or lda is less than n, an error flag is set and passed to the error handler. For the routine SBMV,
if lda is less than k plus one, an error flag is set and passed to the error handler.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and
mixed precision are permitted.

- Fortran 95 binding:

  Symmetric Band:
  SUBROUTINE sbmv( a, x, y [, uplo [, alpha [, beta [, prec ] ] ] ] )

  Symmetric Packed:
  SUBROUTINE spmv( ap, x, y [, uplo [, alpha [, beta [, prec ] ] ] ] )
  <type>(<wp>), INTENT(IN) :: <aa>
  <type>(<wp>), INTENT(IN) :: x(:)
  <type>(<wp>), INTENT(INOUT) :: y(:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_prec_type), INTENT(IN), OPTIONAL :: alpha, beta
  where
  <aa> ::= a(:,:) or ap(:)
  and
  SB a has shape (k+1,n)
  SP ap has shape (n*(n+1)/2)
  x and y have shape (n)

  The types of alpha, a or ap, x, beta, and y are governed by the rules of mixed precision
  arguments set down in section 4.3: the types of a or ap and x can optionally differ from that
  of y, alpha and beta.

  The functionality of symv is covered by symm.

- Fortran 77 binding:
Symmetric:

SUBROUTINE BLAS-xSYMV(_a_b){_x}( UPLO, N, ALPHA, A, LDA, X, INCX,
    $                    BETA, Y, INCY [, PREC] )

Symmetric Band:

SUBROUTINE BLAS-xSBMV(_a_b){_x}( UPLO, N, K, ALPHA, A, LDA, X, INCX,
    $                    BETA, Y, INCY [, PREC] )

Symmetric Packed:

SUBROUTINE BLAS-xSPMV(_a_b){_x}( UPLO, N, ALPHA, AP, X, INCX, BETA,
    $                    Y, INCY [, PREC] )

all:

INTEGER       INCX, INCY, K, LDA, N, UPLO [, PREC]
<type>         ALPHA, BETA
<type>         A( LDA, * ) or AP( * )
<type>         X( * )
<type>         Y( * )

The types of ALPHA, A or AP, X, Y and BETA are governed according to the rules of mixed
precision arguments set down in section 4.3. The prefix x is the floating point type of the
arguments, but if _a_b is present then _a is the type of A or AP, and _b is the type of X. The
suffix _X is present if and only if PREC is present. One or both of the suffixes _a_b and _X must
be present.

• C binding:

Symmetric:

void BLAS-xsymv(_a_b){_x}( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, SCALAR_IN alpha, const ARRAY a, int lda,
    const ARRAY x, int incx, SCALAR_IN beta, ARRAY y,
    int incy [, enum blas_prec_type prec] );

Symmetric Band:

void BLAS-xsbmv(_a_b){_x}( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, SCALAR_IN alpha, const ARRAY a,
    int lda, const ARRAY x, int incx, SCALAR_IN beta,
    ARRAY y, int incy [, enum blas_prec_type prec] );

Symmetric Packed:

void BLAS-xspmv(_a_b){_x}( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, SCALAR_IN alpha, const ARRAY ap,
    const ARRAY x, int incx, SCALAR_IN beta, ARRAY y,
    int incy [, enum blas_prec_type prec] );

The types of alpha, a or ap, x, y, and beta are governed according to the rules of mixed
precision arguments set down in section 4.3. The prefix x is the floating point type of the
arguments, but if _a_b is present then _a is the type of argument a or ap and _b is the type of
argument x. The suffix _X is present if and only if prec is present. One or both of the suffixes
_a_b and _X must be present.

{HE,HB,HP}MV (Hermitian matrix vector product)  \[ y \leftarrow \alpha Ax + \beta y \] with \( A = A^H \)
The routines multiply a vector \( x \) by a Hermitian matrix \( A \), scales the resulting vector and adds it to the scaled vector operand \( y \). If \( n \) is less than or equal to zero or if beta is equal to one and alpha is equal to zero, this routine returns immediately. The imaginary part of the diagonal entries of the matrix operand are supposed to be zero and should not be referenced. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler. For the routine HEMV, if lda is less than one or lda is less than \( n \), an error flag is set and passed to the error handler. For the routine HBMV, if lda is less than \( k + 1 \) plus one, an error flag is set and passed to the error handler.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

  Hermitian Band:
  
  ```fortran
  SUBROUTINE hbmv{a}_{-a}( a, x, y [, uplo] [, alpha] [, beta] [, prec] )
  ```

  Hermitian Packed:
  
  ```fortran
  SUBROUTINE hpmv{a}_{-a}( ap, x, y [, uplo] [, alpha] [, beta] [, prec] )
  ```

  ```fortran
  COMPLEX(<wp>), INTENT(IN) :: <aa>
  COMPLEX(<wp>), INTENT(IN) :: x(:)
  COMPLEX(<wp>), INTENT(INOUT) :: y(:)
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  TYPE(blas_prec_type), INTENT(IN), OPTIONAL :: prec
  where
  <aa> ::= a(:, :) or ap(:)
  and
  HB  a has shape \((k+1,n)\)
  HP  ap has shape \((n*(n+1)/2)\)
  \( x \) and \( y \) have shape \((n)\)

  The types of alpha, a or ap, x, beta, and y are governed by the rules of mixed precision arguments set down in section 4.3: the types of a or ap and x can optionally differ from that of y, alpha and beta.

  The functionality of hmv is covered by hemm.

- Fortran 77 binding:

  Hermitian:
  
  ```fortran
  SUBROUTINE BLAS_xHEMV{a}_{-a}{-x}( UPLO, N, ALPHA, A, LDA, X, INCX,
  $                             BETA, Y, INCY [, PREC] )
  ```

  Hermitian Band:
  
  ```fortran
  SUBROUTINE BLAS_xHEMV{a}_{-a}{-x}( UPLO, N, K, ALPHA, A, LDA, X, INCX,
  $                             BETA, Y, INCY [, PREC] )
  ```

  Hermitian Packed:
  
  ```fortran
  SUBROUTINE BLAS_xHPMV{a}_{-a}{-x}( UPLO, N, ALPHA, AP, X, INCX,
  $                             BETA, Y, INCY [, PREC] )
  ```

  all:
The types of ALPHA, A or AP, X, Y, and BETA are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \( x \) is the floating point type of the arguments, but if \( .a_b \) is present then \( .a \) is the type of A or AP and \( .b \) is the type of X. The suffix \( .X \) is present if and only if \( \text{PREC} \) is present. One or both of the suffixes \( .a_b \) and \( .X \) must be present.

- C binding:

Hermitian:

```c
void BLAS_xhemv{_a_b}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
                            int n, CSCALAR_IN alpha, const CARRAY a, int lda,
                            const CARRAY x, int incx, CSCALAR_IN beta, CARRAY y,
                            int incy [ , enum blas_prec_type prec ]);
```

Hermitian Band:

```c
void BLAS_xhbmv{_a_b}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
                            int n, int k, CSCALAR_IN alpha, const CARRAY a,
                            int lda, const CARRAY x, int incx, CSCALAR_IN beta,
                            CARRAY y, int incy [ , enum blas_prec_type prec ]);
```

Hermitian Packed:

```c
void BLAS_xhpmv{_a_b}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
                            int n, CSCALAR_IN alpha, const CARRAY ap,
                            const CARRAY x, int incx, CSCALAR_IN beta, CARRAY y,
                            int incy [ , enum blas_prec_type prec ]);
```

The types of alpha, a or ap, x, y, and beta are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \( x \) is the floating point type of the arguments, but if \( .a_b \) is present then \( .a \) is the type of argument a or ap and \( .b \) is the type of argument x. The suffix \( .X \) is present if and only if \( \text{PREC} \) is present. One or both of the suffixes \( .a_b \) and \( .X \) must be present.

\{TR,TB,TP\}MV (Triangular matrix vector product) \( x \leftarrow c^T x \), \( x \leftarrow c^T x \) or \( x \leftarrow c^T x \)

The routines multiply a vector \( x \) by a general triangular matrix \( T \) or its transpose, or its conjugate transpose, and copies the resulting vector in the vector operand \( x \). If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to one, an error flag is set and passed to the error handler. For the routine TRMV, if ldt is less than one or ldt is less than n, an error flag is set and passed to the error handler. For the routine TBMV, if ldt is less than k plus one, an error flag is set and passed to the error handler.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.
• Fortran 95 binding:

Triangular Band:

```fortran
```

Triangular Packed:

```fortran
```

```fortran
<type>(<wp>), INTENT(IN) :: <tt>
<type>(<wp>), INTENT(INOUT) :: x(:)
<type>(<wp>), INTENT(IN), OPTIONAL :: alpha
```

```fortran
TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
```

```fortran
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
```

```fortran
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
```

```fortran
TYPE (blas_prec_type), INTENT(IN), OPTIONAL :: prec
```

where

```fortran
<tt> :: t(:,:) or tp(:)
```

and

```fortran
x has shape (n)
```

```fortran
TB  t has shape (k+1,n)
```

```fortran
TP  tp has shape (n*(n+1)/2)
```

(k=band width)

The types of alpha, t or tp, and x are governed by the rules of mixed precision arguments set down in section 4.3: the type of t or tp can optionally differ from that of x and alpha.

The functionality of trmv is covered by trmm.

• Fortran 77 binding:

Triangular:

```fortran
SUBROUTINE BLAS_xTRMV{a}{_X}( UPL0, TRANS, DIAG, N, ALPHA, T, LDT, X,
$   INCX [, PREC] )
```

Triangular Band:

```fortran
SUBROUTINE BLAS_xTBMV{a}{_X}( UPL0, TRANS, DIAG, N, K, ALPHA, T, LDT,
$   X, INCX [, PREC] )
```

Triangular Packed:

```fortran
SUBROUTINE BLAS_xTPMV{a}{_X}( UPL0, TRANS, DIAG, N, ALPHA, TP, X, INCX
$   [, PREC] )
```

all:

```fortran
INTEGER     DIAG, INCX, K, LDT, N, TRANS, UPL0 [, PREC]
```

```fortran
<TYPE>      ALPHA
```

```fortran
<TYPE>      T( LDT, * ) or TP( * )
```

```fortran
<TYPE>      X( * )
```

The types of ALPHA, T or TP, and X are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of T or TP. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a and _X must be present.

• C binding:
Triangular:
void BLAS_xtrmv{_a}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, SCALAR_IN alpha, const ARRAY t, int ldt,
ARRAY x, int incx [, enum blas_prec_type prec ]);

Triangular Band:
void BLAS_xtbmv{_a}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, int k, SCALAR_IN alpha, const ARRAY t, int ldt,
ARRAY x, int incx [, enum blas_prec_type prec ]);

Triangular Packed:
void BLAS_xtpmv{_a}{_x}( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, SCALAR_IN alpha, const ARRAY tp,
ARRAY x, int incx [, enum blas_prec_type prec ]);

The types of alpha, t or tp, and x are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of argument t or tp. The suffix _x is present if and only if prec is present. One or both of the suffixes _a and _x must be present.

GESUM_MV (Summed matrix vector multiplies)  \( y \leftarrow \alpha A x + \beta B x \)

This routine adds the product of two scaled matrix vector products. It can be used to compute the residual of an approximate eigenvector and eigenvalue of the generalized eigenvalue problem \( A x = \lambda B x \). If m or n is less than or equal to zero or if beta is equal to one and alpha is equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if incx or incy is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than m, or ldb is less than one or ldb is less than m, an error flag is set and passed to the error handler.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

  SUBROUTINE ge_sum_mv( a, x, b, y [, alpha] [, beta] [, prec])
  \(<\text{type}(\langle\text{wp}\rangle), \text{INTENT (IN)} :: a(:,:), b(:,:))\)
  \(<\text{type}(\langle\text{wp}\rangle), \text{INTENT (IN)} :: x(:))\)
  \(<\text{type}(\langle\text{wp}\rangle), \text{INTENT (OUT)} :: y(:))\)
  \(<\text{type}(\langle\text{wp}\rangle), \text{INTENT (IN), OPTIONAL :: alpha, beta}\>
  \(<\text{type}(\langle\text{bl} as\_\text{prec}\_\text{type}\rangle), \text{INTENT (IN), OPTIONAL :: prec}\>

  where
  x has shape (n);
  y has shape (m);
  a and b have shape (m,n) for general matrices
The types of alpha, a, x, beta, b, and y are governed according to the rules of mixed precision arguments set down in section 4.3: the types of a and b can optionally differ from that of x, y, alpha and beta. Arguments a and b must have the same type.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xGE_SUM_MV{a_b}_{x}( M, N, ALPHA, A, LDA, X, INCX,
  $             BETA, B, LDB, Y, INCY
  $                     [, PREC])
INTEGER        INCX, INCY, LDA, LDB, M, N [, , PREC]
<type>         ALPHA, BETA
<type>         A( LDA, * ), B( LDB, * )
<type>         X( * )
<type>         Y( * )
```

The types of alpha, a, x, beta, b, and y are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of a and B, and _b is the type of x. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a_b and _x must be present.

- C binding:

```c
void BLAS_xge_sum_mv{a_b}_{x}( enum blas_order_type order, int m, int n,
  SCALAR_IN alpha, const ARRAY a, int lda,
  const ARRAY x, int incx, SCALAR_IN beta,
  const ARRAY B, int ldb, ARRAY y, intincy
  [, enum blas_prec_type prec] );
```

The types of alpha, a, x, beta, b, and y are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a_b is present then _a is the type of a and b, and _b is the type of x. The suffix _x is present if and only if prec is present. One or both of the suffixes _a_b and _x must be present.

---

{TR,TB,TP}SV (Triangular solve)

\[ x \leftarrow \alpha T^{-1} x, \quad y \leftarrow \alpha T^{-T} x \]

These functions solve one of the systems of equations \( x \leftarrow \alpha T^{-1} x \) or \( y \leftarrow \alpha T^{-T} x \), where \( x \) and \( y \) are vectors and the matrix \( T \) is a unit, non-unit, upper or lower triangular (or triangular banded or triangular packed) matrix. If \( n \) is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler. If ltd is less than one or ltd is less than \( n \), an error flag is set and passed to the error handler.

Extended precision and mixed precision are permitted.

Advice to implementors. Note that no check for singularity, or near singularity is specified for these triangular equation-solving functions. The requirements for such a test depend on the application, and so we felt that this should not be included, but should instead be performed before calling the triangular solver.

To implement this function when the internal precision requested is higher than the precision of \( x \), temporary workspace is needed to compute and store \( x \) internally to higher precision.

(End of advice to implementors.)
This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

  Triangular Band:
  
  `[SUBROUTINE tbsv( t, x [, uplo] [, transt] [, diag] [, alpha] [, prec] )]

  Triangular Packed:
  
  <type>(<wp>), INTENT(IN) :: <tt>
  <type>(<wp>), INTENT(INOUT) :: x(:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha
  TYPE (blas_prec_type), INTENT(IN), OPTIONAL :: prec

  where
  
  `<tt> ::= t(:,:), or tp(:)

  and

  x has shape (n)
  TB t has shape (k+1,n)
  TP tp has shape (n*(n+1)/2)
  (k=band width)

  The types of alpha, t or tp, and x are governed by the rules of mixed precision arguments set down in section 4.3; the type of t or tp can optionally differ from that of x and alpha.

  The functionality of trsv is covered by trsm.

- Fortran 77 binding:

  Triangular:
  
  `[SUBROUTINE BLAS_xTRSV{a}{x}( uplo, trans, diag, n, alpha, t, ldt, ]
  $  x, incx [, prec] )]

  Triangular Band:
  
  `[SUBROUTINE BLAS_xTBSV{a}{x}( uplo, trans, diag, n, k, alpha, t, ]
  $  ldt, x, incx [, prec] )]

  Triangular Packed:
  
  `[SUBROUTINE BLAS_xTPSV{a}{x}( uplo, trans, diag, n, alpha, tp, x, ]
  $  incx [, prec] )]

  all:
  
  INTEGER DIAG, INCX, K, LDT, N, TRANS, UPLO [, PREC]
  <type> ALPHA
  <type> T( LDT, * ) or TP( * )
  <type> X( * )

  The types of ALPHA, T or TP, and X are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of T or TP. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a and _X must be present.
• C binding:

    Triangular:
    void BLAS_xtrsv{<a>}{<x>}( enum blas_order_type order, enum blas_uplo_type uplo,
                      enum blas_trans_type trans, enum blas_diag_type diag,
                      int n, SCALAR_IN alpha, const ARRAY t, int ldt,
                      ARRAY x, int incx [, enum blas_prec_type prec] );

    Triangular Band:
    void BLAS_xtbsv{<a>}{<x>}( enum blas_order_type order, enum blas_uplo_type uplo,
                      enum blas_trans_type trans, enum blas_diag_type diag,
                      int n, int k, SCALAR_IN alpha, const ARRAY t, int ldt,
                      ARRAY x, int incx [, enum blas_prec_type prec] );

    Triangular Packed:
    void BLAS_xtpsv{<a>}{<x>}( enum blas_order_type order, enum blas_uplo_type uplo,
                      enum blas_trans_type trans, enum blas_diag_type diag,
                      int n, SCALAR_IN alpha, const ARRAY tp, ARRAY x,
                      int incx [, enum blas_prec_type prec] );

The types of alpha, t or tp, and x are governed according to the rules of mixed precision
arguments set down in section 4.3. The prefix x is the floating point type of the arguments,
but if _a is present then _a is the type of argument t or tp. The suffix _x is present if and
only if prec is present. One or both of the suffixes _a and _x must be present.

4.5.5 Mixed and Extended Precision Matrix-Matrix Operations

In the following section, \(a_\cdot(X)\) denotes \(X\), or \(X^T\) or \(X^H\) where \(X\) is a matrix.

GEMM (General Matrix Matrix Product) \[ C \leftarrow \alpha a_\cdot(A) a_\cdot(B) + \beta C \]

The routine performs a general matrix matrix multiply \(C \leftarrow \alpha a_\cdot(A) a_\cdot(B) + \beta C\) where \(\alpha\) and
\(\beta\) are scalars, and \(A\), \(B\), and \(C\) are general matrices. This routine returns immediately if \(m\) or \(n\) or
\(k\) is less than or equal to zero. If \(lda\) is less than one or less than \(m\), or if \(ldb\) is less than one or less
than \(k\), or if \(ldc\) is less than one or less than \(m\), an error flag is set and passed to the error handler.

This interface encompasses the Legacy BLAS routine xGEMM.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and
mixed precision are permitted.

• Fortran 95 binding:

    SUBROUTINE gemm( a, b, c [, transa] [, , transb] [, , alpha] [, , beta] &
                      [ , , prec] )
                      <type>(<wp>), INTENT(IN) :: <aa>
                      <type>(<wp>), INTENT(IN) :: <bb>
                      <type>(<wp>), INTENT(INOUT) :: <cc>
                      TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transa, transb
                      <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
<aa> ::= a(:,::) or a(:)
<bb> ::= b(:,::) or b(:)
<cc> ::= c(:,::) or c(:)
and
c, rank 2, has shape (m,n)
a has shape (m,k) if transa = blas_no_trans (the default)
   (k,m) if transa /= blas_no_trans
   (m) if rank 1
b has shape (k,n) if transb = blas_no_trans (the default)
   (n,k) if transb /= blas_no_trans
   (n) if rank 1
c, rank 1, has shape (m)
a has shape (m,n) if transa = blas_no_trans (the default)
   (n,m) if transa /= blas_no_trans
b has shape (n)

<table>
<thead>
<tr>
<th>Rank a</th>
<th>Rank b</th>
<th>Rank c</th>
<th>transa</th>
<th>transb</th>
<th>Operation</th>
<th>Arguments</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>N</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>N</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>N</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>T</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>H</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>H</td>
<td>H</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>N</td>
<td>-</td>
<td>c ← αAb + βc</td>
<td>real or complex</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>T</td>
<td>-</td>
<td>c ← αATb + βc</td>
<td>real or complex</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>H</td>
<td>-</td>
<td>c ← αAHb + βc</td>
<td>complex</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>C ← αabT + βc</td>
<td>real or complex</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-</td>
<td>H</td>
<td>C ← αabH + βc</td>
<td>complex</td>
</tr>
</tbody>
</table>

The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The functionality of xGEMV is also covered by this generic procedure.
The types of a, b, c, alpha and beta are governed according to the rules of mixed precision arguments set down in section 4.3: the types of a and b can optionally differ from that of c, alpha and beta.

• Fortran 77 binding:

General:

SUBROUTINE BLAS_xGEMM{...}_x{...}( TRANSA, TRANSB, M, N, K, ALPHA, A, LDA, $ 
   B, LDB, BETA, C, ldc [, prec] )  
INTEGER K, LDA, LDB, ldc, M, N, TRANSA, TRANSB [, prec]
<type>   ALPHA, BETA
<type>   A ( LDA, * )
4.5. LANGUAGE BINDINGS

The types of ALPHA, A, B, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \( x \) is the floating point type of the arguments, but if \( .a .b \) is present then \( .a \) is the type of A and \( .b \) is the type of B. The suffix \( .x \) is present if and only if PREC is present. One or both of the suffixes \( .a .b \) and \( .x \) must be present.

- C binding:

```c
void BLAS_xgemm(_a_, _b_){_x_}( enum blas_order_type order, 
enum blas_trans_type transa, 
enum blas_trans_type transb, int m, int n, int k, 
SCALAR_IN alpha, const ARRAY a, int lda, 
const ARRAY b, int ldb, 
SCALAR_IN beta, ARRAY c, int ldc 
[, enum blas_prec_type prec] );
```

The types of \( \alpha, a, b, \beta \) and \( c \) are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \( x \) is the floating point type of the arguments, but if \( .a .b \) is present then \( .a \) is the type of argument \( a \) and \( .b \) is the type of argument \( b \). The suffix \( .x \) is present if and only if \( \text{prec} \) is present. One or both of the suffixes \( .a .b \) and \( .x \) must be present.

SYMM (Symmetric Matrix Matrix Product) \( C \gets \alpha AB + \beta C \) or \( C \gets \alpha BA + \beta C \)

This routine performs one of the symmetric matrix matrix operations \( \alpha AB + \beta C \) or \( \alpha BA + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( A \) is a symmetric matrix, and \( B \) and \( C \) are general matrices. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For side equal to \text{blas_left_side}, and if \( \text{lda} \) is less than one or less than \( m \), or if \( \text{ldb} \) is less than one or less than \( m \), or if \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For side equal to \text{blas_right_side}, and if \( \text{lda} \) is less than one or less than \( m \), or if \( \text{ldb} \) is less than one or less than \( n \), or if \( \text{ldc} \) is less than one or less than \( n \), an error flag is set and passed to the error handler.

The interfaces encompass the Legacy BLAS routine \text{xSYM}M with added functionality for complex symmetric matrices.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```fortran
  <type>(<wp>), INTENT(IN) :: a(:, :)
  <type>(<wp>), INTENT(IN) :: b(:)
  <type>(<wp>), INTENT(INOUT) :: c:
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
```
TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
<bb> ::= b(,:) or b(:)
<cc> ::= c(,:) or c(:)
and
c, rank 2, has shape (m,n), b same shape as c
SY  a has shape (m,m) if side = blas_left_side (the default)
    a has shape (n,n) if side /= blas_left_side
C, rank 1, has shape (m), b same shape as c
SY  a has shape (m,m)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>Rank c</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>L</td>
<td>C ← αAB + βC</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>R</td>
<td>C ← αBA + βC</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td>c ← αAb + βc</td>
</tr>
</tbody>
</table>

The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The functionality of xSYMV is covered by symm.

The types of a, b, c, alpha and beta are governed according to the rules of mixed precision arguments set down in section 4.3: the types of a and b can optionally differ from that of c, alpha and beta.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYMV{_a_b}{_X}( SIDE, UPLO, M, N, ALPHA, A, LDA,
$                 B, LDB, BETA, C, LDC [, PREC] )
INTEGER LDA, LDB, LDC, M, N, SIDE, UPLO [, PREC]
[type> ALPHA, BETA
[type> A( LDA, * )
[type> B( LDB, * )
[type> C( LDC, * )
```

The types of ALPHA, A, B, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a_b is present then _a is the type of A and _b is the type of B. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a_b and _X must be present.

- C binding:

```c
void BLAS_xsymm{_a_b}{_X}( enum blas_order_type order,
        enum blas_side_type side,
        enum blas_uplo_type uplo, int m, int n,
        SCALAR_IN alpha, const ARRAY a, int lda,
        const ARRAY b, int ldb, SCALAR_IN beta, ARRAY c,
        int ldc [, enum blas_prec_type prec] );
```
4.5. LANGUAGE BINDINGS

The types of alpha, a, b, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if .a.b is present then .a is the type of argument a, and .b is the type of argument b. The suffix x is present if and only if prec is present. One or both of the suffixes .a.b and x must be present.

HEMM (Hermitian Matrix Matrix Product) 

\[ C \leftarrow \alpha AB + \beta C \] or \[ C \leftarrow \alpha BA + \beta C \]

This routine performs one of the Hermitian matrix matrix operations \( C \leftarrow \alpha AB + \beta C \) or \( C \leftarrow \alpha BA + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( A \) is a Hermitian matrix, and \( B \) and \( C \) are general matrices. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For side equal to blas_left_side, and if \( lda \) is less than one or less than \( m \), or if \( ldb \) is less than one or less than \( m \), or if \( ldc \) is less than one or less than \( m \), an error flag is set and passed to the error handler. For side equal to blas_right_side, and if \( lda \) is less than one or less than \( n \), or if \( ldb \) is less than one or less than \( n \), or if \( ldc \) is less than one or less than \( n \), an error flag is set and passed to the error handler.

The interfaces encompass the Legacy BLAS routine xHEMM.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

  Hermitian:

  ```fortran
  SUBROUTINE hemm( a, b, c [, side] [, uplo] [, alpha] [, beta] [, prec] )
  COMPLEX(<wp>), INTENT(IN) :: a(:,:)
  COMPLEX(<wp>), INTENT(IN) :: <bb>
  COMPLEX(<wp>), INTENT(INOUT) :: <cc>
  TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
  where
  <bb> ::= b(:,:) or b(:)
  <cc> ::= c(:,:) or c(:)
  and
  c, rank 2, has shape (m,n), b same shape as c
  HE a has shape (m,m) if "side" = blas_left_side (the default)
  a has shape (n,n) if "side" /= blas_left_side
  c, rank 1, has shape (m), b same shape as c
  HE a has shape (m,m)
<table>
<thead>
<tr>
<th>Rank b</th>
<th>Rank c</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>L</td>
<td>( C \leftarrow \alpha AB + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>R</td>
<td>( C \leftarrow \alpha BA + \beta C )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>( c \leftarrow \alpha Ab + \beta c )</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
  The table defining the operation as a function of the operator arguments is identical to Chapter 2.
The functionality of xHEMV is covered by hemm.

The types of a, b, c, alpha and beta are governed according to the rules of mixed precision arguments set down in section 4.3: the types of a and b can optionally differ from that of c, alpha and beta.

- Fortran 77 binding:

```
SUBROUTINE BLAS_xHEMV{_a_b}{_X}( SIDE, UPLO, M, N, ALPHA, A, LDA,
   $                       B, LDB, BETA, C, LDC [, PREC] )
INTEGER            LDA, LDB, LDC, M, N, SIDE, UPLO [, PREC]
<ctype>        ALPHA, BETA
<ctype>        A( LDA, * )
<ctype>        B( LDB, * )
<ctype>        C( LDC, * )
```

The types of ALPHA, A, B, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix _x is the floating point type of the arguments, but if _a_b is present then _a is the type of A and _b is the type of B. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a_b and _X must be present.

- C binding:

```
void BLAS_xhemm{_a_b}{_X}( enum blas_order_type order,
                      enum blas_side_type side,
                      enum blas_uplo_type uplo, int m, int n,
                      CScalers_IN alpha, const CARRAY a, int lda,
                      const CARRAY b, int ldb, CScalers_IN beta, CARRAY c,
                      int ldc [, enum blas_prec_type prec] );
```

The types of alpha, a, b, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix _x is the floating point type of the arguments, but if _a_b is present then _a is the type of argument a, and _b is the type of argument b. The suffix _X is present if and only if prec is present. One or both of the suffixes _a_b and _X must be present.

TRMM (Triangular Matrix Matrix Multiply)  \[ B \leftarrow \alpha op(T) B \text{ or } B \leftarrow \alpha Bop(T) \]

These routines perform one of the matrix-matrix operations \( B \leftarrow \alpha op(T) B \text{ or } B \leftarrow \alpha Bop(T) \) where \( \alpha \) is a scalar, \( B \) is a general matrix, and \( T \) is a unit, or non-unit, upper or lower triangular matrix. This routine returns immediately if m or n is less than or equal to zero. For side equal to blas_left_side, and if ldt is less than one or less than m, or if ldb is less than one or less than m, an error flag is set and passed to the error handler. For side equal to blas_right_side, and if ldt is less than one or less than n, or if ldb is less than one or less than m, an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xTRMM.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.
4.5. LANGUAGE BINDINGS

- Fortran 95 binding:

```
SUBROUTINE trmm( t, b [, side] [, uplo] [, transt] [, diag] &
    [, alpha] [, prec] )
    <type>(<wp>), INTENT(IN) :: t(:,:)
    <type>(<wp>), INTENT(INOUT) :: <bb>
    <type>(<wp>), INTENT(IN), OPTIONAL :: alpha
TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
    <bb> :: b(:,,:) or b(:)
and
    b, rank 2, has shape (m,n)
    TR t has shape (m,m) if side = blas_left_side (the default)
    t has shape (n,n) if side /= blas_left_side
    b, rank 1, has shape (m)
    TR t has shape (m,m)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>transa</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>L</td>
<td>B ← αTB</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>L</td>
<td>B ← αT'B</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>L</td>
<td>B ← αTHB</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>R</td>
<td>B ← αBT</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>R</td>
<td>B ← αBT'</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>R</td>
<td>B ← αBTH</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>-</td>
<td>b ← αTb</td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>-</td>
<td>b ← αT'b</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>-</td>
<td>b ← αTHb</td>
</tr>
</tbody>
</table>
```

The table defining the operation as a function of the operator arguments is identical to
Chapter 2.

The functionality of xTRMV is covered by trmm.

The types of alpha, t, and b are governed according to the rules of mixed precision arguments
set down in section 4.3: the type of t can optionally differ from that of b and alpha.

- Fortran 77 binding:

```
SUBROUTINE BLAS_XTRMM(a){X}( SIDE, UPLO, TRANST, DIAG, M, N,
    $                                    ALPHA, T, LDT, B, LDB [, PREC] )
    INTEGER                                DIAG, LDT, LDB, M, N, SIDE, TRANST, UPLO
    $                                    [, PREC]
    <type>                                 ALPHA
    <type>                                 T( LDT, * )
    <type>                                 B( LDB, * )
```
The types of \texttt{ALPHA}, \texttt{T}, and \texttt{B} are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \texttt{x} is the floating point type of the arguments, but if \texttt{a} is present then \texttt{a} is the type of \texttt{T}. The suffix \texttt{X} is present if and only if \texttt{PREC} is present. One or both of the suffixes \texttt{a} and \texttt{X} must be present.

- C binding:

```c
void BLAS_xtrmm{\texttt{a}}{\texttt{X}}(enum blas_order_type order, enum blas_side_type side,
enum blas_uplo_type uplo, enum blas_trans_type transa,
enum blas_diag_type diag, int m, int n,
SCALAR_IN alpha, const ARRAY t, int ldb, ARRAY b,
int ldb [, enum blas_prec_type prec] );
```

The types of \texttt{alpha}, \texttt{t}, and \texttt{b} are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \texttt{x} is the floating point type of the arguments, but if \texttt{a} is present then \texttt{a} is the type of argument \texttt{t}. The suffix \texttt{x} is present if and only if \texttt{prec} is present. One or both of the suffixes \texttt{a} and \texttt{x} must be present.

---

**TRSM (Triangular Solve)**

\[ B \leftarrow aop(T^{-1})B \text{ or } B \leftarrow \alpha B op(T^{-1}) \]

This routine solves one of the matrix equations \( B \leftarrow aop(T^{-1})B \) or \( B \leftarrow \alpha B op(T^{-1}) \) where \( \alpha \) is a scalar, \( B \) is a general matrix, and \( T \) is a unit, or non-unit, upper or lower triangular matrix. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For \texttt{side} equal to \texttt{blas.left_side}, and if \texttt{ldt} is less than one or less than \( m \), or if \texttt{ldb} is less than one or less than \( m \), an error flag is set and passed to the error handler. For \texttt{side} equal to \texttt{blas.right_side}, and if \texttt{ldt} is less than one or less than \( n \), or if \texttt{ldb} is less than one or less than \( m \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine \texttt{xTRSM}.

Extended precision and mixed precision are permitted.

*Advice to implementors.* Note that no check for singularity, or near singularity is specified for these triangular equation-solving functions. The requirements for such a test depend on the application, and so we felt that this should not be included, but should instead be performed before calling the triangular solver.

To implement this function when the internal precision requested is higher than the precision of \( B \), temporary workspace is needed to compute and store \( B \) internally to higher precision. *(End of advice to implementors.)*

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```fortran
SUBROUTINE trsm( t, b [, side] [, uplo] [, transt] [, diag] &
                  [, alpha] [, prec] )
  \texttt{<type>(<wp>)}, \texttt{INTENT(IN)} :: \texttt{t}(::,:)
  \texttt{<type>(<wp>)}, \texttt{INTENT(INOUT)} :: \texttt{<bb>}
  \texttt{TYPE (blas_side_type), INTENT(IN), OPTIONAL :: side}
```
4.5. LANGUAGE BINDINGS

```fortran
TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: transt
TYPE (blas_diag_type), INTENT(IN), OPTIONAL :: diag
<type>(<wp>), INTENT(IN), OPTIONAL :: alpha
TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec

where

<bb> ::= b(:,:) or b(:)

and

b, rank 2, has shape (m,n)
  TR t has shape (m,m) if side = blas_left_side (the default)
  t has shape (n,n) if side /= blas_left_side

b, rank 1, has shape (m)
  TR t has shape (m,m)

<table>
<thead>
<tr>
<th>Rank b</th>
<th>transa</th>
<th>side</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>L</td>
<td>B ← αT⁻¹B</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>L</td>
<td>B ← αT⁻¹B</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>L</td>
<td>B ← αT⁻¹B</td>
</tr>
<tr>
<td>2</td>
<td>N</td>
<td>R</td>
<td>B ← αBT⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>R</td>
<td>B ← αBT⁻¹</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>R</td>
<td>B ← αBT⁻¹</td>
</tr>
<tr>
<td>1</td>
<td>N</td>
<td>-</td>
<td>b ← αT⁻¹b</td>
</tr>
<tr>
<td>1</td>
<td>T</td>
<td>-</td>
<td>b ← αT⁻¹b</td>
</tr>
<tr>
<td>1</td>
<td>H</td>
<td>-</td>
<td>b ← αT⁻¹b</td>
</tr>
</tbody>
</table>
```

The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The functionality of xTRSV is covered by trsm.

The types of t, x and alpha are governed according to the rules of mixed precision arguments set down in section 4.3: the type of t can optionally differ from that of x and alpha.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xTRSM{a}{x}( SIDE, UPLO, TRANS, DIAG, M, N,
  $                      ALPHA, T, LDT, B, LDB [, PREC] )
  INTEGER               DIAG, LDT, LDB, M, N, SIDE, TRANS, UPLO
  $                  [, PREC]
  <type>               ALPHA
  <type>               T( LDT, * )
  <type>               B( LDB, * )
```

The types of ALPHA, T, and B are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if .a is present then .a is the type of T. The suffix .X is present if and only if PREC is present. One or both of the suffixes .a and .X must be present.

- C binding:
void BLAS_xtrsm(_a){x}( enum blas_order_type order, enum blas_side_type side,
  enum blas_uplo_type uplo, enum blas_trans_type transt,
  enum blas_diag_type diag, int m, int n,
  SCALAR_IN alpha, const ARRAY t, int ldt, ARRAY b,
  int ldb [, enum blas_prec_type prec] );

The types of alpha, t, and b are governed according to the rules of mixed precision arguments
set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a
is present then _a is the type of argument t. The suffix _x is present if and only if prec is
present. One or both of the suffixes _a and _x must be present.

SYRK (Symmetric Rank K update) \( C \leftarrow \alpha A^T + \beta C, \quad C \leftarrow \alpha A^T A + \beta C \)

This routine performs one of the symmetric rank k operations \( C \leftarrow \alpha A^T + \beta C \) or \( C \leftarrow \alpha A^T A + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a symmetric matrix, and \( A \) is a general matrix. This
routine returns immediately if \( n \) or \( k \) is less than or equal to zero. If \( \text{ldc} \) is less than one or less
than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \( \text{blas_no_trans} \), and if
\( \text{lda} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \)
equal to \( \text{blas_trans} \), and if \( \text{lda} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xSYRK with added functionality for
complex symmetric matrices.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and
mixed precision are permitted.

- Fortran 95 binding:

```
SUBROUTINE syr( a, c [, uplo] [, trans] [, alpha] [, beta] &
   [, prec] )
  <type>(<wp>), INTENT(IN) :: <aa>
  <type>(<wp>), INTENT(INOUT) :: c(:,:)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
  <aa> ::= a(:,:) or a(:)
and
  c has shape (n,n)
  a has shape (n,k) if trans = blas_no_trans (the default)
  (k,n) if trans /= blas_no_trans
  (n) if rank 1
```

<table>
<thead>
<tr>
<th>Rank a</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha A^T + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^T A + \beta C )</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>( C \leftarrow \alpha a a^T + \beta C )</td>
</tr>
</tbody>
</table>
4.5. LANGUAGE BINDINGS

The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The types of alpha, a, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3: the type of a can optionally differ from those of c, alpha and beta.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYRK({}_a){}_x}( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA, $ C, LDC [, PREC] )
INTEGER K, LDA, LDC, N, TRANS, UPLO [, PREC]
<type> ALPHA, BETA
<type> A( LDA, * )
<type> C( LDC, * )
```

The types of ALPHA, A, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of A. The suffix _x is present if and only if PREC is present. One or both of the suffixes _a and _x must be present.

- C binding:

```c
void BLAS_xsyrk{}_a{}_x}( enum blas_order_type order, enum blas_uplo_type uplo, enum blas_trans_type trans, int n, int k, SCALAR_IN alpha, const ARRAY a, int lda, SCALAR_IN beta, ARRAY c, int ldc [, enum blas_prec_type prec] );
```

The types of alpha, a, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of argument a. The suffix _x is present if and only if prec is present. One or both of the suffixes _a and _x must be present.

| HERK (Hermitian Rank K update) | C ← αAAH + βC, C ← αAH A + βC |

This routine performs one of the Hermitian rank k operations C ← αAAH + βC or C ← αAH A + βC where α and β are scalars, C is a Hermitian matrix, and A is a general matrix. This routine returns immediately if n or k is less than or equal to zero. If ldc is less than one or less than n, an error flag is set and passed to the error handler. For trans equal to blas_no_trans, and if lda is less than one or less than n, an error flag is set and passed to the error handler. For trans equal to blas_trans, and if lda is less than one or less than k, an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xHERK.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:
SUBROUTINE herk( a, c [, uplo] [, trans] [, alpha] [, beta] &
  [, prec] )
  COMPLEX(<wp>), INTENT(IN) :: <aa>
  COMPLEX(<wp>), INTENT(INOUT) :: c(:,:),
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  REAL(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
  TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
  <aa> ::= a(:, :) or a(:)
and
  c has shape (n,n)
  a has shape (n,k) if trans = blas_no_trans (the default)
  (k,n) if trans /= blas_no_trans
  (n) if rank 1

<table>
<thead>
<tr>
<th>Rank a</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>N</td>
<td>$C \leftarrow \alpha A A^H + \beta C$</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>$C \leftarrow \alpha A^H A + \beta C$</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>$C \leftarrow \alpha a a^H + \beta C$</td>
</tr>
</tbody>
</table>

The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The types of alpha, a, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3: the type of a can optionally differ from those of c, alpha and beta.

- Fortran 77 binding:

  SUBROUTINE BLAS_xHERK{a}{X}( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA,
  $\text{C, LDC [, PREC]}$ )
  INTEGER K, LDA, LDC, N, TRANS, UPLO [, PREC]
  <rtype> ALPHA, BETA
  <ctype> A( LDA, * )
  <ctype> C( LDC, * )

  The types of ALPHA, A, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if _a is present then _a is the type of A. The suffix _X is present if and only if PREC is present. One or both of the suffixes _a and _X must be present.

- C binding:

  void BLAS_xherk{a}{X}( enum blas_order_type order, enum blas_uplo_type uplo,
  enum blas_trans_type trans, int n, int k,
  RSCALAR_IN alpha, const CARRAY a, int lda,
  RSCALAR_IN beta, CARRAY c, int ldc
  [, enum blas_prec_type prec] );
The types of alpha, a, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix x is the floating point type of the arguments, but if a is present then a is the type of argument a. The suffix x is present if and only if prec is present. One or both of the suffixes a and x must be present.

SYR2K (Symmetric rank 2k update)  
\[ C \leftarrow (\alpha A)B^T + B(\alpha A)^T + \beta C \]  
\[ C \leftarrow (\alpha A)^T B + B^T(\alpha A) + \beta C \]

These routines perform the symmetric rank 2k operation \( C \leftarrow (\alpha A)B^T + B(\alpha A)^T + \beta C \) or \( C \leftarrow (\alpha A)^T B + B^T(\alpha A) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a symmetric matrix, and \( A \) and \( B \) are general matrices. This routine returns immediately if \( n \) or \( k \) is less than or equal to zero. If \( \text{lda} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \text{blas_no_trans}, and if \( \text{lda} \) is less than one or less than \( n \), or if \( \text{ldb} \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For \( \text{trans} \) equal to \text{blas_trans}, and if \( \text{lda} \) is less than one or less than \( k \), or if \( \text{ldb} \) is less than one or less than \( k \), an error flag is set and passed to the error handler.

These interfaces encompass the Legacy BLAS routine xSYR2K with added functionality for complex symmetric matrices.

Extended precision and mixed precision are permitted.

This routine has the same specification as in Chapter 2, except that extended precision and mixed precision are permitted.

- Fortran 95 binding:

```fortran
SUBROUTINE syr2k( a, b, c [, uplo] [, trans] [, alpha] [, beta] 
                  [, prec] )
    <type>(<wp>), INTENT(IN) :: <aa>
    <type>(<wp>), INTENT(IN) :: <bb>
    <type>(<wp>), INTENT(INOUT) :: c(:,:)
    TYPE (bias_uplo_type), INTENT(IN), OPTIONAL :: uplo
    TYPE (bias_trans_type), INTENT(IN), OPTIONAL :: trans
    <type>(<wp>), INTENT(IN), OPTIONAL :: alpha, beta
    TYPE (bias_prec_type), INTENT (IN), OPTIONAL :: prec

where
    <aa> ::= a(:,:) or a(:)
    <bb> ::= b(:,:) or b(:)
and
    c has shape \((n,n)\)
if \text{trans} = \text{blas_no_trans} (the default)
    a has shape \((n,k)\)
    b has shape \((n,k)\)
if \text{trans} /= \text{blas_no_trans}
    a has shape \((k,n)\)
    b has shape \((k,n)\)
```

<table>
<thead>
<tr>
<th>Rank a</th>
<th>Rank b</th>
<th>trans</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>N</td>
<td>( C \leftarrow \alpha AB^T + \alpha BA^T + \beta C )</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>T</td>
<td>( C \leftarrow \alpha A^T B + \alpha B^T A + \beta C )</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>-</td>
<td>( C \leftarrow \alpha \alpha b^T + \alpha b_a^T + \beta C )</td>
</tr>
</tbody>
</table>
The table defining the operation as a function of the operator arguments is identical to Chapter 2.

The types of alpha, a, b, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3: the types of a and b can optionally differ from those of c, alpha and beta.

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYR2K{._a_}{._b_}{._X} (UPLO, TRANS, N, K, ALPHA, A, LDA,
$ B, LDB, BETA, C, LDC [, PREC] )
INTEGER K, LDA, LDB, LDC, N, TRANS, UPLO [, PREC]
<type> ALPHA, BETA
<type> A ( LDA, * )
<type> B ( LDB, * )
<type> C ( LDC, * )
```

The types of ALPHA, A, B, BETA and C are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix `_x` is the floating point type of the arguments, but if `_a_b` is present then `.a` is the type of `A` and `.b` is the type of `B`. The suffix `_X` is present if and only if PREC is present. One or both of the suffixes `_a_b` and `_X` must be present.

- C binding:

```c
void BLAS_xsyr2k{._a_}{._b_}{._X}( enum blas_order_type order,
 enum blas_uplo_type uplo,
 enum blas_trans_type trans, int n, int k,
 SCALAR_IN alpha, const ARRAY a, int lda,
 const ARRAY b, int ldb,
 SCALAR_IN beta, ARRAY c, int ldc
 [, enum blas_prec_type prec] );
```

The types of alpha, a, b, beta and c are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix `_x` is the floating point type of the arguments, but if `_a_b` is present then `.a` is the type of argument `a` and `.b` is the type of argument `b`. The suffix `_X` is present if and only if `prec` is present. One or both of the suffixes `_a_b` and `_X` must be present.

---

**HER2K (Hermitian rank 2k update)**

\[ C \leftarrow (\alpha A)B^H + B(\alpha A)^H + \beta C \]

\[ C \leftarrow (\alpha A)^H B + B^H (\alpha A) + \beta C \]

These routines perform the Hermitian rank 2k operation \( C \leftarrow (\alpha A)B^H + B(\alpha A)^H + \beta C \) or \( C \leftarrow (\alpha A)^H B + B^H (\alpha A) + \beta C \) where \( \alpha \) and \( \beta \) are scalars, \( C \) is a Hermitian matrix, and \( A \) and \( B \) are general matrices. This routine returns immediately if \( n \) or \( k \) is less than or equal to zero. If \( ld \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For trans equal to `blas_no_trans`, and if \( lda \) is less than one or less than \( n \), or if \( ldb \) is less than one or less than \( n \), an error flag is set and passed to the error handler. For trans equal to `blas_trans`, and if \( lda \) is less than one or less than \( k \), or if \( ldb \) is less than one or less than \( n \), an error flag is set and passed to the error handler.
These interfaces encompass the Legacy BLAS routine xHER2K.
Extended precision and mixed precision are permitted.
This routine has the same specification as in Chapter 2, except that extended precision and
mixed precision are permitted.

- Fortran 95 binding:

```
SUBROUTINE her2k( a, b, c [, uplo] [, trans] [, alpha] [, beta]
    [, prec] )
  COMPLEX(<wp>), INTENT(IN) :: <aa>
  COMPLEX(<wp>), INTENT(IN) :: <bb>
  COMPLEX(<wp>), INTENT(INOUT) :: c(:,;)
  TYPE (blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE (blas_trans_type), INTENT(IN), OPTIONAL :: trans
  COMPLEX(<wp>), INTENT(IN), OPTIONAL :: alpha
  REAL(<wp>), INTENT(IN), OPTIONAL :: beta
  TYPE (blas_prec_type), INTENT (IN), OPTIONAL :: prec
where
  <aa>  ::= a(:,;): or a(:)
  <bb>  ::= b(:,;): or b(:)
and
  c has shape (n,n)
  a and b have shape (n,k) if trans = blas_no_trans (the default)
  (k,n) if trans /= blas_no_trans
  (n) if rank 1
```

The table defining the operation as a function of the operator arguments is identical to
Chapter 2.

The types of alpha, a, b, beta and c are governed according to the rules of mixed precision
arguments set down in section 4.3; the types of a and b can optionally differ from those of c,
alpha and beta.

- Fortran 77 binding:

```
SUBROUTINE BLAS_XHER2K{<a_b>_<X>}( UPLO, TRANS, N, K, ALPHA, A, LDA,
  $  B, LDB, BETA, C, LDC [, PREC] )
INTEGER K, LDA, LDB, LDC, N, TRANS, UPLO [, PREC]
<ctype>        ALPHA
<rtype>        BETA
<ctype>        A( LDA, * )
<ctype>        B( LDB, * )
<ctype>        C( LDC, * )
The types of \texttt{ALPHA}, \texttt{A}, \texttt{B}, \texttt{BETA} and \texttt{C} are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \texttt{x} is the floating point type of the arguments, but if \texttt{.A.B} is present then \texttt{.A} is the type of \texttt{A} and \texttt{.B} is the type of \texttt{B}. The suffix \texttt{X} is present if and only if \texttt{PREC} is present. One or both of the suffixes \texttt{.A.B} and \texttt{X} must be present.

- C binding:

```c
void BLAS_xher2k\{\_A\_B\\{\_X\}\}( enum blas_order_type order,
   enum blas_uplo_type uplo,
   enum blas_trans_type trans, int n, int k,
   CSCALAR_IN alpha, const CARRAY A, int lda,
   const CARRAY b, int ldb,
   RSCALAR_IN beta, CARRAY c, int ldc
   [, enum blas_prec_type prec] );
```

The types of \texttt{alpha}, \texttt{a}, \texttt{b}, \texttt{beta} and \texttt{c} are governed according to the rules of mixed precision arguments set down in section 4.3. The prefix \texttt{x} is the floating point type of the arguments, but if \texttt{.A.B} is present then \texttt{.A} is the type of argument \texttt{a} and \texttt{.B} is the type of argument \texttt{b}. The suffix \texttt{X} is present if and only if \texttt{PREC} is present. One or both of the suffixes \texttt{.A.B} and \texttt{X} must be present.

---

### 4.5.6 Environmental Enquiry

\texttt{FPINFO\_X} (Environmental enquiry)

This routine queries for machine-specific floating point characteristics.

- Fortran 95 binding:

```fortran
INTEGER FUNCTION fpinfo\_x( cmach, prec )
   TYPE (blas\_cmach\_type), INTENT (IN) :: cmach
   TYPE (blas\_prec\_type), INTENT (IN) :: prec
```

- Fortran 77 binding:

```fortran
INTEGER FUNCTION BLAS\_FPINFO\_X( cmach, prec )
   INTEGER cmach, prec
```

- C binding:

```c
int BLAS_fpinfo\_x( enum blas\_cmach\_type cmach,
   enum blas\_prec\_type prec );
```
Annex A

Appendix

This appendix contains overall notation, definitions, and implementation details for the chapters of the BLAS Technical Forum Standard.

A.1 Vector Norms

There are a variety of ways to define the norm of a vector, in particular for vectors of complex numbers, several of which have been used in the existing Level 1 BLAS and in various LAPACK auxiliary routines. Our definitions include all of these in a systematic way.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Name</th>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>one-norm</td>
<td>(|x|_1)</td>
<td>(\sum_i</td>
</tr>
<tr>
<td></td>
<td>two-norm</td>
<td>(|x|_2)</td>
<td>(\sqrt{\sum_i x_i^2})</td>
</tr>
<tr>
<td></td>
<td>infinity-norm</td>
<td>(|x|_\infty)</td>
<td>(\max_i</td>
</tr>
<tr>
<td>Complex</td>
<td>one-norm</td>
<td>(|x|_1)</td>
<td>(\sum_i</td>
</tr>
<tr>
<td></td>
<td>real one-norm</td>
<td>(|x|_{1R})</td>
<td>(\sum_i (</td>
</tr>
<tr>
<td></td>
<td>two-norm</td>
<td>(|x|_2)</td>
<td>(\sqrt{\sum_i</td>
</tr>
<tr>
<td></td>
<td>infinity-norm</td>
<td>(|x|_\infty)</td>
<td>(\max_i</td>
</tr>
<tr>
<td></td>
<td>real infinity-norm</td>
<td>(|x|_{\infty R})</td>
<td>(\max_i(</td>
</tr>
</tbody>
</table>

Table A.1: Vector Norms

Rationale. The reason for the two extra norms of complex vectors, the real one-norm and real infinity-norm, is to avoid the expense of up to \(n\) square roots, where \(n\) is the length of the vector \(x\). The two-norm only requires one square root, so a real version is not needed. The infinity norm only requires one square root in principle, but this would require tests and branches, making it more complicated and slower than the real infinity-norm. When \(x\) is real, the one-norm and real one-norm are identical, as are the infinity-norm and real infinity-norm. We note that the Level 1 BLAS routine ICAMAX, which finds the largest entry of a complex vector, finds the largest value of \(|Re(x_i)| + |Im(x_i)|\). (End of rationale.)
Computing the two-norm or Frobenius-norm of a vector is equivalent. However, this is not the case for computing matrix norms. For consistency of notation between vector and matrix norms, both norms are available.

### A.2 Matrix Norms

Analogously to vector norms as discussed in Section A.1, there are a variety of ways to define the norm of a matrix, in particular for matrices of complex numbers. Our definitions include all of these in a systematic way.

<table>
<thead>
<tr>
<th>Data Type</th>
<th>Name</th>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real</td>
<td>one-norm</td>
<td>$|A|_1$</td>
<td>$\max_j \sum_i</td>
</tr>
<tr>
<td></td>
<td>Frobenius-norm</td>
<td>$|A|_F$</td>
<td>$\sqrt{\sum_i \sum_j a_{ij}^2}$</td>
</tr>
<tr>
<td></td>
<td>infinity-norm</td>
<td>$|A|_\infty$</td>
<td>$\max_i \sum_j</td>
</tr>
<tr>
<td></td>
<td>max-norm</td>
<td>$|A|_{\max}$</td>
<td>$\max_i \max_j</td>
</tr>
<tr>
<td>Complex</td>
<td>one-norm</td>
<td>$|A|_1$</td>
<td>$\max_j \sum_i</td>
</tr>
<tr>
<td></td>
<td>real one-norm</td>
<td>$|A|_{1R}$</td>
<td>$\max_j \sum_i (\Re(a_{ij})^2 + \Im(a_{ij})^2)^{1/2}$</td>
</tr>
<tr>
<td></td>
<td>Frobenius-norm</td>
<td>$|A|_F$</td>
<td>$\sqrt{\sum_i \sum_j a_{ij}^2}$</td>
</tr>
<tr>
<td></td>
<td>infinity-norm</td>
<td>$|A|_\infty$</td>
<td>$\max_i \sum_j</td>
</tr>
<tr>
<td></td>
<td>real infinity-norm</td>
<td>$|A|_{\infty R}$</td>
<td>$\max_i \sum_j (\Re(a_{ij})^2 + \Im(a_{ij})^2)^{1/2}$</td>
</tr>
<tr>
<td></td>
<td>max-norm</td>
<td>$|A|_{\max}$</td>
<td>$\max_i \max_j</td>
</tr>
<tr>
<td></td>
<td>real max-norm</td>
<td>$|A|_{\max R}$</td>
<td>$\max_i \max_j (\Re(a_{ij})^2 + \Im(a_{ij})^2)^{1/2}$</td>
</tr>
</tbody>
</table>

Table A.2: Matrix Norms

In contrast to computing vector norms, computing the two-norm and Frobenius-norm of a matrix are not equivalent. If the user asks for the two-norm of a matrix, where the matrix is 2-by-2 or larger, an error flag is raised. The one exception occurs when the matrix is a single column or a single row. In this case, the two-norm is requested and the Frobenius-norm is returned.

### A.3 Operator Arguments

The following table lists the operator arguments and their associated named constants. For complete details of the meanings of the operator prec, refer to section 4.3.1.

**Example:** Consider the matrix-vector products $x = Ax$, $x = A^T x$ and $x = A^H x$. It is convenient to use the `trans` operator and define $op(A)$ as being $A$, $A^T$ or $A^H$ depending on the value of the `trans` operator argument. Again, the specification of the type and the valid values such an operator should have will be defined in the language-dependent section and may vary from one language binding to another.

It is worthwhile noticing that in some rare cases, the meaning of the `trans` operator argument is extended to a function of the matrix to which it applies. Consider for example the symmetric
<table>
<thead>
<tr>
<th>operator argument</th>
<th>named constant</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>norm</td>
<td>blas_one_norm</td>
<td>1-norm</td>
</tr>
<tr>
<td></td>
<td>blas_real_one_norm</td>
<td>real 1-norm</td>
</tr>
<tr>
<td></td>
<td>blas_two_norm</td>
<td>2-norm</td>
</tr>
<tr>
<td></td>
<td>blas_frobenius_norm</td>
<td>Frobenius-norm</td>
</tr>
<tr>
<td></td>
<td>blas_inf_norm</td>
<td>infinity-norm</td>
</tr>
<tr>
<td></td>
<td>blas_real_inf_norm</td>
<td>real infinity-norm</td>
</tr>
<tr>
<td></td>
<td>blas_max_norm</td>
<td>max-norm</td>
</tr>
<tr>
<td></td>
<td>blas_real_max_norm</td>
<td>real max-norm</td>
</tr>
<tr>
<td>sort</td>
<td>blas_increasing_order</td>
<td>sort in increasing order</td>
</tr>
<tr>
<td></td>
<td>blas_decreasing_order</td>
<td>sort in decreasing order</td>
</tr>
<tr>
<td>side</td>
<td>blas_left_side</td>
<td>operate on the left-hand side</td>
</tr>
<tr>
<td></td>
<td>blas_right_side</td>
<td>operate on the right-hand side</td>
</tr>
<tr>
<td>uplo</td>
<td>blas_upper</td>
<td>reference upper triangle only</td>
</tr>
<tr>
<td></td>
<td>blas_lower</td>
<td>reference lower triangle only</td>
</tr>
<tr>
<td>transz</td>
<td>blas_no_trans</td>
<td>operate with $x$</td>
</tr>
<tr>
<td></td>
<td>blas_trans</td>
<td>operate with $x^T$</td>
</tr>
<tr>
<td></td>
<td>blas_conj_trans</td>
<td>operate with $x^H$</td>
</tr>
<tr>
<td>conj</td>
<td>blas_conj</td>
<td>operate with $\bar{x}$</td>
</tr>
<tr>
<td></td>
<td>blas_no_conj</td>
<td>operate with $x$</td>
</tr>
<tr>
<td>diag</td>
<td>blas_non_unit_diag</td>
<td>non-unit triangular</td>
</tr>
<tr>
<td></td>
<td>blas_unit_diag</td>
<td>unit triangular</td>
</tr>
<tr>
<td>jrot</td>
<td>blas_jrot_inner</td>
<td>inner rotation $c &gt; \frac{1}{\sqrt{2}}$</td>
</tr>
<tr>
<td></td>
<td>blas_jrot_outer</td>
<td>outer rotation $0 \leq c \leq \frac{1}{\sqrt{2}}$</td>
</tr>
<tr>
<td></td>
<td>blas_jrot_sorted</td>
<td>sorted rotation $\text{abs}(a) \geq \text{abs}(b)$</td>
</tr>
<tr>
<td>order</td>
<td>blas_colmajor</td>
<td>assume column-major ordering</td>
</tr>
<tr>
<td></td>
<td>blas_rowmajor</td>
<td>assume row-major ordering</td>
</tr>
<tr>
<td>index_base</td>
<td>blas_zero_base</td>
<td>assumes zero-based indexing</td>
</tr>
<tr>
<td></td>
<td>blas_one_base</td>
<td>assumes one-based indexing</td>
</tr>
<tr>
<td>prec</td>
<td>blas_prec_single</td>
<td>internal computation performed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>in single precision</td>
</tr>
<tr>
<td></td>
<td>blas_prec_double</td>
<td>internal computation performed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>in double precision</td>
</tr>
<tr>
<td></td>
<td>blas_prec_indigenous</td>
<td>internal computation performed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>in the widest hardware-supported</td>
</tr>
<tr>
<td></td>
<td></td>
<td>format available</td>
</tr>
<tr>
<td></td>
<td>blas_prec_extra</td>
<td>internal computation performed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>in format wider than 80-bits</td>
</tr>
</tbody>
</table>

Table A.3: Operator Arguments

Rank-$k$ update operations, $C \leftarrow C + AA^T$ and $C \leftarrow C + A^T A$ where $C$ is a symmetric matrix. The value of the trans operator refers to the product $AA^T$. It follows that these operations can be specified by $C \leftarrow C + \text{op}(AA^T)$ where $\text{op}(AA^T)$ is $AA^T$ or $A^T A$ depending on the input value of the trans argument.

All possible values of the operator argument trans are not always meaningful. For example, in
the symmetric rank-$k$ update operations defined above, when the matrix $C$ is complex symmetric, the only valid values of $\text{op}(AA^T)$ are $AA^T$ or $A^T A$. Similarly, when the matrix $C$ is complex Hermitian, the only valid values of $\text{op}(AA^H)$ are $AA^H$ or $A^H A$. Such restrictions are detailed for each dense and banded BLAS function to which they apply.

Some BLAS routines have more than one `trans` operator argument because such an argument is needed for each matrix to which it applies. For example, a general matrix-multiply operation can be specified as $C \leftarrow \text{op}(A)\text{op}(B)$ where $A$, $B$ and $C$ are general matrices. A `trans` argument is needed for each of the input matrices $A$ and $B$; by convention we denote those formal arguments `transA` and `transB`.

**Rationale.** As mentioned above, section (1.4) does not specify how the objects manipulated by the BLAS routines are stored. This important aspect of the interface specification is deferred to the language-dependent specification sections. In particular, the operator arguments do not indicate whether only half or all entries of triangular, symmetric and Hermitian matrices are stored, or even how these entries are stored. The intent is to provide each language binding with the opportunity to choose the appropriate data structures for each object. Note that a given language binding specification may provide multiple functions performing the same operation on operands stored differently. For example, triangular matrices may be stored within conventional two-dimensional arrays or in packed storage, where the triangle may be packed by rows or columns. Consequently, a BLAS routine specified in the functionality tables may induce multiple functions in a particular language binding, say for instance, to provide the user with the same operation on objects that are stored differently. (*End of rationale.*)

It follows that, in general, a mathematical operation involving a matrix $A$, where $A$ could be general or banded, triangular, symmetric or Hermitian, induces the language-independent specification of multiple routines. However, this language-independent section ignores the fact that a given language binding may choose to provide multiple storage schemes for some specific classes of matrices, such as triangular matrices.

### A.4 Fortran 95 Modules

Several Fortran 95 modules are provided, allowing for the flexible inclusion of only select portions of the document. The modules `blas_dense`, `blas_sparse`, and `blas_extended`, are provided for Chapters 2, 3, and 4, respectively.

```
http://www.netlib.org/blas/blast-forum/blas_dense.f90
http://www.netlib.org/blas/blast-forum/blas_sparse.f90
http://www.netlib.org/blas/blast-forum/blas_extended.f90
```

Each of these modules in turn contains a USE statement to include the module of operator arguments (in `blas_operator_arguments` for Chapters 2 and 4, and `blas_sparseNamedConstants` for Chapter 3), and the respective module(s) of explicit interfaces for that chapter.

For Chapters 2 and 4, one derived type is specified for each category of operator arguments (such as `trans`) and some parameters are defined of this type (for the different settings). For consistency, the suffix _type is used to name all of the derived types. This suffix is needed in some cases to differentiate between the type and one of the parameters (for example, `blas_trans_type` is a type and `blas_trans` is a parameter of this type). The Sparse BLAS chapter represents its operator arguments and a list of matrix properties (see section 3.5.1) as named constants.
Advice to implementors. For Chapter 2, all the entities (derived types, named constants and BLAS procedures) must be accessible to the user via the module blas_dense.

There are many ways to create this module. However the following three conditions MUST be adhered to:

- all entities can be accessed by the module
- the generic names must be the same as in the Fortran 95 bindings
- the specific name must be standard. The standard that we recommend is “suffix _d, _z, _s and _c” for double precision, double complex, real and complex.

For example the Fortran 95 bindings gives the generic name gemm. This is a generic procedure for the following 12 specific procedures:

- gemm_d corresponds to BLAS_DGEMM (legacy DGEMM)
- gemm_z corresponds to BLAS_ZGEMM (legacy ZGEMM)
- gemm_s corresponds to BLAS_SGEMM (legacy SGEMM)
- gemm_c corresponds to BLAS_DCGEMM (legacy CGEMM)
- gemv_d corresponds to BLAS_DGEMV (legacy DGEMV)
- gemv_z corresponds to BLAS_ZGEMV (legacy ZGEMV)
- gemv_s corresponds to BLAS_SGEMV (legacy SGEMV)
- gemv_c corresponds to BLAS_DCGEMV (legacy CGEMV)
- ger_d corresponds to BLAS_DGER (legacy DGER)
- ger_z corresponds to BLAS_ZGER (legacy ZGER)
- ger_s corresponds to BLAS_SGER (legacy ZGERU, ZGERC)
- ger_c corresponds to BLAS_CGER (legacy CGERU, CGERC)

A specific procedure could be an external procedure or a module procedure.

One approach for creating the module blas_dense is to:

- create one file for each procedure
- create the interface blocks for the generic names using one or more modules
- create the module blas_dense from the modules in the last step and other modules such as blas_operator_args

Assuming we are using external procedures, the following files could be used as templates to create the module blas_dense. The interface blocks are grouped according to the grouping in section 2.8.1. The files are:

- http://www.netlib.org/blas/blast-forum/blas_operator_arguments.f90
  file containing the module blas_operator_arguments
- http://www.netlib.org/blas/blast-forum/blas_precision.f90
  file containing the module used to specify the precision (not visible to the user)
- http://www.netlib.org/blas/blast-forum/blas_dense_red_op.f90
  file containing the interface blocks for the reduction operations (section 2.8.2)
- http://www.netlib.org/blas/blast-forum/blas_dense_gen_trans.f90
  file containing the interface blocks for the generate transformations procedures (section 2.8.3)
• http://www.netlib.org/blas/blast-forum/blas_dense_vec_op.f90
  file containing the interface blocks for the vector operations (section 2.8.4)
• http://www.netlib.org/blas/blast-forum/blas_dense_vec_mov.f90
  file containing the interface blocks for the data movement with vectors (section 2.8.5)
• http://www.netlib.org/blas/blast-forum/blas_dense_mat_vec_op.f90
  file containing the interface blocks for the matrix-vector operations (section 2.8.6)
• http://www.netlib.org/blas/blast-forum/blas_dense_mat_op.f90
  file containing the interface blocks for the matrix operations (section 2.8.7)
• http://www.netlib.org/blas/blast-forum/blas_dense_mat_mat_op.f90
  file containing the interface blocks for the matrix-matrix operations (section 2.8.8)
• http://www.netlib.org/blas/blast-forum/blas_dense_mat_mov.f90
  file containing the interface blocks for the data movement with matrices (section 2.8.9)
• http://www.netlib.org/blas/blast-forum/blas_dense_fpinfo.f90
  file containing the interface blocks for the environmental enquiry (section 2.8.10)
• http://www.netlib.org/blas/blast-forum/blas_dense.f90
  file containing the module blas_dense that imports the information from all other modules and makes them available.

The specifications for all specific procedures MUST be as they appear in the above files. The only change is the way that the precision is specified. (*End of advice to implementors.*)

## A.5 Fortran 77 Include File

One Fortran 77 include file is provided, blas_namedconstants.h. This include file contains the values of all named constants, and applies to Chapters 2, 3, and 4.

http://www.netlib.org/blas/blast-forum/blas_namedconstants.h

Operator arguments norm, sort, side, uplo, trans, conj, diag, jrot, index_base, and prec are represented in the Fortran 77 interface as INTEGERS. These operator arguments assume the named constant values as defined in section A.3. The Sparse BLAS chapter defines a list of matrix properties (see section 3.5.1) that must also be defined.

*Advice to implementors.* This specification is a deviation from the Legacy BLAS, where these operator arguments were defined as CHARACTER*1. (*End of advice to implementors.*)

## A.6 C Include Files

Several C include files are provided, allowing for the flexible inclusion of only select portions of the document. The file blas.h contains the enumerated types and all prototypes for Chapters 2, 3, and 4. The files blas_dense.h, blas_sparse.h, and blas_extended.h, include the values of the operator arguments (enumerated types) and the function prototypes for the respective chapter.

http://www.netlib.org/blas/blast-forum/blas.h
http://www.netlib.org/blas/blast-forum/blas_dense.h
http://www.netlib.org/blas/blast-forum/blas_sparse.h
http://www.netlib.org/blas/blast-forum/blas_extended.h
The file `blas_enum.h` contains the values of all enumerated types, applying to all chapters. The files `blas_dense_proto.h`, `blas_sparse_proto.h`, and `blas_extended_proto.h`, contain the respective function prototypes for Chapters 2, 3, and 4.

http://www.netlib.org/blas/blast-forum/blas_enum.h
http://www.netlib.org/blas/blast-forum/blas_dense_proto.h
http://www.netlib.org/blas/blast-forum/blas_sparse_proto.h
http://www.netlib.org/blas/blast-forum/blas_extended_proto.h

All operator arguments are handled by enumerated types in the C interface. This allows for tighter error checking, and provides less opportunity for user error. In addition to the operator arguments of norm, sort, side, uplo, trans, conj, diag, jrot, index_base, and prec, this interface adds another such argument to all routines involving two dimensional arrays, order. order designates if the array elements are stored in row-major or column-major ordering. Refer to section 2.6.6 for further details. The Sparse BLAS chapter defines a list of matrix properties (see section 3.5.1) that must also be defined.
Annex B

Legacy BLAS

B.1 Introduction

This chapter addresses additional language bindings for the original Level 1, 2, and 3 BLAS. The Level 1, 2, and 3 BLAS will hereafter be referred to as the Legacy BLAS.

B.2 C interface to the Legacy BLAS

This section gives a detailed discussion of the proposed C interface to the legacy BLAS. Every mention of “BLAS” in this chapter should be taken to mean the legacy BLAS. Each interface decision is discussed in its own section. Each section also contains a Considered methods subsection, where other solutions to that particular problem are discussed, along with the reasons why those options were not chosen. These Considered methods subsections are indented and italicized in order to distinguish them from the rest of the text.

It is largely agreed among the group (and unanimous among the vendors) that user demand for a C interface to the BLAS is insufficient to motivate vendors to support a completely separate standard. This proposal therefore confines itself to an interface which can be readily supported on top of the already existing Fortran 77 callable BLAS (i.e., the legacy BLAS).

The interface is expressed in terms of ANSI/ISO C. Very few platforms fail to provide ANSI/ISO C compilers at this time, and for those platforms, free ANSI/ISO C compilers are almost always available (e.g., gcc).

B.2.1 Naming scheme

The naming scheme consists of taking the Fortran 77 routine name, making it lower case, and adding the prefix cblas_. Therefore, the routine Dgemm becomes cblas_dgemm.

Considered methods

Various other naming schemes have been proposed, such as adding c_ or c to the name. Most of these schemes accomplish the requirement of separating the Fortran 77 and C name spaces. It was argued, however, that the addition of the blas prefix unifies the naming scheme in a logical and useful way (making it easy to search for BLAS use in a code, for instance), while not placing too great a burden on the typist. The letter c is used to distinguish this language interface from possible future interfaces.
B.2.2 Indices and I_AMAX

The Fortran 77 BLAS return indices in the range \(1 \leq I \leq N\) (where \(N\) is the number of entries in the dimension in question, and \(I\) is the index), in accordance with Fortran 77 array indexing conventions. This allows functions returning indices to be directly used to index standard arrays. The C interface therefore returns indices in the range \(0 \leq I < N\) for the same reason.

The only BLAS routine which returns an index is the function I_AMAX. This function is declared to be of type CBLAS_INDEX, which is guaranteed to be an integer type (i.e., no cast is required when assigning to any integer type). CBLAS_INDEX will usually correspond to size_t to ensure any array can be indexed, but implementors might choose the integer type which matches their Fortran 77 INTEGER, for instance. It is defined that zero is returned as the index for a zero length vector (e.g., For \(N = 0\), I_AMAX will always return zero).

B.2.3 Character arguments

All arguments which were characters in the Fortran 77 interface are handled by enumerated types in the C interface. This allows for tighter error checking, and provides less opportunity for user error. The character arguments present in the Fortran 77 interface are: SIDE, UPLO, TRANSPOSE, and DIAG. This interface adds another such argument to all routines involving two dimensional arrays, ORDER. The standard dictates the following enumerated types:

```c
enum CBLAS_ORDER     {ClasRowMajor=101, ClasColMajor=102};
enum CBLAS_TRANSPOSE  {ClasNoTrans=111, ClasTrans=112, ClasConjTrans=113};
enum CBLAS_UPLO       {ClasUpper=121, ClasLower=122};
enum CBLAS_DIAG       {ClasNonUnit=131, ClasUnit=132};
enum CBLAS_SIDE       {ClasLeft=141, ClasRight=142};
```

Considered methods

The other two most commonly suggested methods were accepting these arguments as either char * or char. It was noted that both of these options require twice as many comparisons as normally required to branch (so that the character may be either upper or lower case). Both methods also suffered from ambiguity (what does it mean to have DIAG=’H’, for instance). If char was chosen, the words could not be written out as they can for the Fortran 77 interface (you couldn’t write ”NoTranspose”). If char * were used, some compilers might fail to optimize string constant use, causing unnecessary memory usage.

The main advantage of enumerated data types, however, is that much of the error checking can be done at compile time, rather than at runtime (i.e., if the user fails to pass one of the valid options, the compiler can issue the error).

There was much discussion as to whether the integer values should be specified, or whether only the enumerated names should be so specified. The group could find no substantive way in which specifying the integer values would restrict an implementor, and specifying the integer values was seen as an aid to inter-language calls.

B.2.4 Handling of complex data types

All complex arguments are accepted as void *. A complex element consists of two consecutive memory locations of the underlying data type (i.e., float or double), where the first location contains the real component, and the second contains the imaginary part of the number.
In practice, programmers' methods of handling complex types in C vary. Some use various data structures (some examples are discussed below). Others accept complex numbers as arrays of the underlying type.

Complex numbers are accepted as void pointers so that widespread type casting will not be required to avoid warning or errors during compilation of complex code.

An ANSI/ISO committee is presently working on an extension to ANSI/ISO C which defines complex data types. The definition of a complex element is the same as given above, and so the handling of complex types by this interface will not need to be changed when ANSI/ISO C standard is extended.

Considered methods

Probably the most strongly advocated alternative was defining complex numbers via a structure such as

```c
struct NON_PORTABLE_COMPLEX {float r; float i;}; The main problem with this solution is the lack of portability. By the ANSI/ISO C standard, elements in a structure are not guaranteed to be contiguous. With the above structure, padding between elements has been experimentally observed (on the CRAY T3D), so this problem is not purely theoretical.

To get around padding problems within the structure, a structure such as

```c
struct NON_PORTABLE_COMPLEX {float v[2];} has been suggested. With this structure there will obviously be no padding between the real and imaginary parts. However, there still exists the possibility of padding between elements within an array. More importantly, this structure does not lend itself nearly as well as the first to code clarity.

A final proposal is to define a structure which may be addressed the same as the one above (i.e., ptr->r, ptr->i), but whose actual definition is platform dependent. Then, hopefully, various vendors will either use the above structure and ensure via their compilers its contiguousness, or they will create a different structure which can be accessed in the same way.

This requires vendors to support something which is not in the ANSI C standard, and so there is no way to ensure this would take place. More to the point, use of such a structure turns out to not offer much in the way of real advantage, as discussed in the following section.

All of these approaches require the programmer to either use the specified data type throughout the code which will call the BLAS, or to perform type casting on each BLAS call. When complex numbers are accepted as void pointers, no type casting or data type is dictated, with the only restriction being that a complex number have the definition given above.

B.2.5 Return values of complex functions

BLAS routines which return complex values in Fortran 77 are instead recast as subroutines in the C interface, with the return value being an output parameter added to the end of the argument list. This allows the output parameter to be accepted as void pointers, as discussed above.

Further, the name is suffixed by .sub. There are two main reasons for this name change. First, the change from a function to a subroutine is a significant change, and thus the name should reflect this. More importantly, the "traditional" name space is specifically reserved for use when the forthcoming ANSI/ISO C extension is finalized. When this is done, this C interface will be extended
to include functions using the "traditional" names which utilize the new ANSI/ISO complex type to return the values.

Considered methods

This is the area where use of a structure is most desired. Again, the most common suggestion is a structure such as

```c
struct NON_PORTABLE_COMPLEX {float r; float i;};
```

If one is willing to use this structure throughout one's code, then this provides a natural and convenient mechanism. If, however, the programmer has utilized a different structure for complex, this ease of use breaks down. Then, something like the following code fragment is required:

```c
NON_PORTABLE_COMPLEX ctmp;
float cdot[2];

cmp = cblas_cdotc(n, x, 1, y, 1);
cdot[0] = ctmp.r;
cdot[1] = ctmp.i;
```

which is certainly much less convenient than: cblas_cdotc_sub(n, x, 1, y, 1, cdot).

It should also be noted that the primary reason for having a function instead of a subroutine is already invalidated by C's lack of a standard complex type. Functions are most useful when the result may be used directly as part of an in-line computation. However, since ANSI/ISO C lacks support for complex arithmetic primitives or operator overloading, complex functions cannot be standardly used in this way. Since the function cannot be used as a part of a larger expression, nothing is lost by recasting it as a subroutine; indeed a slight performance win may be obtained.

B.2.6 Array arguments

Arrays are constrained to being contiguous in memory. They are accepted as pointers, not as arrays of pointers.

All BLAS routines which take one or more two dimensional arrays as arguments receive one extra parameter as their first argument. This argument is of the enumerated type

```c
enum CBLAS_ORDER {CblasRowMajor=101, CblasColMajor=102};
```

If this parameter is set to CblasRowMajor, it is assumed that elements within a row of the array(s) are contiguous in memory, while elements within array columns are separated by a constant stride given in the stride parameter (this parameter corresponds to the leading dimension [e.g. LDA] in the Fortran 77 interface).

If the order is given as CblasColMajor, elements within array columns are assumed to be contiguous, with elements within array rows separated by stride memory elements.

Note that there is only one CBLAS_ORDER parameter to a given routine: all array operands are required to use the same ordering.

Considered methods

This solution comes after much discussion. C users appear to split roughly into two camps. Those people who have a history of mixing C and Fortran 77 (in particular making use of the Fortran 77 BLAS from C), tend to use column-major arrays in order to allow ease of inter-language operations. Because of the flexibility of pointers, this is
not appreciably harder than using row-major arrays, even though C “natively” possesses row-major arrays.

The second camp of C users are not interested in overt C/Fortran 77 interoperability, and wish to have arrays which are row-major, in accordance with standard C conventions. The idea that they must recast their row-oriented algorithms to column-major algorithms is unacceptable; many in this camp would probably not utilize any BLAS which enforced a column-major constraint.

Because both camps are fairly widely represented within the target audience, it is impossible to choose one solution to the exclusion of the other.

Column-major array storage can obviously be supported directly on top of the legacy Fortran 77 BLAS. Recent work, particularly code provided by D.P. Manley of DEC, has shown that row-major array storage may also be supported in this way with little cost. Appendix B.2.12 discusses this issue in detail. To preview it here, we can say the level 1 and 3 BLAS require no extra operations or storage to support row-major operations on top of the legacy BLAS. Level 2 real routines also require no extra operations or storage. Some complex level 2 routines involving the conjugate transpose will require extra storage and operations in order to form explicit conjugates. However, this will always involve vectors, not the matrix. In the worst case, we will need \( n \) extra storage, and \( 3n \) sign changes.

One proposal was to accept arrays as arrays of pointers, instead of as a single pointer. The problems with this approach are manifold. First, the existing Fortran 77 BLAS could not be used, since they demand contiguous (though strided) storage. Second, this approach requires users of standard C 2D arrays or 1D arrays to allocate and assign the appropriate pointer array.

Beyond this, many of the vectors used in level 1 and level 2 BLAS come from rows or columns of two dimensional arrays. Elements within columns of row-major arrays are not uniformly strided, which means that a \( n \)-element column vector would need \( n \) pointers to represent it. This then leads to vectors being accepted as arrays of pointers as well.

Now, assuming both our one and two dimensional arrays are accepted as arrays of pointers, we have a problem when we wish to perform sub-array access. If we wish to pass an \( m \times n \) subsection of a this array of pointers, starting at row \( i \) and column \( j \), we must allocate \( m \) pointers, and assign them in a section of code such as:

```c
float **A, **subA;

subA = malloc(m*sizeof(float*));
for (k=0; k != m; k++) subA[k] = A[i+k] + j;
cblas_rout(... subA ...);
```

The same operation must be done if we wish to use a row or column as a vector. This is not only an inconvenience, but can add up to a non-negligible performance loss as well.

A fix for these problems is that one and two dimensional arrays be passed as arrays of pointers, and then indices are passed in to indicate the sub-portion to access. Thus you have a call that looks like: `cblas_rout(... A, i, j, ...)`. This solution still requires some additional tweaks to allow using two dimensional array rows and columns as vectors. Users presently using C 2D arrays or 1D arrays would have to malloc the
array of pointers as shown in the preceding example in order to use this kind of interface. At any rate, a library accepting pointers to pointers cannot be supported on top of the Fortran 77 BLAS, while one supporting simple pointers can.

If the programmer is utilizing the pointer to pointer style of array indexing, it is still possible to use this library providing that the user ensures that the operand matrix is contiguous, and that the rows are constantly strided. If this is the case, the user may pass the operand matrix to the library in precisely the same way as with a 2D C array:
cblas_rout(... &A[i][j] ...);

Example 1: making a library call with a C 2D array:

double A[50][25]; /* standard C 2D array */
cblas_rout(CblasRowMajor, ... &A[i][j], 25, ...);

Example 2: Legal use of pointer to pointer style programming and the CBLAS

double **A, *p;

A = malloc(M);
p = malloc(M*N* sizeof(double));
for (i=0; i < M; i++) A[i] = &p[i*N];
cblas_rout(CblasRowMajor, ... &A[i][j], N, ...);

Example 3: Illegal use of pointer to pointer style programming and the CBLAS

double **A, *p;

A = malloc(M);
p = malloc(M*N* sizeof(double));
for (i=0; i < M; i++) A[i] = malloc(N* sizeof(double));
cblas_rout(CblasRowMajor, ... &A[i][j], N, ...);

Note that Example 3 is illegal because the rows of A have no guaranteed stride.

B.2.7 Aliasing of arguments

Unless specified otherwise, only input-only arguments (specified with the const qualifier), may be legally aliased on a call to the C interface to the BLAS.

Considered methods

The ANSI C standard allows for the aliasing of output arguments. However, allowing this often carries a substantial performance penalty. This, along with the fact that Fortran 77 (which we hope to call for optimized libraries) does not allow aliasing of output arguments, led us to make this restriction.
B.2.8 C interface include file

The C interface to the BLAS will have a standard include file, called cblas.h, which minimally contains the definition of the CBLAS types and ANSI/ISO C prototypes for all BLAS routines. It is not an error to include this file multiple times. Refer to section B.2.11 for an example of a minimal cblas.h.

**ADVICE TO THE IMPLEMENTOR:**

*Note that the vendor is not constrained to using precisely this include file; only the enumerated type definitions are fully specified. The implementor is free to make any other changes which are not apparent to the user. For instance, all matrix dimensions might be accepted as size_t instead of int, or the implementor might choose to make some routines inline.*

B.2.9 Error checking

The C interface to the legacy BLAS must supply error checking corresponding to that provided by the reference Fortran 77 BLAS implementation.

B.2.10 Rules for obtaining the C interface from the Fortran 77

- The Fortran 77 routine name is changed to lower case, and prefixed by cblas_

- All routines which accept two dimensional arrays (i.e., level 2 and 3), acquire a new parameter of type CBLAS_ORDER as their first argument, which determines if the two dimensional arrays are row or column major.

- *Character arguments* are replaced by the appropriate enumerated type, as shown in Section B.2.3.

- *Input arguments* are declared with the const modifier.

- *Non-complex scalar input arguments* are passed by value. This allows the user to put in constants when desired (e.g., passing 10 on the command line for N).

- *Complex scalar input arguments* are passed as void pointers, since they do not exist as a predefined data type in ANSI/ISO C.

- *Array arguments* are passed by address.

- *Output scalar arguments* are passed by address.

- *Complex functions* become subroutines which return the result via a void pointer, added as the last parameter. The name is suffixed with _sub.

B.2.11 cblas.h include file

The cblas.h include file can be found on the BLAS Technical Forum webpage:

http://www.netlib.org/blas/blast-forum/cblas.h
B.2. C INTERFACE TO THE LEGACY BLAS

B.2.12 Using Fortran 77 BLAS to support row-major BLAS operations

This section is not part of the standard per se. Rather, it exists as an advice to the implementor on how row-major BLAS operations may be implemented using column-major BLAS. This allows vendors to leverage years of Fortran 77 BLAS development in producing the C BLAS.

Before this issue is examined in detail, a few general observations on array storage are helpful. We must distinguish between the matrix and the array which is used to store the matrix. The matrix, and its rows and columns, have mathematical meaning. The array is simply the method of storing the matrix, and its rows and columns are significant only for memory addressing.

Thus we see we can store the columns of a matrix in the rows of an array, for instance. When this occurs in the BLAS, the matrix is said to be stored in transposed form.

A row-major array stores elements along a row in contiguous storage, and separates the column elements by some constant stride (often the actual length of a row). Column-major arrays have contiguous columns, and strided rows. The importance of this is to note that a row-major array storing a matrix in the natural way, is a transposed column-major array (i.e., it can be thought of as a column-major array where the rows of the matrix are stored in the columns of the array).

Similarly, an upper triangular row-major array corresponds to a transposed lower triangular column-major array (the same is true in reverse [i.e., lower-to-upper], obviously). To see this, simply think of what a upper triangular matrix stored in a row-major array looks like. The first \( n \) entries contain the first matrix row, followed by a non-negative gap, followed by the second matrix row.

If this same array is viewed as column-major, the first \( n \) entries are a column, instead of a row, so that the columns of the array store the rows of the matrix (i.e., it is transposed). This means that if we wish to use the Fortran 77 (column-major) BLAS with triangular matrices coming from C (possibly row-major), we will be reversing the setting of \( \text{UPLO} \), while simultaneously reversing the setting of \( \text{TRANS} \) (this gets slightly more complicated when the conjugate transpose is involved, as we will see).

Finally, note that if a matrix is symmetric or Hermitian, its rows are the same as its columns, so we may merely switch \( \text{UPLO} \), without bothering with \( \text{TRANS} \).

In the BLAS, there are two separate cases of importance. One dimensional arrays (storage for vectors) have the same meaning in both C and Fortran 77, so if we are solving a linear algebra problem who's answer is a vector, we will need to solve the same problem for both languages. However, if the answer is a matrix, in terms of calling routines which use column-major storage from one using row-major storage, we will want to solve the transpose of the problem.

To get an idea of what this means, consider a contrived example. Say we have routines for simple matrix-matrix and matrix-vector multiply. The vector operation is \( y \leftarrow A \times x \), and the matrix operation is \( C \leftarrow A \times B \). Now say we are implementing these as calls from row-major array storage to column-major storage. Since the matrix-vector multiply's answer is a vector, the problem we are solving remains the same, but we must remember that our C array \( A \) is a Fortran 77 \( A^T \). On the other hand, the matrix-matrix multiply has a matrix for a result, so when the differing array storage is taken into account, the problem we want to solve is \( C^T \leftarrow B^T \times A^T \).

This last example demonstrates another general result. Some level 3 BLAS contain a \( \text{SIDE} \) parameter, determining which side a matrix is applied on. In general, if we are solving the transpose of this operation, the side parameter will be reversed.

With these general principles, it is possible to show that all that row-major level 3 BLAS can be expressed in terms of column-major BLAS without any extra array storage or extra operations. In the level 2 BLAS, no extra storage or array accesses are required for the real routines. Complex routines involving the conjugate transpose, however, may require a \( n \)-element temporary, and up
to $3n$ more operations (vendors may avoid all extra workspace and operations by overloading the 
TRANS option for the level 2 BLAS; letting it also allow conjugation without doing the transpose). 
The level 1 BLAS, which deal exclusively with vectors, are unaffected by this storage issue.

With these ideas in mind, we will now show how to support a row-major BLAS on top of a 
column major BLAS. This information will be presented in tabular form. For brevity, row-major 
storage will be referred to as coming from C (even though column-major arrays can also come from 
C), while column-major storage will be referred to as F77.

Each table will show a BLAS invocation coming from C, the operation that the BLAS should 
perform, the operation required once F77 storage is taken into account (if this changes), and the call 
to the appropriate F77 BLAS. Not every possible combination of parameters is shown, since many 
are simply reflections of another (i.e., when we are applying the Upper, NoTranspose becomes 
Lower, Transpose rule, we will show it for only the upper case. In order to make the notation 
more concise, let us define $\overline{x}$ to be $\text{conj}(x)$.

Level 2 BLAS

**GEMV**

C call  
\[
\text{cblas}\_cgemv(CblasRowMajor, CblasNoTrans, m, n, \alpha, A, lda, x, incx, \beta, y, incy)
\]

\[
y \leftarrow \alpha A x + \beta y
\]

F77 call  
\[\text{CGEMV}('T', n, m, \alpha, A, lda, x, incx, \beta, y, incy)\]

C call  
\[
\text{cblas}\_cgemv(CblasRowMajor, CblasTrans, m, n, \alpha, A, lda, x, incx, \beta, y, incy)
\]

\[
y \leftarrow \alpha A^T x + \beta y
\]

F77 call  
\[\text{CGEMV}('N', n, m, \alpha, A, lda, x, incx, \beta, y, incy)\]

C call  
\[
\text{cblas}\_cgemv(CblasRowMajor, CblasConjTrans, m, n, \alpha, A, lda, x, incx, \beta, y, incy)
\]

\[
y \leftarrow \alpha A^H x + \beta y \Rightarrow (\overline{y} \leftarrow \alpha A^T x + \overline{\beta y})
\]

F77 call  
\[\text{CGEMV}('N', n, m, \overline{\alpha}, A, lda, \overline{x}, 1, \overline{\beta}, \overline{y}, incy)\]

Note that we switch the value of transpose to handle the row/column major ordering difference. 
In the last case, we will require $n$ elements of workspace so that we may store the conjugated vector 
$\overline{x}$. Then, we set $y = \overline{y}$, and make the call. This gives us the conjugate of the answer, so we once 
again set $y = \overline{y}$. Therefore, we see that to support the conjugate transpose, we will need to allocate 
an $n$-element vector, and perform $2m + n$ extra operations.

**SYM**

**SYMV requires no extra workspace or operations.**

C call  
\[
\text{cblas}\_csymv(CblasRowMajor, CblasUpper, n, \alpha, A, lda, x, incx, \beta, y, incy)
\]

\[
y \leftarrow \alpha Ax + \beta y \Rightarrow y \leftarrow \alpha A^T x + \beta y
\]

F77 call  
\[\text{CSYMV}('L', n, \alpha, A, lda, x, incx, \beta, y, incy)\]

**HEMV**

**HEMV routine requires $3n$ conjugations, and $n$ extra storage.**

C call  
\[
\text{cblas}\_chemv(CblasRowMajor, CblasUpper, n, \alpha, A, lda, x, incx, \beta, y, incy)
\]

\[
y \leftarrow \alpha Ax + \beta y \Rightarrow y \leftarrow \alpha A^H x + \beta y \Rightarrow (\overline{y} \leftarrow \alpha A^T x + \overline{\beta y})
\]

F77 call  
\[\text{CHEMV}('L', n, \overline{\alpha}, A, lda, \overline{x}, incx, \overline{\beta}, \overline{y}, incy)\]
B.2. C INTERFACE TO THE LEGACY BLAS

TRMV/TRSV

C call    cblas_strmv(CblasRowMajor, CblasUpper, CblasNoTrans, diag, n, A, lda, x, incx)
op \quad x \leftarrow Ax
F77 call   CTRMV('L', 'T', diag, n, A, lda, x, incx)

C call    cblas_strmv(CblasRowMajor, CblasUpper, CblasTrans, diag, n, A, lda, x, incx)
op \quad x \leftarrow A^Tx
F77 call   CTRMV('L', 'N', diag, n, A, lda, x, incx)

C call    cblas_strmv(CblasRowMajor, CblasUpper, CblasConjTrans, diag, n, A, lda, x, incx)
op \quad x \leftarrow A^Hx \Rightarrow (x = A\overline{x})
F77 call   CTRMV('L', '\overline{N}', diag, n, A, lda, \overline{\tau}, incx)

Again, we see that we will need some extra operations when we are handling the conjugate transpose. We conjugate x before the call, giving us the conjugate of the answer we seek. We then conjugate this again to return the correct answer. This routine therefore needs 2n extra operations for the complex conjugate case.

The calls with the C array being Lower are merely the reflection of these calls, and thus are not shown. The analysis for TRMV is the same, since it involves the same principle of what a transpose of a triangular matrix is.

GER/GERU

This is our first routine that has a matrix as the solution. Recalling that this means we solve the transpose of the original problem, we get:

C call    cblas_sgeru(CblasRowMajor, m, n, alpha, x, incx, y, incy, A, lda)
C op \quad A \leftarrow \alpha xy^T + A
F77 op \quad A^T \leftarrow \alpha xy^T + A^T
F77 call   CGERU(n, m, alpha, y, incy, x, incx, A, lda)

No extra storage or operations are required.

GERC

C call    cblas_sgerc(CblasRowMajor, m, n, alpha, x, incx, y, incy, A, lda)
C op \quad A \leftarrow \alpha xy^H + A
F77 op \quad A^T \leftarrow \alpha(xy^H)^T + A^T = \alpha xy^T + A^T
F77 call   CGERU(n, m, alpha, y, incy, x, incx, A, lda)

Note that we need to allocate n-element workspace to hold the conjugated y, and we call GERU, not GERC.

HER

C call    cblas_scher(CblasRowMajor, CblasUpper, n, alpha, x, incx, A, lda)
C op \quad A \leftarrow \alpha xx^H + A
F77 op \quad A^T \leftarrow \alpha xx^T + A^T
F77 call   CHER('L', n, alpha, \tau, 1, A, lda)

Again, we have an n-element workspace and n extra operations.
HER2
C call cblascher2(BlasRowMajor, BlasUpper, n, α, x, incx, y, incy, A, lda)
C op \[ A \leftarrow \alpha xy^H + y(\alpha x)^H + A \]
F77 op \[ A^T \leftarrow \alpha \overline{y} x^T + \overline{\alpha x} y^T + A^T = \alpha \overline{y}(x)^H + \overline{x}(\alpha y)^H + A^T \]
F77 call CHER2('L', n, \alpha, y, 1, x, 1, A, lda)

So we need \(2n\) extra workspace and operations to form the conjugates of \(x\) and \(y\).

SYR
C call cblasssyr(BlasRowMajor, BlasUpper, n, α, x, incx, A, lda)
C op \[ A \leftarrow \alpha xx^T + A \]
F77 op \[ A^T \leftarrow \alpha x^T x^T + A^T \]
F77 call SSYR('L', n, \alpha, x, incx, A, lda)

No extra storage or operations required.

SYR2
C call cblasssyr2(BlasRowMajor, BlasUpper, n, α, x, incx, y, incy, A, lda)
C op \[ A \leftarrow \alpha xy^T + yz^T + A \]
F77 op \[ A^T \leftarrow \alpha \overline{y} x^T + \alpha x^T \overline{y} + A^T \]
F77 call SSYR2('L', n, \alpha, y, incy, x, incx, A, lda)

No extra storage or operations required.
B.2. C INTERFACE TO THE LEGACY BLAS

GEMM

C call: cblas_gemm(BlasRowMajor, BlasNoTrans, BlasNoTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha AB + \beta C$
F77 op: $C^T \leftarrow \alpha B^T A^T + \beta C^T$
F77 call: Cgemm( 'N', 'N', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasNoTrans, BlasTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A B^T + \beta C$
F77 op: $C^T \leftarrow \alpha A^T B + \beta C^T$
F77 call: Cgemm( 'C', 'N', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasNoTrans, BlasConjTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A^T B + \beta C$
F77 op: $C^T \leftarrow \alpha A^T B + \beta C^T$
F77 call: Cgemm( 'T', 'N', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasTrans, BlasNoTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A^T B + \beta C$
F77 op: $C^T \leftarrow \alpha A^T B + \beta C^T$
F77 call: Cgemm( 'T', 'N', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasConjTrans, BlasNoTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A^T B^H + \beta C$
F77 op: $C^T \leftarrow \alpha A^T B^H + \beta C^T$
F77 call: Cgemm( 'C', 'N', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasConjTrans, BlasTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A^H B + \beta C$
F77 op: $C^T \leftarrow \alpha A^H B^T + \beta C^T$
F77 call: Cgemm( 'N', 'C', n, m, k, α, B, ldb, A, lda, β, C, ldc)

C call: cblas_gemm(BlasRowMajor, BlasConjTrans, BlasConjTrans, m, n, k, α, lda, B, ldb, β, C, ldc)
C op: $C \leftarrow \alpha A^H B^H + \beta C$
F77 op: $C^T \leftarrow \alpha A^H B^T + \beta C^T$
F77 call: Cgemm( 'C', 'C', n, m, k, α, B, ldb, A, lda, β, C, ldc)
### SYMM/HEMM

<table>
<thead>
<tr>
<th>C call</th>
<th>cblas chemm(CblasRowMajor, CblasLeft, CblasUpper, m, n, α, A, lda, B, ldb, β, C, ldc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C ← αAB + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αBT A^T + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CHEMM('R', 'L', n, m, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C call</td>
<td>cblas chemm(CblasRowMajor, CblasRight, CblasUpper, m, n, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C</td>
<td>C ← αBA + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αA^T B + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CHEMM('L', 'L', n, m, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
</tbody>
</table>

### SYRK

<table>
<thead>
<tr>
<th>C call</th>
<th>cblas csyrk(CblasRowMajor, CblasUpper, CblasNoTrans, n, k, α, A, lda, β, C, ldc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C ← αAA^T + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αAA^T + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CSYRK('L', 'T', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C call</td>
<td>cblas csyrk(CblasRowMajor, CblasUpper, CblasTrans, n, k, α, A, lda, β, C, ldc)</td>
</tr>
<tr>
<td>C</td>
<td>C ← αA^T A + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αA^T A + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CSYRK('L', 'N', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
</tbody>
</table>

In reading the above descriptions, it is important to remember a few things. First, the symmetric matrix is \( C \), and thus we change UPL0 to accommodate the differing storage of \( C \). TRANSPOSE is then varied to handle the storage effects on \( A \).

### HERK

<table>
<thead>
<tr>
<th>C call</th>
<th>cblas cherk(CblasRowMajor, CblasUpper, CblasNoTrans, n, k, α, A, lda, β, C, ldc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C ← αAA^H + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αAA^T + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CHERK('L', 'C', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C call</td>
<td>cblas cherk(CblasRowMajor, CblasUpper, CblasConjTrans, n, k, α, A, lda, β, C, ldc)</td>
</tr>
<tr>
<td>C</td>
<td>C ← αA^H A + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αA^T A + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CHERK('L', 'N', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
</tbody>
</table>

### SYR2K

<table>
<thead>
<tr>
<th>C call</th>
<th>cblas csyr2k(CblasRowMajor, CblasUpper, CblasNoTrans, n, k, α, A, lda, B, ldb, β, C, ldc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>C ← αAB^T + αBA^T + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αBA^T + αAB^T + βCT = αAB^T + αBA^T + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CSYR2K('L', 'T', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C call</td>
<td>cblas csyr2k(CblasRowMajor, CblasUpper, CblasTrans, n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
<tr>
<td>C</td>
<td>C ← αA^T B + αB^T A + βC</td>
</tr>
<tr>
<td>F77 op</td>
<td>( C^T \leftarrow αB^T A + αA^T B + βCT = αA^T B + αB^T A + βCT )</td>
</tr>
<tr>
<td>F77 call</td>
<td>CSYR2K('L', 'N', n, k, α, A, lda, B, ldb, β, C, ldc)</td>
</tr>
</tbody>
</table>
Note that we once again wind up with an operation that looks the same from C and Fortran 77, saving that the C operations wishes to form $C^T$, instead of $C$. So once again we flip the setting of UPL0 to handle the difference in the storage of $C$. We then flip the setting of TRANS to handle the storage effects for $A$ and $B$.

**HER2K**

C call \( \text{cblas cher2k} \)(CblasRowMajor, CblasUpper, CblasNoTrans, n, k, α, A, lda, B, ldb, β, C, ldc)

C op \( C \leftarrow αAB^H + βC \)

F77 op \( C^T \leftarrow αBT + βC^T = αAB^T + βC \)

F77 call \( \text{HER2K('L', 'C', n, k, A, lda, B, ldb, β, C, ldc)} \)

**TRMM/TRSM**

Because of their identical use of the SIDE, UPL0, and TRANSA parameters, TRMM and TRSM share the same general analysis. Remember that $A$ is a triangular matrix, and thus when we handle its storage by flipping UPL0, we implicitly change its TRANS setting as well. With this in mind, we have:

C call \( \text{cblas ctrmm} \)(CblasRowMajor, CblasLeft, CblasUpper, CblasNoTrans, diag, m, n, α, A, lda, B, ldb)

C op \( B \leftarrow αA \)

F77 op \( BT \leftarrow αA'B^T \)

F77 call \( \text{TRMM('R', 'L', 'N', diag, m, n, α, A, lda, B, ldb)} \)

C call \( \text{cblas ctrmm} \)(CblasRowMajor, CblasLeft, CblasUpper, CblasNoTrans, diag, m, n, α, A, lda, B, ldb)

C op \( B \leftarrow αA'T \)

F77 op \( BT \leftarrow αA'B^T \)

F77 call \( \text{TRMM('R', 'L', 'T', diag, m, n, α, A, lda, B, ldb)} \)

Banded routines

The above techniques can be used for the banded routines only if a C (row-major) banded array has some sort of meaning when expanded as a Fortran banded array. It turns out that when this is done, you get the transpose of the C array, just as in the dense case.

In Fortran 77, the banded array is an array whose rows correspond to the diagonals of the matrix, and whose columns contain the selected portion of the matrix column. To rephrase this, the diagonals of the matrix are stored in strided storage, and the relevant pieces of the columns of the matrix are stored in contiguous memory. This makes sense: in a column-based algorithm, you will want your columns to be contiguous for efficiency reasons.

In order to ensure our columns are contiguous, we will structure the banded array as shown below. Notice that the first $K_U$ rows of the array store the superdiagonals, appropriately spaced.
to line up correctly in the column direction with the main diagonal. The last $K_L$ rows contain the
subdiagonals.

| -------- | Super diagonal KU |
| -------- | Super diagonal 2 |
| -------- | Super diagonal 1 |
| -------- | main diagonal (D) |
| -------- | Sub diagonal 1 |
| -------- | Sub diagonal 2 |
| -------- | Sub diagonal $K_L$ |

If we have a row-major storage, and thus a row-oriented algorithm, we will similarly want our rows to be contiguous in order to ensure efficiency. The storage scheme that is thus dictated is shown below. Notice that the first $K_L$ columns store the subdiagonals, appropriately padded to line up with the main diagonal along rows.

$$
\begin{array}{cccc}
K_L & D & KU \\
\mid & & & \\
\mid & & & \\
\mid & & & \\
\mid & & & \\
\mid & & & \\
\mid & & & \\
\end{array}
$$

Now, let us contrast these two storage schemes. Both store the diagonals of the matrix along the non-contiguous dimension of the matrix. The column-major banded array stores the matrix columns along the contiguous dimension, whereas the row-major banded array stores the matrix rows along the contiguous storage.

This gives us our first hint as to what to do: rows stored where columns should be, indicated, in the dense routines, that we needed to set a transpose parameter. We will see that we can do this for the banded routines as well.

We can further note that in the column-major banded array, the first part of the non-contiguous dimension (i.e. the first rows) store superdiagonals, whereas the first part of the non-contiguous dimension of row-major arrays (i.e., the first columns) store the subdiagonals.

We now note that when you transpose a matrix, the superdiagonals of the matrix become the subdiagonals of the matrix transpose (and vice versa).

Along the contiguous dimension, we note that we skip $K_U$ elements before coming to our first entry in a column-major banded array. The same happens in our row-major banded array, except that the skipping factor is $K_L$.

All this leads to the idea that when we have a row-major banded array, we can consider it as a transpose of the Fortran 77 column-major banded array, where we will swap not only $m$ and $n$, but also $K_U$ and $K_L$. An example should help demonstrate this principle. Let us say we have the matrix $A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 \end{bmatrix}$

If we express this entire array in banded form (a fairly dumb thing to do, but good for example purposes), we get $K_U = 3$, $K_L = 1$. In row-major banded storage this becomes:

$$
C_b = \begin{bmatrix} X & 1 & 3 & 5 & 7 \\ 2 & 4 & 6 & 8 & X \end{bmatrix}
$$
So, we believe this should be the transpose if interpreted as a Fortran 77 banded array. The matrix transpose, and its Fortran 77 banded storage is shown below:

\[
A^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix} \Rightarrow F_b = \begin{bmatrix} X & 2 \\ 1 & 4 \\ 3 & 6 \\ 5 & 8 \\ 7 & X \end{bmatrix}
\]

Now we simply note that since \( C_b \) is row major, and \( F_b \) is column-major, they are actually the same array in memory.

With the idea that row-major banded matrices produce the transpose of the matrix when interpreted as column-major banded matrices, we can use the same analysis for the banded BLAS as we used for the dense BLAS, noting that we must also always swap \( K_U \) and \( K_L \).

Packed routines

Packed routines are much simpler than banded. Here we have a triangular, symmetric or Hermitian matrix which is packed so that only the relevant triangle is stored. Thus if we have an upper triangular matrix stored in column-major packed storage, the first element holds the relevant portion of the first column of the matrix, the next two elements hold the relevant portion of the second column, etc.

With an upper triangular matrix stored in row-major packed storage, the first \( N \) elements hold the first row of the matrix, the next \( N - 1 \) elements hold the next row, etc.

Thus we see in the Hermitian and symmetric cases, to get a row-major packed array correctly interpreted by Fortran 77, we will simply switch the setting of \texttt{UPLO}. This will mean that the rows of the matrix will be read in as the columns, but this is not a problem, as we have seen before. In the symmetric case, since \( A = A^T \) the column and rows are the same, so there is obviously no problem. In the Hermitian case, we must be sure that the imaginary component of the diagonal is not used, and it assumed to be zero. However, the diagonal element in a row when our matrix is upper will correspond to the diagonal element in a column when our matrix is called lower, so this is handled as well.

In the triangular cases, we will need to change both \texttt{UPLO} and \texttt{TRANS}, just as in the dense routines.

With these ideas in mind, the analysis for the dense routines may be used unchanged for packed.
Annex C

Journal of Development
C.1 Environmental Routine for Effective use of Cache, Pipelining and Registers

C.1.1 Introduction

It is well known that effective use of cache and registers can make a substantial difference in the performance of codes written to do the core algorithms of linear algebra. Existing computer languages could make it possible to write portable code without losing efficiency by providing a set of functions to probe the system parameters. A user would then construct a program to use the parameters to develop an optimal implementation on the target computer. With many optimizing compilers, a preprocessor could be used to get much of these benefits. It would not be expected to do as good a job, and would still require some selection of parameters for each new machine. The ideas apply to PC's, workstations, and to the code running on a single node of most new parallel machines.

Computers have been evolving in such a way that arithmetic speeds far exceed the rate with which operands from the main memory can be obtained for processing. Because it is cost effective to do so, many new parallel machines are using microprocessors for which this is the case. Since it is frequently the case that when something is used, it is going to be used again reasonably soon, systems are designed to use one or more levels of cache, (i.e. fast memory) where data used recently can be reused more quickly. This approach works quite well for most users, but when working with large dense matrices, data is never in the cache when needed. This can easily cost a factor of two or three in performance and in many cases significantly more.

We regard a computer's registers as a special case of a very small cache, since operations done on registers proceed more quickly than those using an operand that is in the fastest cache. (RISC architectures require that operands be in the registers.) But the differences between registers and cache memory require that different mechanisms be defined to make most effective use of both.

At every level of cache (including the registers), one wants to do as much computing as possible before the data in the cache is flushed for other data references. There is great scope for cleverness here in the design of algorithms. Since matrix multiplication is simple to understand we use it as an example to clarify how the ideas presented here might be used.

We believe that with the kind of features described here, it would not be difficult for those who know about such things to write compilers that would make it possible for those who know about such things to write portable code with no significant loss of efficiency. In addition it would be possible to write applications that can take advantage of these parameters and enhance the performance on a wide range of applications.

The most notable effort to deal with these problems is the Level 3 BLAS,[24] (it should be noted that we make no pretense to discussing all the factors that may be important to obtaining high performance on modern processors.

C.1.2 Language Extensions for the Cache

The cache system of a computer can be characterized by the following:

We expect $\sigma_{L} > \sigma_{L-1}$ and $\tau_{L} > \tau_{L-1}, L = 1, 2, \ldots, kache$, where we adopt the convention that $L_{0}$ corresponds to these values for the registers. We define $\sigma_{kache}$ to be the largest amount of memory available, which in the case of virtual memory includes disk space. We make no further reference to the $\tau$'s, although there are certainly cases when such information might be of use.

In the case of Fortran, these values could be provided by environmental intrinsic functions, which take an argument of the type for which space is desired. In the case of C, these could be provided as part of the standard math.h header file. In the case of compilers being used in environments with different cache characteristics, it would be useful to have some means to specify
this information in a configuration file. Finally if this were done using a preprocessor, one would provide the information for the system being used to the preprocessor.

Extensions that would allow one to gain the efficiency using the BLAS without writing machine specific code.

C.1.3 For Efficient LA Software

There are at least two things that might be done. Provide information through an interface about the cache structure of the machine, number of caches, their sizes, their access times, the size of the cache line, issues connected with pipelining, etc. Allow the programmer to declare variables and arrays (very small arrays, but still arrays) that are to be kept in registers. It is up to the compiler to pick the size of the arrays, and to unwind loops that refer to such “register” arrays. The compiler makes available the size of the arrays that it selects so that the programmer can reference them for purposes of writing loops, or for any other purpose.

C.1.4 Advantages of this approach

Code need only be written once, no dependence on computer vendors, or when a machine first comes out efficient implementations of important software will be available. If one has the stomach for it, it should be possible to write algorithms exactly as you would like to have them. Thus for example, one doesn’t have to organize things to use a matrix multiplication if that is not the most effective way to do things.

C.1.5 Disadvantages of this approach

Code is significantly more difficult to write. It may require some cooperation from compiler vendors. What is proposed here can be done by a preprocessor if one can supply it with the cache information (probably not too hard) and the size that would be appropriate for the register arrays. This can be determined on a machine by machine basis by trying a few possibilities.

<table>
<thead>
<tr>
<th>Characterizing a Cache System</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of levels in the cache architecture.</td>
</tr>
<tr>
<td>The size of the L(^{th}) cache.</td>
</tr>
<tr>
<td>The access time to get data from the L(^{th}) cache divided by the time to copy one floating point register to another.</td>
</tr>
<tr>
<td>Number of consecutive items that get replaced when a new item is fetched to cache L.</td>
</tr>
<tr>
<td>Cache mapping: set associatively, direct</td>
</tr>
</tbody>
</table>
### C.1. ENVIRONMENTAL ROUTINE FOR EFFECTIVE USE OF CACHE, PIPELINING AND REGISTERS

<table>
<thead>
<tr>
<th>Machine Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of floating point registers</td>
</tr>
<tr>
<td>Number of floating point units</td>
</tr>
<tr>
<td>Number of caches</td>
</tr>
<tr>
<td>Cache size</td>
</tr>
<tr>
<td>Type of cache e.g. 2 way set associative, least recently used</td>
</tr>
<tr>
<td>Cache line size</td>
</tr>
<tr>
<td>Cycle time</td>
</tr>
<tr>
<td>Page size</td>
</tr>
<tr>
<td>Size of TLB (translation look-aside buffer)</td>
</tr>
<tr>
<td>Cycles for floating point operations, add, multiply, division</td>
</tr>
<tr>
<td>Number of processors</td>
</tr>
<tr>
<td>Fused floating point ops; multiply/add</td>
</tr>
<tr>
<td>Cycles from memory to stages in the cache</td>
</tr>
<tr>
<td>Pre-fetch and how many outstanding requests</td>
</tr>
<tr>
<td>Bandwidth from/to memory</td>
</tr>
<tr>
<td>Latency from memory/cache</td>
</tr>
<tr>
<td>Number of instructions issued per cycle</td>
</tr>
</tbody>
</table>

#### Five Parameters Associated with Memory Hierarchy

- **Access Time**: Time for the CPU to fetch a value from memory – including delays through any intermediate levels.
- **Memory Size**: The amount of memory of a given type in a system.
- **Cost Per Unit (byte)**: Cost per unit times size roughly equals total cost.
- **Transfer Bandwidth**: Units (bytes) per second transferred to the next level.
- **Unit of Transfer**: Number of units moved between adjacent levels in a single move.

#### Memory Hierarchy

- **Desired**: no cost, very fast, large non-volatile
- **Actual Registers**: cache, DRAM, Disk, tape, CD, Flash & EPROM
- **Registers**: fast, local, volatile, VERY expensive, very small
- **Cache**: fast, expensive, volatile, small
- **DRAM**: medium size & speed, volatile
- **Distributed Memory**
- **Disk**: slow, low cost per byte, non-volatile
- **Tape**: very slow, very low cost, durable, non-volatile
- **CD**: slow, "read only", good data density, non-volatile
- **Flash & EPROM**: small, not as fast a DRAM, non-volatile
C.2 Distributed-memory Dense BLAS

This document summarizes the discussions that took place during the meetings of the BLAS Technical Forum concerning the distributed-memory BLAS. The committee did not reach an agreement on how to specify an interface for such a set of routines, however it was felt that this document should keep a record of those discussions.

There has been much interest in the past few years in developing versions of the BLAS for distributed-memory computers [50, 28, 3, 29, 13, 17, 15, 18, 8, 51]. Some of this research proposed parallelizing the BLAS [45, 50, 17, 15, 11, 18, 49, 51], and some implemented a few important BLAS routines [50, 43, 17, 15, 11, 18, 49, 51], such as matrix-matrix multiplication [31, 5, 36, 44, 4, 16, 52] or triangular system solve [34, 40, 41, 10].

Based on this research work, it was agreed that an interface for the distributed-memory BLAS should have the following features:

- The calling sequence definitions should be simple and similar in all targeted programming languages.
- The interface should be effective for the development of large and high-quality dense linear algebra software for distributed-memory computers.
- The interface should permit broad functionality to enable, facilitate and encourage the development of current and related research projects.

The main advantages of establishing a distributed-memory dense BLAS standard are portability and ease-of-use. In a distributed-memory environment in which the higher level routines and/or abstractions are built upon lower level message-passing and computational routines the benefits of standardization are particularly apparent. Furthermore, the definition of distributed-memory dense basic linear algebra subprograms provides vendors with a clearly defined base set of routines that they can implement efficiently, or in some cases provide hardware support for, thereby enhancing scalability.

The goal of the distributed-memory dense BLAS interface simply stated should be to develop a widely used standard for writing message-passing programs performing dense basic linear algebra operations. As such the interface should establish a practical, portable, efficient and flexible standard for distributed-memory dense basic linear algebra operations.

A complete list of goals follows.

- Design an Application Programming Interface (API) (not necessarily for compilers or a system implementation library) well suited for distributed-memory dense basic linear algebra computations.
- Allow efficient communication and computation: minimizing communication startup overhead and volume, while maximizing load balance and local computational performance.
- Allow for re-use of existing message-passing interface standard [30] as well as local basic linear algebra computational kernels such as the *de facto* standard BLAS.
- Allow for implementations that can be used in a heterogeneous environment.
- Define an interface that is not too different from current practice and provide extensions that allow greater flexibility.
C.2. DISTRIBUTED-MEMORY DENSE BLAS

- Define an interface that can be implemented on many vendors' platforms, with no significant changes in the underlying system software.

- Semantics of the interface should be language-and data-distribution independent.

- The interface should be designed to allow for thread-safety.

Such a distribute-memory BLAS standard should be intended for use by all those who want to write portable programs performing dense linear algebra operations in Fortran 77, Fortran 90, High Performance Fortran (HPF), C or C++. This includes individual application programmers, developers of dense linear algebra software designed to run on parallel machines, and creators of computational environment and tools. In order to be attractive to this wide audience, the standard must provide a simple, easy-to-use interface for the basic user while not semantically precluding the high-performance computation and communication operations available on advanced computers.

The attractiveness of the distributed-memory dense BLAS at least partially stems from its wide portability as well as the common occurrence of dense linear algebra operations in numerical simulations. These programs may run on distributed-memory multiprocessors, networks or clusters of workstations, and combinations of all of these. In addition, shared-memory implementations are possible. The message passing paradigm will not be made obsolete by architectures combining the shared- and distributed-memory views, or by increases in network speeds. It thus should be both possible and useful to implement such a standard on a great variety of machines, including those ''machines'', parallel or not, connected by a communication network.

The distributed-memory dense BLAS interface should provide many features intended to improve performance on scalable parallel computers with specialized interprocessor communication hardware. Thus, we expect that native, high-performance implementations of this interface could be provided on such machines. At the same time, implementations of such a standard on top of MPI or PVM will provide portability to workstation clusters and heterogeneous networks of workstations.

During the discussions it was agreed that the distributed-memory BLAS should include

- A set of basic dense linear algebra computational operations

- Data-redistribution operations

- Environmental management and inquiry

- Bindings for various widely used programming languages, including high level languages such as High Performance Fortran (HPF)

The distributed-memory dense BLAS interface should specify routines that operate on in-core dense matrices. On entry, these routines assume that the data has been distributed on the processors according to a specific data decomposition scheme that dictates the local storage of the data when it resides in the processors' memory. The data layout information as well as the local storage scheme for these different matrix operands is conveyed to the routines via a descriptor that could be a simple array of integers. The standard could mandate that the first entry of this array identifies the type of the descriptor, i.e., the data distribution scheme it describes. This allows to specify the distributed-memory dense BLAS interface independently from the data distribution.

The distributed-memory dense BLAS are executed by processes, rather than physical processors. In general there may be several processes running on a processor, in which case it is assumed that the runtime system handles the scheduling of processes. In the absence of such a runtime system, the distributed-memory dense BLAS assume one process per processor. A process is defined as a
basic unit or thread of execution that minimally includes a stack, registers, and memory. Multiple processes may share a physical processor. The term processor refers to the actual hardware. Each process is treated as if it were a processor: a process executing a program or subprogram calling the distributed-memory dense BLAS must exist for the lifetime of the program's or subprogram's run. Its execution should affect other processes' execution only through the use of message-passing calls. With this in mind, the term process is used in all sections of this chapter unless otherwise specified.

The distributed-memory dense BLAS are thus executed by a collection of processes, that are enclosed in a communication context. Similarly, a communication context or simply context is associated with every global matrix. The use of a context provides the ability to have separate "universes" of message passing. This means that a collection of processes can safely communicate even if other (possibly overlapping) sets of processes are also communicating. Thus, a context is a powerful mechanism for avoiding unintentional nondeterminism in message passing and provides support for the design of safe, modular software libraries. In MPI, this concept is referred to as a communicator.

A context partitions the communication space. A message sent from one context cannot be received in another context. The use of separate communication contexts by distinct libraries (or distinct library routine invocations) insulates communication internal to a specific library routine from external communication that may be going on within the user's program.

In most respects, the terms process collection and context can be used interchangeably. For example, one may say that an operation is performed "in context X" or "in process collection X". The slight difference here is that the user may define two identical sets of processes (say, two $1 \times 3$ process grids, both of which use processes 0, 1, and 2), but each will be enclosed in its own context, so that they are distinct in operation, even though they are indistinguishable from a process collection standpoint.

Another example of the use of context might be to define a normal two-dimensional process grid within which most computation takes place. However, in certain portions of the code it may be more convenient to access the processes as a one-dimensional process grid, whereas at other times one may wish, for instance, to share information among nearest neighbors. In such cases, one will want each process to have access to three contexts: the two-dimensional process grid, the one-dimensional process grid, and a small process grid that contains the process and its nearest neighbors. Therefore, communication contexts allow one to

- create arbitrary groups of processes;
- create an indeterminate number of overlapping and/or disjoint collections of processes, and
- isolate a set of processes so that communication interference does not occur.

A distributed-memory dense BLAS function should create a grid of processes and its enclosing context. This routine returns a context handle, which is a simple integer, assigned by the message-passing library used by a given implementation of the distributed-memory dense BLAS to identify the communication context. Subsequent distributed-memory dense BLAS will be passed these handles, which allow to determine from which context/process collection a routine is being called. The user should never alter or change these handles; they are opaque data objects that are only meaningful for the distributed-memory dense BLAS routines.

A defined context consumes resources. It is therefore advisable to release contexts when they are no longer needed. When the entire distributed-memory dense BLAS system is shut down, all outstanding contexts are automatically freed.
Some systems, such as MPI, supply their own version of context. For portability reasons, one
thus cannot assume that the communication contexts used by the user’s program are usable by the
distributed-memory dense BLAS. Therefore, the following interface allows to form a distributed-
memory dense BLAS context in reference to a user’s context.

The standard could mandate that the second entry of each descriptor is set to the value of the
communication context identifying the collection of processes onto which the data is distributed.

The routines of the distributed-memory dense BLAS could require that all global data (vectors
or matrices) be distributed across the processes prior to invoking the routines. The data layout
and local storage scheme are specified by a particular implementation, but are strictly speaking
not part of the standard. Global data is mapped to the local memories of processes assuming an
implementation-dependent data distribution scheme.

A descriptor is associated with each global array. This descriptor stores the information
required to establish the mapping between each global array entry and its corresponding process
and memory location. Array descriptors corresponding to distinct layouts are differentiated by
their first entry called the type of the descriptor. The data residing in the processes’ memory
is specified by a pointer. Splitting the local data from the layout’s description allows to specify
language-dependent interfaces for programming languages providing only simple data structures
such as Fortran 77 without affecting the functionality or ease-of-use of the interface.

Most of the distributed-memory dense BLAS should operate within the same communication
context, i.e., all distributed operands should be distributed on the same process grid. These op-
erations are said to be intra-context operations. Only, very few distributed-memory dense BLAS
perform inter-context operations, in which case this feature should clearly be mentioned in the
procedure functionality.

Furthermore, all distributed operands involved in the operation should be distributed accordin-
gly to the same decomposition scheme. In other words, the type entry of each descriptor must
be equal.

All distributed-memory dense BLAS operations could be collective, that is, all processes in
the context need to invoke the procedure even if certain processes are not involved in the operation.
This situation may happen for example when some processes don’t own any data to be operated
on.

The distributed-memory dense BLAS manages system memory that is used for buffering
messages and for storing internal representations of various distributed-memory dense BLAS objects
such as communication contexts, local arrays of data, etc. This memory is not directly accessible
to the user, and objects stored there are either private or opaque: their size and shape is not
visible to the user. Opaque objects are accessed via handles, which exist in user space. Private
objects cannot be accessed by the user, and are allocated and released within the same routine.

Opaque objects are allocated and deallocated by calls that are specific to each object type.
These are listed in the sections where the object are described. The calls accept a handle argument
of matching type. In an allocate call this is an output argument that returns a valid reference to
the object. In a call to deallocate this is an input/output argument which returns with an “invalid
handle” constant for each object type. Comparisons to this constant are used to test for validity
of the handle.
A call to deallocate invalidates the handle and marks the object for deallocation. The object is not accessible to the user after the call. However, distributed-memory dense BLAS need not deallocate the object immediately. Any operation pending (at the time of the deallocate) that involves this object will complete normally; the object will be deallocated afterwards.

An opaque object and its handle are significant only at the process where the object was created, and cannot be transferred to another process.

This design hides the internal representation used for distributed-memory dense BLAS internal data structures, thus allowing similar calls in C and Fortran. It also avoids conflicts with the typing rules in these languages, and easily allows for future extensions of functionality. Note that the objects handles defined in the distributed-memory dense BLAS are exclusively used by the underlying message passing library. The user data itself remains directly accessible in the user's program.

The explicit separating of handles in user space, objects in system space, allows space-reclaiming, deallocation calls to be made at appropriate points in the user program. If the opaque objects were in user space, one would have to be very careful not to go out of scope before any pending operation requiring that object completed. The specified design allows an object to be marked for deallocation, the user program can then go out of scope, and the object itself still persists until any pending operations are complete. Again such a design cannot be applied in general to the user's data.

The requirement that handles support assignment/comparison is made since such operations are common. This restricts the domain of possible implementations. Moreover, such a design has been adopted by most message passing library interfaces such as the MPI.

The intended semantics of opaque objects is that each opaque object is separate from each other; each call to allocate such an object copies copies all the information required for that object. Implementations may avoid excessive copying by substituting referencing for copying. For example, a derived datatype may contain references to its components, rather then copies of its components. In such cases, the implementation must maintain reference counts, and allocate and deallocate objects such that the visible is as if the objects were copied. Such a design is particularly suitable for communication contexts, because the amount of data one has to keep track of is small. However, applying the same concept to the user's data forces the introduction of routines to manage logical templates, adding complexity, and was therefore ruled out.

There are several important language bindings issues not addressed by this document. This section does not discuss the interoperability of message passing between languages. It is fully expected that many implementations should have such features.

A descriptor is associated with each distributed matrix. The entries of the descriptor uniquely determine the mapping of the matrix entries onto the local processes' memories. Since vectors may be seen as a special case of distributed matrices or proper submatrices, the larger scheme just defined encompasses their description as well.

The local storage convention of the distributed matrix operands in every process's memory does not need to be specified by the standard. It is however recommended that convenient data structure are chosen by a given implementation allowing to rely on the sequential BLAS to perform the local computations within a process.

The distributed-memory dense BLAS should not provide mechanisms for dealing with failures in the communication and computation systems. If the distributed-memory dense BLAS is built on an unreliable underlying mechanism, then it is the job of the implementor(s) of the distributed-memory dense BLAS subsystem to insulate the user from this unreliability, or to reflect unrecoverable errors as failures. Whenever possible, such failures will be reflected as errors in the relevant communication or computation call.
Of course, distributed-memory dense BLAS programs can still be erroneous. A program error can occur when a distributed-memory dense BLAS routine is called with an invalid value for any of its arguments. The routine must report this fact and terminate the execution of the program. Each routine, on detecting an error, should call a common error-handling routine, passing to it the current communication context, the name of the routine and the number of the first argument that is in error. For efficiency purposes, the distributed-memory dense BLAS only perform a local validity check of their argument list. If an error is detected in at least one process of the current context, the program execution is stopped.

A global validity check of the input arguments passed to a distributed-memory dense BLAS routine must be performed in the user-level calling procedure. To demonstrate the need and cost of global checking, as well as the reason why this type of checking should not be performed in the distributed-memory dense BLAS, consider the following example: the value of a global input argument is legal but differs from one process to another. The results are unpredictable. In order to detect this kind of error situation, a synchronization point would be necessary, which may result in a significant performance degradation. Since every process must call the same routine to perform the desired operation successfully, it is natural and safe to restrict somewhat the amount of checking operations performed in the distributed-memory dense BLAS routines.

Specialized implementations may call system-specific exception-handling facilities, either via an auxiliary routine or directly from the routine. In addition, the testing programs can take advantage of this exception-handling mechanism by simulating specific erroneous input argument lists and then verifying that particular errors are correctly detected.

Resource errors can also occur when a program exceeds the amount of available system resources. The occurrence of this type of error depends on the amount of available resources in the system and the resource allocation mechanism used; this may differ from system to system. A high-quality implementation will provide generous limits on the important resources so as to alleviate the portability problem this represents.
C.3 Fortran 95 Thin BLAS

C.3.1 Introduction

This paper presents a proposal for a specification of Fortran 95 thin BLAS.

A preliminary version of the F90 Blas proposal has been circulated informally (but not very widely) for about 4 years, and code which implements that version has been available in the Fortran 90 software repository maintained by NAG Ltd (http://www.nag.co.uk).

This proposal is designed to cover — as far as seems sensible — roughly the same functionality as the original Level 1, 2 and 3 (Fortran 77) BLAS. It does not address sparse matrices or vectors, nor does it explicitly address issues of parallel computation.

Many of the Fortran 77 Level 1 BLAS can be replaced by simple array expressions and assignments in Fortran 95, without loss of convenience or performance (assuming a reasonable degree of optimisation by the compiler). Fortran 95 also allows groups of related Level 2 and Level 3 BLAS to be merged together, each group being covered by a single interface.

C.3.2 Design of Fortran 95 Interfaces

Our proposed design utilizes the following features of the Fortran 95 language.

**Generic interfaces:** all routines are accessed through *generic* interfaces. A single generic name covers several specific instances whose arguments may differ in the following properties:

- **data type** (real or complex). However, the relevant arguments must be either all real or all complex. (We do not, for example, cater for multiplying a real matrix by a complex matrix, though this functionality could easily be added to the design if there was a need for it.)

- **precision** (or equivalently, kind-value). However, all real or complex arguments must have the same precision.

- **rank** Some arguments may either have rank 2 (to store a matrix) or rank 1 (to store a vector).

- **different argument list** Some of the arguments may not appear in a specific instance. In this case a pre-defined value or a pre-defined action is assumed. The following table contains the pre-defined value or action for the argument that may not be used. Some of these arguments are key arguments that will be described later.

<table>
<thead>
<tr>
<th>argument</th>
<th>value or action if the argument is not used</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>1.0 or (1.0,0.0)</td>
</tr>
<tr>
<td>beta</td>
<td>0.0 or (0.0,0.0)</td>
</tr>
<tr>
<td>op_x</td>
<td>operate with x</td>
</tr>
<tr>
<td>lower</td>
<td>reference upper triangle only</td>
</tr>
<tr>
<td>right_side</td>
<td>operate on the left-hand side</td>
</tr>
<tr>
<td>unit_diag</td>
<td>non-unit triangular</td>
</tr>
</tbody>
</table>

**Assumed-shape arrays:** all array arguments are *assumed-shape* arrays, which must have the exact shape required to store the corresponding matrix or vector. Hence arguments to specify array-dimensions or problem-dimensions are not required. The routines assume that the supplied arrays have valid and consistent shapes (see Section C.3.5). Zero dimensions (implying empty arrays) are allowed.
Key arguments: in the Fortran 77 BLAS, we use character arguments to specify different options for the operation to be performed. In this proposal we suggest using key arguments. A key argument is a dummy argument whose actual argument must be a named constant defined by BLAS. The following table lists the key arguments, the related BLAS named constants and the equivalent F77 BLAS values.

<table>
<thead>
<tr>
<th>dummy argument</th>
<th>named constant</th>
<th>meaning</th>
<th>F77 argument</th>
</tr>
</thead>
<tbody>
<tr>
<td>op_x</td>
<td>not used</td>
<td>operate with x</td>
<td>TRANSx = 'N'</td>
</tr>
<tr>
<td></td>
<td>blas_trans</td>
<td>operate with transpose x</td>
<td>TRANSx = 'T'</td>
</tr>
<tr>
<td></td>
<td>blas_conj</td>
<td>operate with conjugate x</td>
<td>TRANSx = 'C'</td>
</tr>
<tr>
<td></td>
<td>blas_conj_trans</td>
<td>operate with conjugate-transpose x</td>
<td>TRANSx = 'H'</td>
</tr>
<tr>
<td>lower</td>
<td>not used</td>
<td>reference upper triangle only</td>
<td>UPLO = 'U'</td>
</tr>
<tr>
<td></td>
<td>blas_lower</td>
<td>reference lower triangle only</td>
<td>UPLO = 'L'</td>
</tr>
<tr>
<td>right_side</td>
<td>not used</td>
<td>operate on the left-hand side</td>
<td>SIDE = 'L'</td>
</tr>
<tr>
<td></td>
<td>blas_right</td>
<td>operate on the right-hand side</td>
<td>SIDE = 'R'</td>
</tr>
<tr>
<td>unit_diag</td>
<td>not used</td>
<td>non-unit triangular</td>
<td>DIAG = 'N'</td>
</tr>
<tr>
<td></td>
<td>blas_unit_diag</td>
<td>unit triangular</td>
<td>DIAG = 'U'</td>
</tr>
</tbody>
</table>

C.3.3 Interfaces for Real Data

The primary aim of this paper is to convey the flavour of the different generic interfaces. Therefore we first describe the interfaces as they apply to real data. The extra complications which arise when they apply to complex data will be considered in Section C.3.4.

We summarize each interface in the form of a subroutine statement (or in one case a function statement), in which all the arguments might appear. (This is a convenient way to think of the interface, although such a statement using the generic interface name never appears in the code.) Arguments which need not be supplied are enclosed in square brackets, for example:

```fortran
subroutine trmm( [alpha,] a, [op_a,] b, [lower,] [right_side,] [unit_diag] )
```

This is followed by a table which lists the different variants of the operation, depending either on some of the arguments or on the key arguments.

The following table shows the values used in the tables and the related named constant for the key arguments.

<table>
<thead>
<tr>
<th>dummy argument</th>
<th>value in table</th>
<th>named constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>op_x</td>
<td>'T'</td>
<td>blas_trans</td>
</tr>
<tr>
<td></td>
<td>'C'</td>
<td>blas_conj</td>
</tr>
<tr>
<td></td>
<td>'C/T'</td>
<td>blas_conj_trans</td>
</tr>
<tr>
<td>right_side</td>
<td>'R'</td>
<td>blas_right</td>
</tr>
</tbody>
</table>

Routines using conventional storage for matrices

By conventional storage, we mean storing a matrix in a 2-dimensional array,

```fortran
subroutine gemm( [alpha,] a, [op_a,] b, [op_b], [beta,] c )
```
<table>
<thead>
<tr>
<th>rank of a</th>
<th>rank of b</th>
<th>rank of c</th>
<th>op_a</th>
<th>op_b</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td>( C \leftarrow \alpha AB + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td>'T'</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
<td>'T'</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td>'T'</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td>'T'</td>
<td></td>
</tr>
</tbody>
</table>

subroutine symm( [alpha,] a, b, [beta,] c, [lower,] [right_side] )
subroutine hemm( [alpha,] a, b, [beta,] c, [lower,] [right_side] )

where \( A \) is a symmetric matrix.

<table>
<thead>
<tr>
<th>rank of b</th>
<th>rank of c</th>
<th>right_side</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>'R'</td>
<td>( C \leftarrow \alpha AB + \beta C )</td>
<td>SYMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td>( C \leftarrow \alpha BA + \beta C )</td>
<td>SYMM</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>'T'</td>
<td>( c \leftarrow \alpha Ab + \beta c )</td>
<td>SYMV</td>
</tr>
</tbody>
</table>

where \( C \) is a symmetric matrix.

<table>
<thead>
<tr>
<th>rank of a</th>
<th>op_a</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>( C \leftarrow \alpha AA' + \beta C )</td>
<td>SYRK</td>
</tr>
<tr>
<td>2</td>
<td>'T'</td>
<td>( C \leftarrow \alpha AT A + \beta C )</td>
<td>SYRK</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>( C \leftarrow \alpha a AT + \beta C )</td>
<td>SYR1</td>
</tr>
</tbody>
</table>

where \( C \) is a symmetric matrix.

<table>
<thead>
<tr>
<th>rank of a</th>
<th>rank of b</th>
<th>op_a</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>( C \leftarrow \alpha AB + \alpha BA' + \beta C )</td>
<td>SYR2K</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td>( C \leftarrow \alpha AT B + \alpha B' AT + \beta C )</td>
<td>SYR2K</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>'T'</td>
<td>( C \leftarrow \alpha ab' + \alpha b + \beta C )</td>
<td>SYR2</td>
</tr>
</tbody>
</table>

where \( C \) is a symmetric matrix.

subroutine trmm( [alpha,] a, [op_a,] b, [lower,] [right_side,] [unit_diag] )

<table>
<thead>
<tr>
<th>rank of b</th>
<th>op_a</th>
<th>right_side</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>'T'</td>
<td>( B \leftarrow \alpha AB )</td>
<td>TRMM</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>'R'</td>
<td>( B \leftarrow \alpha A B )</td>
<td>TRMM</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>'T'</td>
<td>( B \leftarrow \alpha AB A' )</td>
<td>TRMM</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>'T'</td>
<td>( b \leftarrow \alpha Ab )</td>
<td>TRMV</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>'T'</td>
<td>( b \leftarrow \alpha A B )</td>
<td>TRMV</td>
</tr>
</tbody>
</table>

subroutine trsm( [alpha,] a, [op_a,] b, [lower,] [right_side,] [unit_diag] )
C.3. FORTRAN 95 THIN BLAS

<table>
<thead>
<tr>
<th>rank of b</th>
<th>op_a</th>
<th>right_side</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>( B \leftarrow \alpha A^{-1} B )</td>
<td>_TRSM</td>
</tr>
<tr>
<td>2</td>
<td>'T'</td>
<td></td>
<td>( B \leftarrow \alpha A^{-T} B )</td>
<td>_TRSM</td>
</tr>
<tr>
<td>2</td>
<td>'R'</td>
<td></td>
<td>( B \leftarrow \alpha BA^{-1} )</td>
<td>_TRSM</td>
</tr>
<tr>
<td>2</td>
<td>'T'</td>
<td>'R'</td>
<td>( B \leftarrow \alpha BA^{-T} )</td>
<td>_TRSM</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>( b \leftarrow \alpha A^{-1} b )</td>
<td>_TRSV</td>
</tr>
<tr>
<td>1</td>
<td>'T'</td>
<td></td>
<td>( b \leftarrow \alpha A^{-T} b )</td>
<td>_TRSV</td>
</tr>
</tbody>
</table>

Routines using packed storage for matrices

By *packed* storage, we mean storing the upper or lower triangle of a symmetric or triangular matrix in a 1-dimensional array (i.e. a vector).

subroutine spmv( [alpha,] a, b, [beta,] c, [lower] )
subroutine hpmv( [alpha,] a, b, [beta,] c, [lower] )

\[
\text{operation} \quad \frac{c \leftarrow \alpha Ab + \beta c}{\text{F77 BLAS}} \quad _{\text{SPMV}}
\]

where \( A \) is a symmetric matrix.

subroutine spr1( [alpha,] a, [beta,] c, [lower] )
subroutine hpr1( [alpha,] a, [beta,] c, [lower] )

\[
\text{operation} \quad \frac{C \leftarrow \alpha a^t + \beta C}{\text{F77 BLAS}} \quad _{\text{SPR1}}
\]

where \( C \) is a symmetric matrix.

subroutine syr2( [alpha,] a, b, [beta,] c, [lower] )
subroutine he2r( [alpha,] a, b, [beta,] c, [lower] )

\[
\text{operation} \quad \frac{C \leftarrow \alpha ab^t + \alpha ba^t + \beta C}{\text{F77 BLAS}} \quad _{\text{SYR2}}
\]

where \( C \) is a symmetric matrix.

subroutine tpmv( [alpha,] a, [op_a,] b, [lower,] [unit_diag] )

\[
\text{op_a} \quad \text{operation} \quad \text{F77 BLAS}
\]
\[
\text{'T'} \quad b \leftarrow \alpha Ab \quad _{\text{TPMV}}
\]

where \( A \) is a triangular matrix.

subroutine tpsv( [alpha,] a, [op_a,] b, [lower,] [unit_diag] )

\[
\text{op_a} \quad \text{operation} \quad \text{F77 BLAS}
\]
\[
\text{'T'} \quad b \leftarrow \alpha A^{-1} b \quad _{\text{TPSV}}
\]

where \( A \) is a triangular matrix.
Routines for band matrices

subroutine gbmv( [alpha,] a, [op_a,] b, [beta,] c, kd )

<table>
<thead>
<tr>
<th>op_a</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>'T'</td>
<td>C ← αAb + βc</td>
<td>_GBMV</td>
</tr>
</tbody>
</table>

where A is a general band matrix with kd superdiagonals supplied.

subroutine sbmv( [alpha,] a, b, [beta,] c, [lower ] )
subroutine hbbmv( [alpha,] a, b, [beta,] c, [lower ] )

<table>
<thead>
<tr>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>c ← αAb + βc</td>
<td>_SPMV</td>
</tr>
</tbody>
</table>

where A is a symmetric band matrix.

subroutine tbmv( [alpha,] a, [op_a,] , [lower,] [unit_diag] )

<table>
<thead>
<tr>
<th>op,a</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>'T'</td>
<td>b ← αAb</td>
<td>_TBMV</td>
</tr>
</tbody>
</table>

where A is a triangular band matrix.

subroutine tpsv( [alpha,] a, [op_a,] , [lower,] [unit_diag] )

<table>
<thead>
<tr>
<th>op,a</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>'T'</td>
<td>b ← αA⁻¹b</td>
<td>_TPSV</td>
</tr>
</tbody>
</table>

where A is a triangular band matrix.

Level 1 routines

function nrm2( x )

Operation: return \( \|x\|_2 \).

subroutine swap( x, y )

Operation: \( x \leftrightarrow y \).

subroutine rot( x, y, c, s )

subroutine rotg( a, b, c, s )
C.3. FORTRAN 95 THIN BLAS

C.3.4 Interfaces for Complex Data

In this section we show the subroutine gemm for complex arguments. The generic interface is that described for real arguments.

<table>
<thead>
<tr>
<th>rank of a</th>
<th>rank of b</th>
<th>rank of c</th>
<th>op_a</th>
<th>op_b</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>&quot;T&quot;</td>
<td></td>
<td>$C \leftarrow \alpha AB + \beta C$</td>
<td>_GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td></td>
<td>$C \leftarrow \alpha AB^H + \beta C$</td>
<td>_GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td></td>
<td>$C \leftarrow \alpha A^TB + \beta C$</td>
<td>_GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td>'T'</td>
<td>$C \leftarrow \alpha A^TB^T + \beta C$</td>
<td>_GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td>'C/T'</td>
<td>$C \leftarrow \alpha A^H B + \beta C$</td>
<td>_GEMM</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>'T'</td>
<td></td>
<td>$c \leftarrow \alpha Ab + \beta c$</td>
<td>_GEMV</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>'C/T'</td>
<td></td>
<td>$c \leftarrow \alpha A^T b + \beta c$</td>
<td>_GEMV</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td>$C \leftarrow \alpha ab^T + \beta C$</td>
<td>_GERU</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
<td>$C \leftarrow \alpha ab^H + \beta C$</td>
<td>_GERC</td>
</tr>
</tbody>
</table>

C.3.5 Error checking

We propose that the Fortran 95 thin BLAS perform no checks on their arguments.

C.3.6 Comparison with the Fortran 77 BLAS

We consider in more detail each Level of BLAS in turn. In referring to particular operations performed by the BLAS, we use the traditional BLAS names, except that we omit the initial letter (S, D, C, Z) which indicates the data type — for example, SWAP. The resulting names are also the generic names which we propose for the Fortran 95 interfaces.

Level 1

We include in this proposal only the following:

SWAP
ROT
NRM2
ROTG

ROT and ROTG have been extended to cover complex rotations.

Level 2

We propose to combine many of the Level 2 BLAS with the corresponding Level 3 BLAS in a single generic interface, the different instances being distinguished by the ranks of some of the arguments. In order to do this, we propose to remove some minor inconsistencies between the specifications of the Level 2 and Level 3 routines:

We propose adding one new routine:
to generate an elementary reflector (that is, a Householder matrix), following the same specification as the LAPACK auxiliary routine xLARTG.

The scope of the proposed BLAS has been extended slightly compared with the Fortran 77 BLAS: for example, we propose Level 1 BLAS for generating an elementary reflector (Householder matrix), and for generating and applying complex plane rotations; we also propose Level 2 BLAS for complex symmetric matrices. On the other hand, many of the Fortran 77 Level 1 BLAS can be replaced in Fortran 95 by simple array constructs, and they have been omitted.

For the thin BLAS we propose that the code does not do any checks on the arguments.

We propose generic interfaces that cover — wherever relevant — both Level 2 and Level 3 BLAS (for example, xTRSV and xTRSM), and have modified the specification of some Level 2 BLAS to make them more consistent with the Level 3 BLAS (for example, xTRSV now has an argument alpha).

For each procedure we specify a number of arguments that must be supplied and another set of arguments that need not be supplied. We specify a value or action for each argument which need not be supplied.

We propose that the thin BLAS contain a specific instance for each possible case and no checks or branching is used within the code.

We propose that the early implementations for the thin BLAS will contain simple calls to the reliable and tested F77 BLAS.

For example, the generic gemm will consist of the following specific procedures:

- 36 specific procedures each of which calls the F77 BLAS procedure ZGEMM (3 settings for each of op_a and op_b, and 2 settings for each of alpha and beta).
- 12 specific procedures each of which calls the F77 BLAS procedure ZGEMV (3 settings for op_a, and 2 settings for each of alpha and beta).
- 4 specific procedures each of which calls the F77 BLAS procedure ZGERU (2 settings for each of alpha and beta).
- 4 specific procedures each of which calls the F77 BLAS procedure ZGERC (2 settings for each of alpha and beta).
- 36 specific procedures each of which calls the F77 BLAS procedure DGEMM (3 settings for each of op_a and op_b, and 2 settings for each of alpha and beta). Only 16 procedures are needed, but we allow for op_a = blas_conj_trans for similarity with the complex case.
- 12 specific procedures each of which calls the F77 BLAS procedure DGEMV (3 settings for op_a, and 2 settings for each of alpha and beta). Only 8 procedures are needed, but we allow for op_a = blas_conj_trans for similarity with the complex case.
- 4 specific procedures each of which calls the F77 BLAS procedure DGER (2 settings for each of alpha and beta).
- 4 specific procedures each of which calls the F77 BLAS procedure DGER (2 settings for each of alpha and beta). These are similar to the previous case but have been added to allow op_b = blas_conj (as in the complex case for ZGERC).

Appendix C.3.8 contains a list of these specific procedures (only double precision procedures are listed).

A proposed document for this procedure is given in a separate document.
C.3. \textit{FORTRAN 95 THIN BLAS}

C.3.7 Conclusion

Our principal purpose in presenting this specification at this meeting is to provide additional input to the discussion about different levels of genericity in the interface to linear algebra routines. The thin BLAS are designed principally as building-blocks for software developers and for the BLAS itself.
C.3.8 Further Details: Specific procedures for gemm

This appendix contains a list of the specific procedures for the generic procedure gemm.

1 36 procedures each calls the F77 BLAS subroutine ZGEMM
  a, b and c are rank-2

<table>
<thead>
<tr>
<th>alpha</th>
<th>op_a</th>
<th>a</th>
<th>op_b</th>
<th>b</th>
<th>beta</th>
<th>c</th>
<th>operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>zgemm_301 (alpha, blas_conj_trans, a, blas_conj_trans, b, beta, c)</td>
<td>C &lt; alpha A(H) B(H) + beta C</td>
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<tr>
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<tr>
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<td>C &lt; alpha A(H) B(T) + beta C</td>
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<tr>
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</tr>
<tr>
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<tr>
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</tr>
<tr>
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<tr>
<td>zgemm_308 (alpha, blas_trans, a, blas_conj_trans, b, c)</td>
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</tr>
<tr>
<td>zgemm_309 (alpha, blas_trans, a, blas_trans, b, beta, c)</td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>zgemm_313 (alpha, a, blas_conj_trans, b, beta, c)</td>
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<tr>
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<tr>
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<tr>
<td>zgemm_317 (alpha, a, b, beta, c)</td>
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<tr>
<td>zgemm_318 (alpha, a, b, c)</td>
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<tr>
<td>zgemm_319 (a, blas_conj_trans, a, blas_conj_trans, b, beta, c)</td>
<td>C &lt; A(H) B(H) + beta C</td>
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<tr>
<td>zgemm_320 (a, blas_conj_trans, a, blas_conj_trans, b, c)</td>
<td>C &lt; A(H) B(H) + C</td>
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<tr>
<td>zgemm_321 (a, blas_conj_trans, a, blas_trans, b, beta, c)</td>
<td>C &lt; A(H) B(T) + beta C</td>
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<tr>
<td>zgemm_322 (a, blas_conj_trans, a, blas_trans, b, c)</td>
<td>C &lt; A(H) B(T) + C</td>
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<tr>
<td>zgemm_323 (a, blas_conj_trans, a, b, beta, c)</td>
<td>C &lt; A(H) B + beta C</td>
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<tr>
<td>zgemm_324 (a, blas_conj_trans, a, b, c)</td>
<td>C &lt; A(H) B + C</td>
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<tr>
<td>zgemm_325 (a, blas_trans, a, blas_conj_trans, b, beta, c)</td>
<td>C &lt; A(T) B(H) + beta C</td>
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<tr>
<td>zgemm_326 (a, blas_trans, a, blas_conj_trans, b, c)</td>
<td>C &lt; A(T) B(H) + C</td>
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<tr>
<td>zgemm_327 (a, blas_trans, a, blas_trans, b, beta, c)</td>
<td>C &lt; A(T) B(T) + beta C</td>
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<tr>
<td>zgemm_328 (a, blas_trans, a, blas_trans, b, c)</td>
<td>C &lt; A(T) B(T) + C</td>
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<tr>
<td>zgemm_329 (a, a, beta, c)</td>
<td>C &lt; A(T) B + beta C</td>
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<tr>
<td>zgemm_330 (a, a, b, c)</td>
<td>C &lt; A(T) B + C</td>
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<tr>
<td>zgemm_331 (a, a, blas_conj_trans, b, beta, c)</td>
<td>C &lt; A B(H) + beta C</td>
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<tr>
<td>zgemm_332 (a, a, blas_conj_trans, b, c)</td>
<td>C &lt; A B(H) + C</td>
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<tr>
<td>zgemm_333 (a, a, blas_trans, b, beta, c)</td>
<td>C &lt; A B(T) + beta C</td>
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<tr>
<td>zgemm_334 (a, a, blas_trans, b, c)</td>
<td>C &lt; A B(T) + C</td>
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</tr>
<tr>
<td>zgemm_335 (a, b, beta, c)</td>
<td>C &lt; A B + beta C</td>
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</tr>
<tr>
<td>zgemm_336 (a, b, c)</td>
<td>C &lt; A B + C</td>
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<td></td>
</tr>
</tbody>
</table>

2 12 procedures each calls the F77 BLAS subroutine ZGEMV
  a is rank-2, and b and c are rank-1

<table>
<thead>
<tr>
<th>alpha</th>
<th>op_a</th>
<th>a</th>
<th>op_b</th>
<th>b</th>
<th>beta</th>
<th>c</th>
<th>operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>zgemv_201 (alpha, blas_conj_trans, a, b, beta, c)</td>
<td>c &lt; alpha A(H) b + beta c</td>
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<tr>
<td>zgemv_202 (alpha, blas_conj_trans, a, b, c)</td>
<td>c &lt; alpha A(H) b + c</td>
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</tr>
<tr>
<td>zgemv_203 (alpha, blas_trans, a, b, beta, c)</td>
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</tr>
<tr>
<td>zgemv_204 (alpha, blas_trans, a, b, c)</td>
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<tr>
<td>zgemv_205 (alpha, a, b, beta, c)</td>
<td>c &lt; alpha A b + beta c</td>
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<tr>
<td>zgemv_206 (alpha, a, b, c)</td>
<td>c &lt; alpha A b + c</td>
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</tr>
<tr>
<td>zgemv_207 (a, blas_conj_trans, a, b, beta, c)</td>
<td>c &lt; A(H) b + beta c</td>
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<tr>
<td>zgemv_208 (a, blas_conj_trans, a, b, c)</td>
<td>c &lt; A(H) b + c</td>
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<td>zgemv_209 (a, blas_trans, a, b, beta, c)</td>
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<tr>
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<tr>
<td>zgemv_211 (a, b, beta, c)</td>
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<tr>
<td>zgemv_212 (a, b, c)</td>
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</table>

3 4 procedures each calls the F77 BLAS subroutine ZGERU
C.3. FORTRAN 95 THIN BLAS

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<tr>
<td>zgeru_201</td>
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<td>zgeru_202</td>
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<td>zgeru_204</td>
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4 procedures each calls the F77 BLAS subroutine ZGERU

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36 procedures each calls the F77 BLAS subroutine DGEMM

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<td>dgemm_302</td>
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<tr>
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<th>b,</th>
<th>c</th>
<th>C &lt; A B(T) + C</th>
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<tbody>
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<td>b, beta, c</td>
<td>C &lt; A B + beta C</td>
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<tr>
<td>dgemm_336</td>
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12 procedures each calls the F77 BLAS subroutine DGEMV

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</tr>
<tr>
<td>dgemv_201</td>
</tr>
<tr>
<td>dgemv_202</td>
</tr>
</tbody>
</table>

... ...

| a,   | b, beta, c | c < A b + beta c |
|----------------|------|---|-----------|
| dgemv_211     | a,   | b,   | c  | C < A b + c |
| dgemv_212     | a,   | b,   | c  | C < A b + c |

... ...

| a,   | b, beta, c | c < A b + beta c |
|----------------|------|---|-----------|
| dgemv_201     | a,   | b, beta, c | C < alpha a b(T) + beta C |
| dgemv_202     | a,   | b,   | c  | C < alpha a b(T) + C |
| dgemv_203     | a,   | b, beta, c | C < a b(T) + beta C |
| dgemv_204     | a,   | b,   | c  | C < a b(T) + C |
| dgemv_205     | alpha, blasconj | a, | b, beta, c | C < alpha a b(H) + beta C |
| dgemv_206     | alpha, blasconj | a, | b,   | c  | C < alpha a b(H) + C |
| dgemv_207     | blasconj | a, | b, beta, c | C < a b(H) + beta C |
| dgemv_208     | blasconj | a, | b,   | c  | C < a b(H) + C |

8 procedures each calls the F77 BLAS subroutine DGER

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<td>dger_207</td>
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<td>dger_208</td>
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</table>

Procedure gemm
Description

gemm is a generic procedure which performs one of following operations:

<table>
<thead>
<tr>
<th>rank of a</th>
<th>rank of b</th>
<th>rank of c</th>
<th>op_a</th>
<th>op_b</th>
<th>operation</th>
<th>F77 BLAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td>C</td>
<td>( C \leftarrow aAB + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td>C</td>
<td>( C \leftarrow aAB^T + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td>C</td>
<td>( C \leftarrow aA^T B + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'T'</td>
<td>C</td>
<td>( C \leftarrow aA^T B^T + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td>C</td>
<td>( C \leftarrow aAB^H + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>'C/T'</td>
<td>C</td>
<td>( C \leftarrow aA^H B + \beta C )</td>
<td>GEMM</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>'T'</td>
<td>C</td>
<td>( C \leftarrow aA b + \beta c )</td>
<td>GEMV</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>'T'</td>
<td>C</td>
<td>( C \leftarrow aA^H b + \beta c )</td>
<td>GEMV</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>'C'</td>
<td>C</td>
<td>( C \leftarrow aab + \beta C )</td>
<td>GER</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>'C'</td>
<td>C</td>
<td>( C \leftarrow aab^H + \beta C )</td>
<td>GER</td>
</tr>
</tbody>
</table>

(If \( A \) is real, then \( A^H = A^T \).)

Usage

CALL gemm([alpha], [op_a], a, [op_b], b, [beta], c)

One or more of the arguments in square brackets can be dropped. The order of the supplied arguments must remain unchanged.

Interfaces

Distinct interfaces are provided for each of the combinations of the following cases:

Real / complex data

Real data: \( \alpha, a, b, \beta a \) and \( c \) are of type \( \text{real(kind) wp)} \).
Complex data: \( \alpha, a, b, \beta a \) and \( c \) are of type \( \text{complex(kind) wp)} \).

different ranks

\text{f77_gemm}: \( a, b \) and \( c \) are rank-2 arrays.
\text{f77_gemv}: \( a \) is a rank-2 array while \( b \) and \( c \) are rank-1 arrays.
\text{f77 ger}: \( c \) is a rank-2 array while \( a \) and \( b \) are rank-1 arrays.

Arguments

All array arguments are assumed-shape arrays. The extent in each dimension must be exactly that required by the problem. Notation such as \( 'x(n)' \) is used in the argument descriptions to specify that the array \( x \) must have exactly \( n \) elements.

The procedure derives the values of the following problem parameters from the shape of the supplied arrays.

\( m \) — the first dimension of \( c \), if \( c \) is rank-2 (\( m = \text{SIZE}(c,1) \)), or the size of \( c \) if it is rank-1 (\( m = \text{SIZE}(c) \))
\( n \) — the second dimension of \( c \) if it is rank-2 (\( n = \text{SIZE}(c,2) \))
\( k \) — the intermediate dimension

Mandatory arguments

One or more of the arguments \( \alpha, \text{op}_a, \text{op}_b \) and \( \beta \) can be dropped. The order of the supplied arguments must remain unchanged.

\( \alpha \) — real(kind=wp) / complex(kind=wp), intent(in)

Input: the value of \( \alpha \) if different from one.

Note: if \( \alpha \) is exactly one, you need not supply this argument.
C.3. FORTRAN 95 THIN BLAS

\begin{verbatim}
  op_a — a "key" argument, intent(in)
  Input: if op_a is supplied, it specifies whether the operation involves the transpose $A^T$ or its conjugate-transpose $A^H$ (= $A^T$ if $A$ is real). In this case op_a must have one of the following values (which are named constants, each of a different derived type, defined by the BLAS, and accessible from the module blas):
  - blas_trans: if the operation involves the transpose $A^T$ rather than the matrix $A$;
  - blas_conj_trans: if the operation involves the conjugate-transpose $A^H$ rather than the matrix $A$.
  Note: for real matrices, blas_conj_trans is equivalent to blas_trans.
  Constraints: op_a must not be supplied if $A$ is rank-1 or if the operation does not involve the transpose or the conjugate-transpose of $A$.

  a(m) / a(p, q) — real(kind=wp)/ complex(kind=wp), intent(in)
  Input: the matrix $A$ or vector $a$.
  If $A$ is rank-2 then:
  - if op_a is not supplied, the shape of $a$ must be $(m, k)$;
  - if op_a is supplied, the shape of $a$ must be $(k, m)$.

  op_b — a "key" argument, intent(in)
  Input: if op_b is supplied and $B$ is rank-2, it specifies whether the operation involves the transpose $B^T$ or its conjugate-transpose $B^H$ (= $B^T$ if $B$ is real). In this case op_b must have one of the following values (which are named constants, each of a different derived type, defined by the BLAS, and accessible from the module blas):
  - blas_trans: if the operation involves the transpose $B^T$ rather than the matrix $B$;
  - blas_conj_trans: if the operation involves the conjugate-transpose $B^H$ rather than the matrix $B$.
  If op_b is supplied and $B$ is rank-1, it specifies that the operation involves the conjugate of $b^T$ ($b^H$) rather than $b^T$. In this case op_b must have be blas_conj (which is a named constant of a derived type, defined by the BLAS, and accessible from the module blas).
  Note: for real matrices, blas_conj_trans is equivalent to blas_trans. For real arrays blas_conj does not have any effect.
  Constraints: op_b must not be supplied if the operation does not involve the conjugate of $b$, the transpose of $B$ or the conjugate-transpose of $B$.

  b(r) / b(r, s) — real(kind=wp)/ complex(kind=wp), intent(in)
  Input: the matrix $B$ or vector $b$.
  If $B$ is rank-1 then:
  - if $B$ is rank-1, the shape of $b$ must be $(m)$;
  - if $B$ is rank-2, the shape of $b$ must be SIZE(op_a(a), 2).
  If $B$ is rank-2 then:
  - if op_b is not supplied, the shape of $b$ must be $(k, n)$;
  - if op_b is supplied, the shape of $b$ must be $(n, k)$.

  beta — real(kind=wp)/ complex(kind=wp), intent(in)
  Input: the value of $\beta$ if different from zero.
  Note: if $\beta$ is exactly zero, you need not supply this argument.

  c(m) / c(m, n) — real(kind=wp)/ complex(kind=wp), intent(inout)
  Input: the matrix $C$ or vector $c$. If beta is not supplied $c$ need not be initialized.
  Output: the matrix $C$ or vector $c$ after applying the operation.

Examples of usage

One or more of the arguments alpha, op_a, op_b and beta can be dropped. The order of the supplied arguments must remain unchanged.

To perform $C \leftarrow \alpha AB^H$ use the call:

\begin{verbatim}
  CALL gemm (alpha, a, blas_conj_trans, b, c)
\end{verbatim}

To perform $C \leftarrow AB + \beta C$ use the call:

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\end{verbatim}
\end{verbatim}
\end{reamble}
CALL gemm (a,b,beta,c)

To perform $C \leftarrow \alpha AB + \beta C$ use the call:

CALL gemm (alpha,a,b,beta,c)

To perform $C \leftarrow A^T B^H + \beta C$ use the call:

CALL gemm (blas_trans,a,blas_her,b,beta,c)

To perform $c \leftarrow \alpha A^T b + \beta c$ use the call:

CALL gemm (alpha,blas_trans,a,b,beta,c)

To perform $C \leftarrow ab^H + \beta C$ use the call:

CALL gemm (a,b,blas_conj,beta,c)
C.4 Interval BLAS

C.4.1 Introduction

Interval computation refers to performing computations with intervals. Computing with intervals guarantees that interval results contain the set of all possible correct answers. Valid implementations of interval arithmetic produce correct bounds on the set of all possible correct answers, including the effects of accumulated roundoff errors. Recent advances in interval algorithms have generated interest in using these methods in increasing numbers of applications. This motivates us to establish the standard for interval BLAS described in this chapter.

Intervals

A nonempty mathematical interval \([a, b]\) is the set \(\{x \in \mathbb{R} | a \leq x \leq b\}\) where \(a \leq b\). A machine interval \([a^*, b^*]\) is a mathematical interval whose endpoints are machine representable numbers. We say that \([a^*, b^*]\) is a machine representation of \([a, b]\) if \([a^*, b^*]\) contains \([a, b]\) i.e. \(a^* \leq a\) and \(b \leq b^*\). We say that the machine interval \([a^*, b^*]\) is a tight representation of a mathematical interval \([a, b]\) if and only if \(a^*\) is the greatest machine representable number which is less than or equal to \(a\), and \(b^*\) is the least machine representable number which is greater than or equal to \(b\).

The empty interval \(\emptyset\), which does not contain any real number, is required in the interval BLAS. For machines in compliance with the IEEE-standard, we recommend the use of \([\text{NaN}_{\text{empty}}, \text{NaN}_{\text{empty}}]\) to represent the empty interval, where \(\text{NaN}_{\text{empty}}\) is a unique non-default quiet not-a-number that is used to represent the empty interval only.

Notation

Both scalar (floating point number) and interval arguments are used for the specification of routines in this chapter. Interval vectors and interval matrices are vectors and matrices whose entries are intervals. The notation used in this chapter is consistent with other chapters, but we use boldface letters to specify interval arguments. We also use overline and underline to specify the greatest lower bound and the least upper bound of an interval variable, respectively. For example, if \(\mathbf{x}\) is an interval vector, then \(\mathbf{x} = [\underline{x}, \overline{x}]\).

Interval arithmetic

Interval arithmetic on mathematical intervals is defined as follows.

Let \(\mathbf{a}\) and \(\mathbf{b}\) be two mathematical intervals. Let \(\text{op}\) be one of the arithmetic operations \(+, -, \times, \div\). Then \(\mathbf{a} \text{ op } \mathbf{b} \equiv \{a \text{ op } b : a \in \mathbf{a}, b \in \mathbf{b}\}\), provided that \(0 \notin \mathbf{b}\) if \(\text{op}\) represents \(\div\).

Advice to users: The above definition of division implies that the user is responsible to trapping and dealing with any division by an interval containing zero.

Table C.1 gives explicit implementations of these four basic interval arithmetic operations and other operations on mathematical intervals used in this chapter. We use the notation \(\mathbf{a} = [\underline{a}, \overline{a}]\) and \(\mathbf{b} = [\underline{b}, \overline{b}]\).

All operations in the interval BLAS are necessarily performed on machine intervals. Arithmetic on machine intervals must satisfy the following property:
<table>
<thead>
<tr>
<th>Operation</th>
<th>$\mathbf{a} \neq \emptyset$ and $\mathbf{b} \neq \emptyset$</th>
<th>$\mathbf{a} = \emptyset$ or/and $\mathbf{b} = \emptyset$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition $\mathbf{a} + \mathbf{b}$</td>
<td>$[a + b, \pi + b]$</td>
<td>$\emptyset$ or/and $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Subtraction $\mathbf{a} - \mathbf{b}$</td>
<td>$[a - b, \pi - b]$</td>
<td>$\mathbf{a} = \emptyset$ or/and $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Multiplication $\mathbf{a} \times \mathbf{b}$</td>
<td>$\min{ab, a\pi, a\pi, b\pi}$, $\max{ab, a\pi, a\pi, b\pi}$</td>
<td>$\emptyset$ or/and $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Cancellation $\mathbf{a} \odot \mathbf{b}$</td>
<td>$[a - b, \pi - b]$ if $(a - b) \leq (\pi - b)$; Otherwise, $\emptyset$</td>
<td>$\emptyset$ or/and $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Division $\frac{\mathbf{a}}{\mathbf{b}}$, $(0 \neq \mathbf{b})$</td>
<td>$\min{\frac{a}{b}, \frac{a}{\pi}, \frac{\pi}{b}, \frac{\pi}{\pi}}$, $\max{\frac{a}{b}, \frac{a}{\pi}, \frac{\pi}{b}, \frac{\pi}{\pi}}$</td>
<td>$\mathbf{b}$ if $\mathbf{a} = \emptyset$; or $\mathbf{a}$ if $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Convex Hull $\mathbf{a}, \mathbf{b}$</td>
<td>$\min{\pi, \beta}$</td>
<td>$\emptyset$ or/and $\mathbf{b} = \emptyset$</td>
</tr>
<tr>
<td>Intersection $\mathbf{a} \cap \mathbf{b}$</td>
<td>$\max{a, b}, \min{\pi, \beta}$ if $\max{a, b} \leq \min{\pi, \beta}$; Otherwise, $\emptyset$</td>
<td>True if $\mathbf{a} \cap \mathbf{b} = \emptyset$; False, otherwise</td>
</tr>
<tr>
<td>Disjoint</td>
<td><strong>True</strong> if $\mathbf{a} \cap \mathbf{b} = \emptyset$; <strong>False</strong>, otherwise</td>
<td><strong>NaN</strong>_empty</td>
</tr>
<tr>
<td>Absolute value $</td>
<td>\mathbf{a}</td>
<td>$</td>
</tr>
<tr>
<td>Midpoint $\mathbf{a}$</td>
<td>$(a + \pi)/2$</td>
<td><strong>NaN</strong>_empty</td>
</tr>
<tr>
<td>Width $\mathbf{a}$</td>
<td>$\pi - a$</td>
<td><strong>NaN</strong>_empty</td>
</tr>
</tbody>
</table>

Table C.1: Elementary interval operations

**Containment Condition:** Let $\mathbf{a} = [a, \pi]$ and $\mathbf{b} = [b, \beta]$ be intervals. Let $\mathbf{c} = [c, \pi]$ be the interval result of computing $\mathbf{a} \text{ op } \mathbf{b}$ where op is defined in Table C.1. If $\mathbf{c}$ is nonempty, then $\mathbf{c}$ must contain the exact mathematical interval $\mathbf{a} \text{ op } \mathbf{b}$.

In other words, interval arithmetic on nonempty machine intervals requires that we round down the lower bound and round up the upper bound to guarantee that the machine interval result contains the true mathematical interval result. This is needed to propagate guaranteed error bounds. A good implementation will round down and round up to the nearest possible floating point numbers, in order to get the narrowest possible machine intervals. But coarser rounding is enough to get a correct implementation. For more information on interval arithmetic specifications, one may refer to [14].

**Advice to implementors:** In implementations of interval BLAS, a warning message should be provided to users whenever there is no finite machine interval that satisfies the containment condition during computations.

With interval arithmetic, one may automatically bound truncation error, round-off error, and even error in the original data to obtain machine intervals that are guaranteed to contain the true mathematical result of a computation. However, simply changing floating point numbers in an algorithm into intervals and all floating point operations into interval operations may result in such wide intervals that the output is useless in practice. For example, $[-100, 200]$ is a correct but probably useless bound for a true result of 3.1416. To apply the interval BLAS routines effectively, appropriate algorithms should be used that attempt to keep interval widths narrow. Many such algorithms are available in the literature. Readers may find a list of reference books, websites, software packages, and applications in [2, 12, 1].

### C.4.2 Functionality

This chapter defines the functionality and language bindings for both the interval BLAS routines, and for selected mathematical operations on: intervals; interval vectors; and, dense, banded, and triangular interval matrices. Neither sparse data structures, nor complex intervals are treated.
Sections C.4.2 – C.4.2 outline the functionality of the proposed routines in tabular form. Sections C.2–C.4.5 present the language bindings for the proposed routines in the functionality tables.

Interval Vector Operations

Table C.2 lists interval vector reduction operations. Table C.3 lists interval vector operations. Table C.4 lists interval vector operations which involve only data movement.

<table>
<thead>
<tr>
<th>Dot product</th>
<th>$\mathbf{r} \leftarrow \beta \mathbf{r} + \alpha \mathbf{x}^T \mathbf{y}$</th>
<th>DOT_J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vector norms</td>
<td>$r \leftarrow |\mathbf{x}|_1, r \leftarrow |\mathbf{x}|_2$</td>
<td>NORM_J</td>
</tr>
<tr>
<td>Sum</td>
<td>$r \leftarrow \sum_i x_i$</td>
<td>SUM_J</td>
</tr>
<tr>
<td>Max magnitude &amp; location</td>
<td>$k, x_k; k = \arg \max_i {</td>
<td>x_i</td>
</tr>
<tr>
<td>Min absolute value &amp; location</td>
<td>$k, x_k; k = \arg \min_i {</td>
<td>x_i</td>
</tr>
<tr>
<td>Sum of squares</td>
<td>$(a, b) \leftarrow \sum_i x_i^2, a \cdot b^2 = \sum_i x_i^2$</td>
<td>SUMSQ_J</td>
</tr>
</tbody>
</table>

Table C.2: Reduction Operations

<table>
<thead>
<tr>
<th>Reciprocal scale</th>
<th>$\mathbf{x} \leftarrow \mathbf{x} / \alpha$</th>
<th>RSCALE_J</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaled interval vector accumulation</td>
<td>$\mathbf{y} \leftarrow \alpha \mathbf{x} + \beta \mathbf{y}$</td>
<td>AXPBY_J</td>
</tr>
<tr>
<td>Scaled interval vector accumulation</td>
<td>$\mathbf{w} \leftarrow \alpha \mathbf{x} + \beta \mathbf{y}$</td>
<td>WAXPBY_J</td>
</tr>
<tr>
<td>Scaled interval vector cancellation</td>
<td>$\mathbf{y} \leftarrow \alpha \mathbf{x} \oplus \beta \mathbf{y}$</td>
<td>CANCEL_J</td>
</tr>
<tr>
<td>Scaled interval vector cancellation</td>
<td>$\mathbf{w} \leftarrow \alpha \mathbf{x} \oplus \beta \mathbf{y}$</td>
<td>WCANCEL_J</td>
</tr>
</tbody>
</table>

Table C.3: Interval Vector Operations

| Copy | $\mathbf{y} \leftarrow \mathbf{x}$ | COPY_J     |
| Swap | $\mathbf{y} \leftrightarrow \mathbf{x}$ | SWAP_J     |
| Permute vector | $\mathbf{x} \leftarrow P\mathbf{x}$ | PERMUTE_J |

Table C.4: Data Movement with Interval Vector Operations
Interval Matrix-Vector Operations

Table C.5 lists interval matrix-vector operations.

<table>
<thead>
<tr>
<th>Matrix vector product</th>
<th>$y \leftarrow \alpha A x + \beta y$</th>
<th>$y \leftarrow \alpha A^T x + \beta y$</th>
<th>$x \leftarrow \alpha T x, x \leftarrow T^T x$</th>
<th>$x \leftarrow \alpha T^{-1} x, x \leftarrow \alpha T^{-T} x$</th>
<th>$A \leftarrow \alpha xy^T + \beta A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GE,GB,SY,SB,SP</td>
<td>GE,GB</td>
<td>TR, TB, TP</td>
<td>TR, TB, TP</td>
<td>GE,SY,SP</td>
</tr>
<tr>
<td></td>
<td>MV_J</td>
<td>MV_J</td>
<td>MV_J</td>
<td>MV_J</td>
<td>R_J</td>
</tr>
</tbody>
</table>

Table C.5: Interval Matrix-vector Operations

Interval Matrix Operations

Table C.6 lists single interval matrix operations and interval matrix operations that involve $O(n^2)$ floating point operations. The matrix $T$ represents an upper or lower triangular interval matrix. $D$ represents a diagonal interval matrix. Table C.7 lists the interval matrix-matrix operations that involve $O(n^3)$ floating point operations and Table C.8 lists those operations that involve only data movement.

| Matrix norms | $r \leftarrow ||A||_1, r \leftarrow ||A||_F$, | $r \leftarrow ||A||_\infty, r \leftarrow ||A||_{\max}$ | GE,GB,SY,SB,SP                  | GE,GB_SY,SB,SP                  | _NORM_J             |
|--------------|-----------------------------------------------|-----------------------------------------------|---------------------------------|---------------------------------|---------------------|
| Diagonal scaling | $A \leftarrow DA, A \leftarrow AD$ | $A \leftarrow D\sqrt{D_2}$ | GE, GB | GE, GB | _DIAG_SCALE_J |
| Two sided diagonal scaling | $A \leftarrow DAD$ | $A \leftarrow A + BD$ | SY, SB | SY, SB | _LRSCALE_J |
| Two sided diagonal scaling | $A \leftarrow DAD$ | $A \leftarrow A + BD$ | GE, GB | GE, GB | _LRSCALE_J |
| Matrix acc and scale | $B \leftarrow \alpha A + \beta B,$ | $B \leftarrow \alpha A^T + \beta B$ | GE,GB,SY,SB,SP                  | GE,GB,SY,SB,SP                  | _ACC_J              |
| Matrix add and scale | $C \leftarrow \alpha A + \beta B$ | $C \leftarrow \alpha A^T + \beta B$ | GE,GB,SY,SB,SP                  | GE,GB,SY,SB,SP                  | _ADD_J              |

Table C.6: Matrix Operations – $O(n^2)$ floating point operations

<table>
<thead>
<tr>
<th>Matrix matrix product</th>
<th>$C \leftarrow \alpha AB + \beta C,$</th>
<th>$C \leftarrow \alpha A^T B + \beta C,$</th>
<th>$C \leftarrow \alpha AB^T + \beta C,$</th>
<th>$C \leftarrow \alpha A^T B^T + \beta C,$</th>
<th>$C \leftarrow \alpha BA + \beta C,$</th>
<th>$C \leftarrow \alpha B^T A + \beta C,$</th>
<th>$C \leftarrow \alpha BA^T + \beta C,$</th>
<th>$C \leftarrow \alpha B^T A^T + \beta C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GE,GB,SY,SB</td>
<td>MM_J</td>
<td>GB</td>
<td>MM_J</td>
<td>MM_J</td>
<td>MM_J</td>
<td>MM_J</td>
<td>MM_J</td>
</tr>
<tr>
<td>Triangular multiply</td>
<td>$B \leftarrow \alpha TB, B \leftarrow \alpha BT$</td>
<td>$B \leftarrow \alpha TB, B \leftarrow \alpha BT$</td>
<td>TR, TB</td>
<td>TR, TB</td>
<td>MM_J</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Triangular solve</td>
<td>$B \leftarrow \alpha T^{-1} B, B \leftarrow \alpha BT^{-1}$</td>
<td>$B \leftarrow \alpha T^{-T} B, B \leftarrow \alpha BT^{-T}$</td>
<td>TR, TB</td>
<td>TR, TB</td>
<td>SM_J</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table C.7: Matrix Operations – $O(n^3)$ floating point operations
### C.4. INTERVAL BLAS

| Matrix copy       | B ← A        | GE,GB,SY,SB,SP,TR,TB,TP | COPY_1 |
| Matrix transpose  | B ← A^T      | GE, GB                  | COPY_1 |
| Permute matrix    | A ← A^T      | GE                      | TRANS_1 |
|                   | A ← PA, A ← AP | GE                        | PERMUTE_1 |

Table C.8: Data Movement with Interval Matrices

### Set Operations Involving Interval Vectors

Table C.9 lists set operations for interval vectors.

| Enclosed       | x is enclosed in y if x ⊆ y | ENCV_1 |
| Interior       | x is enclosed in the interior of y | INTERIORV_1 |
| Disjoint       | x and y are disjoint if x ∩ y = ∅ | DISJ_1, WINTERV_1 |
| Intersection   | y ← x ∩ y, z ← x ∩ y | HULLV_1, WHULLV_1 |
| Hull           | the convex hull of x and y | |

Table C.9: Set Operations for Interval Vectors

### Set Operations Involving Interval Matrices

Table C.10 lists set operations for interval matrices.

| Enclosed       | A is enclosed in B if A ⊆ B | GE,GB,SY,SB,SP,TR,TB,TP | ENCM_1 |
| Interior       | A is enclosed in the interior of B | GE,GB,SY,SB,SP,TR,TB,TP | INTERIORM_1 |
| Disjoint       | A and B are disjoint if A ∩ B = ∅ | GE,GB,SY,SB,SP,TR,TB,TP | DISJM_1 |
| Intersection   | B ← A ∩ B, C ← A ∩ B | GE,GB,SY,SB,SP,TR,TB,TP | INTERM_1, WINTERM_1 |
| Hull           | the convex hull of A and B | GE,GB,SY,SB,SP,TR,TB,TP | HULLM_1, WHULLM_1 |

Table C.10: Set Operations for Interval Matrices

### Utility Functions Involving Interval Vectors

Table C.11 lists some utility operations for interval vectors.

### Utility Functions Involving Interval Matrices

Table C.12 lists some utility operations for interval matrices.
Table C.11: Utility Operations for Interval Vectors

| Empty element | if A has an empty interval element | GE,GB,SY,SB,SP,TR,TB,TP | _EMPTYELEM_J |
| Left endpoint | C ← A | GE,GB,SY,SB,SP,TR,TB,TP | _INFJ |
| Right endpoint | C ← \( \overline{A} \) | GE,GB,SY,SB,SP,TR,TB,TP | _SUPJ |
| Midpoint | C ← \( \frac{(A + \overline{A})}{2} \) | GE,GB,SY,SB,SP,TR,TB,TP | _MIDJ |
| Width | C ← \( \overline{A} - A \) | GE,GB,SY,SB,SP,TR,TB,TP | _WIDTHM_J |
| Construct | A ← B, C | GE,GB,SY,SB,SP,TR,TB,TP | _CONSTRCTM_J |

Table C.12: Utility Operations

C.4.3 Interface Issues

Naming Conventions

The naming conventions are the same as described in section 2.3.1 except that the suffix \( I \) (or \( _i \)) is added to indicate an interval BLAS routine.

Interface Issues for Fortran 95

Design of the Fortran 95 Interfaces

The Fortran 95 binding is defined in a module. The specific interfaces in this module should declare the default interval data type as TYPE(INTERVAL).

Advice to implementors: In the Fortran 95 interfaces, it is assumed that INTERVAL is a derived type. However, in compilers that support an intrinsic interval type, it is recommended that an alternate module that contains appropriately modified declarations also be supplied. For example, TYPE(INTERVAL), INTENT(IN) :: ALPHA could become INTERVAL, INTENT(IN) :: ALPHA in a recommended alternate module.

The Fortran 95 interval BLAS routines are consistent with regard to generic interfaces, precision, rank, assumed-shape arrays, derived types, operator arguments and CMACH values, and error handling as described in section 2.4 of this document. However, in the interval BLAS, \( \alpha \) and \( \beta \) are intervals; and their default values are \( \text{alpha} = [1,1], \text{beta} = [0,0] \). Error handling is as defined in section 2.4.6.
C.4. INTERVAL BLAS

Format of the Fortran 95 bindings

Each interface is summarized in the form of a SUBROUTINE statement (or in few cases a FUNCTION statement), in which all of the potential arguments appear. Arguments which need not be supplied are grouped after the mandatory arguments and enclosed in square brackets, for example:

```
SUBROUTINE axpby_i( x, y [, alpha] [, beta] )
  TYPE(INTEGER) (<wp>), INTENT (IN) :: x (:)
  TYPE(INTEGER) (<wp>), INTENT (INOUT) :: y (:)
  TYPE(INTEGER) (<wp>), INTENT (IN), OPTIONAL :: alpha, beta
```

Variables in interval BLAS routines should be specified as INTEGER, REAL, TYPE(INTEGER) or types defined in MODULE blas_operator_arguments. The precision of a real or interval variable is denoted by <wp> where

```
<wp> ::= KIND(1.0) | KIND(1.0DO)
```

Interface issues for Fortran 77

The interval BLAS Fortran 77 binding is consistent with ANSI standard Fortran 77 except the following:

- Subroutine names are not limited to six significant characters.
- Subroutine names contain one or more underscores.
- Subroutines may use the INCLUDE statement for include files.

In interval BLAS Fortran 77 binding, $\alpha$ and $\beta$ are intervals and their default values are:
\[
\text{ALPHA} = [1.0, 1.0] \text{ and } \text{BETA} = [0.0, 0.0].
\]
Without assuming an intrinsic interval data type, an interval, say $\alpha$, will be declared as REAL or DOUBLE PRECISION ALPHA(2); an interval vector will be stored as REAL or DOUBLE PRECISION X(2,*); and a general interval matrix will be defined as REAL or DOUBLE PRECISION A(2, LDA, *).

Advice to implementors: On Fortran 77 compilers that have an intrinsic interval data type, an interval vector will be stored as INTERVAL X(*), and a general interval matrix will be defined as INTERVAL A(LDA, *).

The Fortran 77 interval BLAS routines are consistent with regard to indexing of vector and matrix operands, operator arguments and CMACH values, array arguments, matrix storage schemes, and error handling as described in section 2.5 of this document but with interval variables. Error handling is as defined in section 2.5.6.

Format of the Fortran 77 bindings

Each interface is summarized in the form of a SUBROUTINE statement (or a FUNCTION statement). For example:

```
SUBROUTINE BLAS_xAXPBY_I( N, ALPHA, X, INCX, BETA, Y, INCY )
  INTEGER INCI, INCY, N
  <type> ALPHA(2), BETA(2)
  <type> X(2,*), Y(2,*)
```

Floating point variables are denoted by the keyword <type> which may be REAL or DOUBLE PRECISION, and should agree with the x letter in the naming convention of the routine.
Interface Issues for C

The interface is expressed in terms of ANSI/ISO C. All interval arguments are accepted as float * or double *. An interval element consists of two consecutive memory locations of the underlying data type (i.e., float or double), where the first location contains the lower bound of the interval, and the second contains the upper bound of the interval.

The C interval BLAS routines are consistent with regard to indexing of vector and matrix operands, operator arguments and CMACH values, array arguments, matrix storage schemes, and error handling that described in section 2.6 of this document but with interval variables. The default value for intervals alpha and beta are alpha = [1.0, 1.0] and beta = [0.0, 0.0].

Error handling is as defined in section 2.6.9.

Format of the C bindings

Each interval BLAS routine is summarized in the form of an ANSI/ISO C prototype. For example:

```c
void BLAS_raxpby_i( int n, <interval> alpha, const <interval_array> x,
                   int incx, <interval> beta, <interval_array> y,
                   int incy)
```

In the C binding, we use the keywords <interval> and <interval_array> to indicate if an argument is a single interval or an interval vector/matrix. In fact, <interval> and <interval_array> can be float * or double *. A real number, not an interval, will be indicated by the keyword SCALAR. A vector/matrix of real numbers, not intervals, will be specified by RARRAY. The precisions of SCALAR, RARRAY can be float or double. They will agree with the x letter in the naming convention of the routine. However, in some routines, not all floating point variables will be the same type.

If this is the case, then a variable may be denoted by the keywords SCALAR_IN or SCALAR_INOUT. SCALAR_IN can be float or double; and SCALAR_INOUT and RARRAY can be float * or double *.

C.4.4 Numerical Accuracy and Environmental Enquiry

The semantics of interval arithmetic require us to have another environmental enquiry function to supplement the routine FPINFO described in sections 1.6 and 2.7. Here we will specify the additional routine FPINFO_I to determine how tightly the containment property of interval arithmetic is maintained.

To establish notation, let \( a = [a, \overline{a}] \) and \( b = [b, \overline{b}] \) be machine intervals, let \( \text{op} \) be one of the operations +, −, , \( \times \) and \( \div \), let \( c = [c, \overline{c}] = a \text{ op } b \) be the exact mathematical interval result of \( a \) op b, and let \( c^* = [c^*, \overline{c^*}] = f(a \text{ op } b) \) be the machine interval computed containing c. Let \( \epsilon_I > 0 \) be defined as the smallest number such that for all \( a, b \) and \( \text{op} \) where overflow and underflow do not occur in computing \( c^* \), then

\[
\begin{align*}
\underline{c} & \geq \min\{c^*(1 + \epsilon_I), c^*(1 - \epsilon_I)\} \\
\overline{c} & \leq \max\{\overline{c^*}(1 + \epsilon_I), \overline{c^*}(1 - \epsilon_I)\}
\end{align*}
\]

In other words, \( \epsilon_I \) measures how much the exact mathematical interval bounds are rounded out to get the machine interval result. When the machine interval is tight, i.e. as narrow as possible, then \( \epsilon_I = BASE_1^{-T} \), where \( BASE \) and \( T \) are values returned by FPINFO. But \( \epsilon_I \) could be larger depending on the implementation, leading us to the following environmental enquiry:
<table>
<thead>
<tr>
<th>Value of CMACH</th>
<th>Name of value returned by FPINFO_I</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>blas_base</td>
<td>BASE</td>
<td>base of the machine</td>
</tr>
<tr>
<td>blas_l.i</td>
<td>T.I</td>
<td>effective number of base BASE digits, such that $\epsilon_l = BASE^{1-T_i}$</td>
</tr>
<tr>
<td>blas_rnd.l</td>
<td>RND.I</td>
<td>when interval arithmetic is implemented with correct IEEE-style directed rounding $\epsilon_l$ as defined above.</td>
</tr>
<tr>
<td>blas_eps.l</td>
<td>EPS.I</td>
<td></td>
</tr>
</tbody>
</table>

C.4.5 Language Bindings

Each specification of a routine will correspond to an operation outlined in the functionality tables. Operations are organized analogous to the order in which they are presented in the functionality tables. The format of the language bindings is as described in section 2.8.

Overview

- Reduction Operations (section C.4.5)
  - DOT.I (Dot product)
  - NORM.I (Interval vector norms)
  - SUM.I (Sum)
  - AMIN.VAL.I (Min absolute value & location)
  - AMAX.VAL.I (Max absolute value & location)
  - SUMSQ.I (Sum of squares)

- Interval Vector Operations (section C.4.5)
  - RSSCALE.I (Reciprocal Scale)
  - AXPBY.I (Scaled vector accumulation)
  - WAXPBY.I (Scaled vector addition)
  - CANCEL.I (Scaled cancellation)
  - WCANCEL.I (Scaled cancellation)

- Data Movement with Interval Vectors (section C.4.5)
  - COPY.I (Interval vector copy)
  - SWAP.I (Interval vector swap)
  - PERMUTE.I (Permute interval vector)

- Interval Matrix-Vector Operations (section C.4.5)
  - {GE,GB}MV.I (Interval matrix vector product)
  - {SY,SB,SP}MV.I (Interval symmetric matrix vector product)
  - {TR,TP}MV.I (Interval triangular matrix vector product)
  - {TR,TP}SV.I (Interval triangular solve)
  - GER.I (Rank one update)
- \{SY,SP\}R_J (Symmetric rank one update)

- **Interval Matrix Operations (section C.4.5)**
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_NORM_J (Interval matrix norms)
  - \{GE,GB\}_DIAG_SCALE_J (Diagonal scaling)
  - \{GE,GB\}_LRSCALE_J (Two-sided diagonal scaling)
  - \{SY,SB,SP\}_LRSCALE_J (Two-sided diagonal scaling of a symmetric interval matrix)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_ACC_J (Matrix accumulation and scale)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_ADD_J (Matrix add and scale)

- **Interval Matrix-Matrix Operations (section C.4.5)**
  - GEMM_J (General interval Matrix Matrix product)
  - SYMM_J (Symmetric interval matrix matrix product)
  - TRMM_J (Triangular interval matrix matrix multiply)
  - TRSM_J (Interval triangular solve)

- **Data Movement with Interval Matrices (section C.4.5)**
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_COPY_J (Matrix copy)
  - GE_TRANS_J (Matrix transposition)
  - GE_PERMUTE_J (Permute an interval matrix)

- **Set Operations Involving Interval Vectors (section C.4.5)**
  - ENCV_J (Checks if an interval vector is enclosed in another interval vector)
  - INTERIORV_J (Checks if an interval vector is enclosed in the interior of another interval vector)
  - DISJV_J (Checks if two interval vectors are disjoint)
  - INTERV_J (Intersection of an interval vector with another)
  - WINTERV_J (Intersection of two interval vectors)
  - HULLV_J (Convex hull of an interval vector with another)
  - WHULLV_J (Convex hull of two interval vectors)

- **Set Operations Involving Interval Matrices (section C.4.5)**
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_ENC_M_J (Checks if an interval matrix is enclosed in another interval matrix)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_INTERIOR_M_J (Checks if an interval matrix is enclosed in the interior of another interval matrix)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_DISJM_J (Checks if two interval matrices are disjoint)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_INTERM_J (Elementwise intersection of an interval matrix with another)
  - \{GE,GB,SY,SB,SP;TR,TB,TP\}_WINTERM_J (Elementwise intersection of two interval matrices)
\[ \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{HULLM.I} \] (Convex hull of an interval matrix with another)

\[ \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{WHULLV.I} \] (Convex hull of two interval matrices)

- Utility Functions Involving Interval Vectors (section C.4.5)
  \- \text{EMPTYELEV.I} (Empty entry and location)
  \- \text{INFV.I} (The left endpoint of an interval vector)
  \- \text{SUPV.I} (The right endpoint of an interval vector)
  \- \text{MIDV.I} (The approximate midpoint of an interval vector)
  \- \text{WIDTHV.I} (The elementwise width of an interval vector)
  \- \text{CONSTRUCTV.I} (Constructs an interval vector from two floating point vectors)

- Utility Functions Involving Interval Matrices (section C.4.5)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{EMPTYELEM.I} \] (Empty entry and location)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{INFM.I} \] (The left endpoint of an interval matrix)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{SUPM.I} \] (The right endpoint of an interval matrix)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{MIDM.I} \] (The approximate midpoint of an interval matrix)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{WIDTHM.I} \] (Elementwise width of an interval matrix)
  \- \{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \}_\text{CONSTRUCTM.I} \] (Constructs an interval matrix from two given floating point matrices)

- Environmental Enquiry (section C.4.5)
  \- \text{FPINFO.I} (Environmental enquiry)

### Reduction Operations

**\text{DOT.I} (Dot Product)**

\[ r \leftarrow \beta r + \alpha x^T y \]

The routine **\text{DOT.I}** adds the scaled dot product of two interval vectors \( x \) and \( y \) into a scaled interval \( r \). The routine returns immediately if \( n \) is less than zero, or, if \textbf{beta} is equal to [1,1] and either \textbf{alpha} is equal to [0,0] or \( n \) is equal to zero. If \textbf{alpha} is equal to [0,0] then \( x \) and \( y \) are not read. Similarly, if \textbf{beta} is equal to [0,0], \( r \) is not referenced. As described in section 2.5.3, the value \textbf{incx} less than zero is permitted. However, if \textbf{incx} is equal to zero, an error flag is set and passed to the error handler.

179, one.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE dot.i( x, y, r [, alpha] [,beta] )
      TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:,), y(:)
      TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL :: alpha, beta
      TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: r
    where
      x and y have shape (n)
  ```
• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xDOT_I( N, ALPHA, X, INCX, BETA, Y, INCY, R )
 INTEGER INCX, INCY, N
<type> ALPHA( 2 ), BETA( 2 ), R( 2 )
<type> X( 2, * ), Y( 2, * )
```

• C binding:

```c
void BLAS_xdot_i( int n, const <interval> alpha, const <interval_array> x,
                 int incx, const <interval> beta, const <interval_array> y,
                 int incy, <interval> r ];
```

Advice to users: The scaling parameters alpha and beta are intervals. If any one
of them is a real number in applications, the user needs to convert it into its interval
representation first, and then use the routine.

---

**NORMJ** (Interval vector norms)

\[ r \leftarrow ||x||_1, ||x||_2, ||x||_\infty \]

The routine **NORMJ** computes the \( || \cdot ||_1 \), \( || \cdot ||_2 \), or \( || \cdot ||_\infty \) of a vector \( x \) depending on the value
passed as the norm operator argument.

If \( n \) is less than or equal to zero, this routine returns immediately with the output scalar \( r \) set to
zero. The resulting scalar \( r \) is always real and its value is as defined in section 2.1.1, provided that
\( |x_i| = \max(|x_i|, |x_i|) \).

As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal
to zero, an error flag is set and passed to the error handler.

• Fortran 95 binding:

```fortran
REAL (<wp>) FUNCTION norm_i ( x [, norm] )
    TYPE(INTERVAL) <wp>, INTENT(IN) :: x::<
    TYPE blas_norm_type, INTENT(IN), OPTIONAL :: norm
where
    x has shape (n)
```

• Fortran 77 binding:

```fortran
<type> FUNCTION BLAS_xNORM_I( NORM, N, X, INCX )
 INTEGER INCX, N, NORM
<type> X( 2, * )
```

• C binding:

```c
void BLAS_xnorm_i( enum blas_norm_type norm, int n, const <interval_array> x,
                   int incx, SCALAR_INOUT r );
```
Advice to implementors: In finite precision floating point arithmetic, an upper bound, preferably the least machine representable upper bound, for the mathematical value should be returned for the norms.

**SUMJ** (Sum the entries of an interval vector)

\[
r \leftarrow \sum_{i=0}^{n-1} x_i
\]

The routine **SUMJ** returns the sum of the entries of an interval vector \( \mathbf{x} \). If \( n \) is less than or equal to zero, this routine returns immediately with the output interval \( \mathbf{r} \) set to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE sum_i( x, r )
  TYPE(INTEGER)(<wp>), INTENT(IN) :: x(:)
  TYPE(INTEGER)(<wp>), INTENT(OUT) :: r
  where
  x has shape (n)
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xSUM_I( N, INCX, X, R )
  INTEGER INCX, N
  <type> X( 2, * )
  <type> R( 2 )
  ```

- C binding:

  ```c
  void BLAS_xsum_i( int n, int incx, const <interval_array> x, <interval> r );
  ```

**AMIN_VALJ** (min\(_{0\leq i<n}\{|x_i|, |x_i| \} \) & location)

\[
k, r \leftarrow \min\{|x_k|, |x_k|\} = \min_{0\leq i<n}\{|x_i|, |x_i|\}
\]

The routine **AMIN_VALJ** finds the index of the component of an interval vector such that the absolute value of the lower or upper bounds of the component is the smallest among the absolute values of the lower and upper bounds of all components of the interval vector. When the value of the \( n \) argument is less than or equal to zero, the routine should initialize the output \( k \) to negative one or zero, and \( r \) to zero. As described in section 2.5.3, the value \( \text{incx} \) less than zero is permitted. However, if \( \text{incx} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE amin_val_i( x, k, r )
  TYPE(INTEGER)(<wp>), INTENT(IN) :: x(:)
  INTEGER, INTENT(OUT) :: k
  REAL (<wp>), INTENT(OUT) :: r
  where
  x has shape (n)
  ```
• Fortran 77 binding:

    SUBROUTINE BLAS_xamin_val_i( N, X, INCX, K, R )
    INTEGER           INCX, K, N
    <type>            X( 2, * )
    <type>            R

• C binding:

    void BLAS_xamin_val_i( int n, const <interval_array> x, int incx, int k,
                           SCALAR_INOUT r );

AMAX_VAL_I (Max absolute value & location) \quad k, r \leftarrow \max\{|x_k|,|x_k|\} = \max_{0\leq i < n} \{||x_i|,|x_i|\}

The routine AMAX_VAL_I finds the index of the component of an interval vector such that the absolute value of the lower or upper bounds of the component has the largest value among the absolute values of the lower and upper bounds of all components of the interval vector. When the value of the n argument is less than or equal to zero, the routine should initialize the output k to negative one or zero, and r to zero. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler.

• Fortran 95 binding:

    SUBROUTINE amax_val_i( x, k, r )
    TYPE INTERVAL(<wp>), INTENT(IN) :: x(:)
    INTEGER, INTENT(INOUT) :: k
    REAL (<wp>), INTENT(INOUT) :: r

where

    x has shape (n)

• Fortran 77 binding:

    SUBROUTINE BLAS_AMAX_VAL_I( N, X, INCX, K, R )
    INTEGER           INCX, K, N
    <type>            X( 2, * )
    <type>            R

• C binding:

    void BLAS_xamax_val_i( int n, const <interval_array> x, int incx, int k,
                           SCALAR_INOUT r );

SUMSQ_I (Sum of squares) \quad (scI, ssq) \leftarrow \sum x_i^2
The routine \texttt{SUMSQ} returns the intervals \texttt{scl} and \texttt{ssq} such that

\[
\text{scl}^2 \cdot \text{ssq} = \text{scale}^2 \cdot \text{sumsq} + \sum_{i=0}^{n-1} x_i^2.
\]

The value of \text{sumsq} is assumed to be at least unity and the value of \text{ssq} will then satisfy \(1.0 \leq \text{ssq} \leq (\text{sumsq} + n)\). It is assumed that \text{scale} is to be non-negative, and \text{scl} returns the value

\[
\text{scl} = \max_{0 \leq i < n} (\text{scale}, |x_i|).
\]

\text{scale} and \text{sumsq} must be supplied on entry in \text{scl} and \text{ssq} respectively. \text{scl} and \text{ssq} are overwritten by \text{scl} and \text{ssq} unchanged. As described in section 2.5.3, the value \text{incx} less than zero is permitted. However, if \text{incx} is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE sumsq_i( x, ssq, scl )
  TYPE(INTERVAL)(<wp>), INTENT(IN) :: x(:)
  TYPE(INTERVAL)(<wp>), INTENT(INOUT) :: ssq, scl
  where
  x has shape (n)
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xSUMSQ_I( N, X, INCX, SSQ, SCL )
  INTEGER INCX, N
  <type> X( 2, * )
  <type> SCL( 2 ), SSQ( 2 )
  ```

- C binding:

  ```c
  void BLAS_xsumsq_i( int n, const <interval_array> x, int incx, <interval> ssq,
                  <interval> scl );
  ```

\text{Interval Vector Operations}
\text{RScale} \_I (Reciprocal Scale of an interval vector)

\[
\text{x} \leftarrow \text{x} / \alpha
\]

The routine \text{RScale} \_I updates the entries of an interval vector \text{x} by the scale interval \(1/\alpha\) provided that \(0 \notin \alpha\). If \text{n} is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value \text{incx} less than zero is permitted. However, if \text{incx} is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE rscale_i( alpha, x )
  TYPE(INTERVAL)(<wp>), INTENT(INOUT) :: x(:)
  TYPE(INTERVAL)(<wp>), INTENT(IN) :: alpha
  where
  x has shape (n)
  ```
AXPBY_I (Scaled vector accumulation)  \[ y \leftarrow \alpha x + \beta y \]

The routine AXPBY_I scales the interval vector \( \mathbf{x} \) by the interval \( \alpha \) and the interval vector \( \mathbf{y} \) by \( \beta \), adds these two vectors to one another and stores the result in the vector \( \mathbf{y} \). If \( n \) is less than or equal to zero, or if \( \alpha \) is equal to \([0,0]\) and \( \beta \) equal to \([1,1]\), this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE axpby_i( x, y [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: x (:)
  <type>(<wp>), INTENT (INOUT) :: y (:)
where
  x and y have shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xAXPBY_I( N, ALPHA, X, INCX, BETA, Y, INCY )
INTEGER       INCX, INCY, N
<type>        ALPHA( 2 ), BETA( 2 )
<type>        X( 2, * ), Y( 2, * )
```

- C binding:

```c
void BLAS_xaxpby_i( int n, <interval> alpha, <interval_array> x, int incx,
                    <interval> beta, <interval_array> y, int incy );
```

WAXPBY_I (Scaled vector addition)  \[ w \leftarrow \alpha x + \beta y \]

The routine WAXPBY_I scales the interval vector \( \mathbf{x} \) by the interval \( \alpha \) and the interval vector \( \mathbf{y} \) by \( \beta \), adds these two vectors to one another and stores the result in the vector \( \mathbf{w} \). If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx or incy or incw less than zero is permitted. However, if either incx or incy or incw is equal to zero, an error flag is set and passed to the error handler.
• Fortran 95 binding:

SUBROUTINE wxapby_i( x, y, w [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: x(:), y(:)
  <type>(<wp>), INTENT (OUT) :: w(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta

where
  x, y and w have shape (n)

• Fortran 77 binding:

SUBROUTINE BLAS_wxapby_i( N, ALPHA, X, INCX, BETA, Y, INCY, W, $
  INTEGER INCW)
  INTEGER INCW, INCX, INCY, N
  <type> ALPHA( 2 ), BETA( 2 )
  <type> W( 2, * ), X( 2, * ), Y( 2, * )

• C binding:

void BLAS_wxapby_i( int n, <interval> alpha, const <interval_array> x,
  int incx, <interval> beta, const <interval_array> y,
  int incy, <interval_array> w, int incw );

CANCEL_I (Scaled cancellation)  \quad \quad y \leftarrow \alpha x \otimes \beta y

The operation cancel, \( \otimes \), between two intervals \( a \) and \( b \) is defined as \( a \otimes b = [a - b, \pi - b] \) if \( (a - b) \leq (\pi - b) \); Otherwise, \( 0 \). The routine CANCEL_I scales the interval vector \( x \) by the interval \( \alpha \) and the interval vector \( y \) by \( \beta \), updates \( y_i \) with \( \alpha x_i \otimes \beta y_i \), \( \forall 0 \leq i < n \). If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler.

• Fortran 95 binding:

SUBROUTINE cancel_i( x, y [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: x(:)
  <type>(<wp>), INTENT (INOUT) :: y(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta

where
  x and y have shape (n)

• Fortran 77 binding:

SUBROUTINE BLAS_xcancel_i( N, ALPHA, X, INCX, BETA, Y, INCY )
  INTEGER INCX, INCY, N
  <type> ALPHA( 2 ), BETA( 2 )
  <type> X( 2, * ), Y( 2, * )
• C binding:

```c
void BLAS_xcancel_i( int n, <interval> alpha, <interval_array> x, int incx,
                     <interval> beta, <interval_array> y, int incy );
```

WCANCEL_I (Scaled cancellation) \( w \leftarrow \alpha x \odot \beta y \)

The operation cancel, \( \odot \), between two intervals \( a \) and \( b \) is defined as \( a \odot b = [a - b, \pi - b] \) if \( (a - b) \leq (\pi - b) \); Otherwise, \( \emptyset \). The routine WCANCEL_I scales the interval vector \( x \) by the interval \( \alpha \) and the interval vector \( y \) by \( \beta \), stores \( \alpha x_i \odot \beta y_i \) in \( w_i \) for \( 0 \leq i < n \). If \( n \) is less than or equal to zero, this routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) or \( \text{incw} \) is equal to zero, an error flag is set and passed to the error handler.

• Fortran 95 binding:

```fortran
SUBROUTINE wcancel_i( x, y, w [, alpha] [, beta] )
  <type>(<wp>), INTENT (IN) :: x(:), y(:)
  <type>(<wp>), INTENT (OUT) :: w(:)
  <type>(<wp>), INTENT (IN), OPTIONAL :: alpha, beta
where
x, y, and w have shape (n)
```

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xWCANCEL_I( N, ALPHA, X, INCX, BETA, Y, INCY, W, $ INTEGER INCW, INCX, INCY, N
  <type> ALPHA( 2 ), BETA( 2 )
  <type> W( 2, * ), X( 2, * ), Y( 2, * )
```

• C binding:

```c
void BLAS_xwcancel_i( int n, <interval> alpha, <interval_array> x, int incx,
                      <interval> beta, <interval_array> y, int incy,
                      <interval_array> w, int incw );
```

Data Movement with Interval Vectors

COPY_I (Interval vector copy) \( y \leftarrow x \)

The routine COPY_I copies the interval vector \( x \) into the interval vector \( y \). If \( n \) is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

• Fortran 95 binding:
SUBROUTINE copy_i( x, y )
  TYPE(INTEGER) (<wp>), INTENT(IN) :: x(:)
  TYPE(INTEGER) (<wp>), INTENT(OUT) :: y(:)
  where
  x and y have shape (n)

- Fortran 77 binding:
  SUBROUTINE BLAS_xCOPY_I( N, X, INCX, Y, INCY )
  INTEGER INCX, INCY, N
  <type> X( 2, * ), Y( 2, * )

- C binding:
  void BLAS_xcopy_i( int n, const <interval_array> x, int incx,
                   <interval_array> y, int incy );

SWAP_I (Interval vector swap) y ↦ x

The routine SWAP_I interchanges the interval vectors x and y. If n is less than or equal to zero,
the routine returns immediately. As described in section 2.5.3, the value incx or incy less than zero
is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the
error handler.

- Fortran 95 binding:
  SUBROUTINE swap_i( x, y )
  TYPE(INTEGER) (<wp>), INTENT(INOUT) :: x(:), y(:)
  where
  x and y have shape (n)

- Fortran 77 binding:
  SUBROUTINE BLAS_xSWAP_I( N, X, INCX, Y, INCY )
  INTEGER INCX, INCY, N
  <type> X( 2, * ), Y( 2, * )

- C binding:
  void BLAS_xswap_i( int n, <interval_array> x, int incx, <interval_array> y, int incy );

PERMUTE_I (Permute interval vector) x ← Px

The routine PERMUTE_I permutes the entries of an interval vector x according to the permutation
vector P. If n is less than or equal to zero, the routine returns immediately. As described in section
2.5.3, the value incx or incp less than zero is permitted. However, if either incx or incp is equal to
zero, an error flag is set and passed to the error handler.
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- Fortran 95 binding:

```fortran
SUBROUTINE permute_i(x, p )
  INTEGER, INTENT(IN) :: p(:)
  TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: x(:)
where
  x and p have shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xPERMUTE_I( N, P, INCP, X, INCX )
  INTEGER INCP, INCX, N
  INTEGER P( * )
  <type> X( 2, * )
```

- C binding:

```c
void BLAS_xpermute_i( int n, const int *p, int incp, <interval_array> x, int incx );
```

Interval Matrix-Vector Operations

\{GE,GB\}MV (Interval matrix-vector multiplication)

\[
y \leftarrow \alpha A x + \beta y, y \leftarrow \alpha A^T x + \beta y
\]

The routines multiply the interval vector \( x \) by a general (or general band) interval matrix \( A \) or its transpose, scales the resulting interval vector and adds it to the scaled interval vector operand \( y \). If \( m \) or \( n \) is less than or equal to zero or if \( \text{beta} \) is equal to \([1,1]\) and \( \text{alpha} \) is equal to \([0,0]\), the routine returns immediately. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if either \( \text{inxc} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler. For the routine \( \text{GEMV} \), if \( lda \) is less than one or if \( lda \) is less than \( m \), an error flag is set and passed to the error handler. For the routine \( \text{GBMV} \), if \( kl \) or \( ku \) is less than zero, or if \( lda \) is less than \( kl \) plus \( ku \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  General:
  ```fortran
  SUBROUTINE gemv_i( a, x, y [, transa ] [, alpha] [, beta] )
  ```

  General Band:
  ```fortran
  SUBROUTINE gbmv_i( a, m, kl, x, y [, transa ] [, alpha] [, beta] )
  ```

  all:
  ```fortran
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), x(:)
  TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: y(:)
  INTEGER INTENT(IN) :: m, kl
  TYPE(blas_trans_type), INTENT(IN), OPTIONAL :: transa
  TYPE(INTERVAL) (<wp>), INTENT(INOUT), OPTIONAL :: alpha, beta
  ```

where

  - \( a \) has shape \((m,n)\) for general matrix
  - \((1,n)\) for general banded matrix \((1 > kl)\)
  - \( x \) and \( y \) have shape \( n \) if \( \text{transa} = \text{blas_no_trans} \) (the default)
  - \( m \) if \( \text{transa} /= \text{blas_no_trans} \)
• Fortran 77 binding:

General:

```fortran
SUBROUTINE BLAS_xGEMV_I( TRANS, M, N, ALPHA, A, LDA, X, INCX, BETA, $ Y, INCY )
```

General Band:

```fortran
SUBROUTINE BLAS_xGBMV_I( TRANS, M, N, KL, KU, ALPHA, A, LDA, X, INCX, $ BETA, Y, INCY )
```

all:

```fortran
INTEGER INCX, INCY, KL, KU, LDA, M, N, TRANS
<type> ALPHA( 2 ), BETA( 2 )
<type> A( 2, LDA, * ), X( 2, * ), Y( 2, * )
```

• C binding:

General:

```c
void BLAS_xgemv_i( enum blas_order_type order, enum blas_trans_type trans, int m, int n, <interval> alpha, const <interval_array> a, 
int lda, const <interval_array> x, int incx, <interval> beta, 
<interval_array> y, int incy );
```

General Band:

```c
void BLAS_xgbmv_i( enum blas_order_type order, enum blas_trans_type trans, int m, int n, int kl, int ku, <interval> alpha, 
const <interval_array> a, int lda, const <interval_array> x, 
int incx, <interval> beta, <interval_array> y, int incy );
```

\{SY,SB,SP\}MV_I (Interval symmetric matrix vector product) \( y \leftarrow \alpha A x + \beta y \) with \( A = A^T \)

The routines multiply an interval vector \( x \) by a symmetric interval matrix \( A \), scales the resulting interval vector and adds it to the scaled interval vector operand \( y \). If \( n \) is less than or equal to zero or if \texttt{beta} is equal to one and \texttt{alpha} is equal to zero, the routine returns immediately. The operator argument \texttt{uplo} specifies if the matrix operand is an upper or lower triangular part of the symmetric matrix. As described in section 2.5.3, the value \texttt{incx} or \texttt{incy} less than zero is permitted. However, if either \texttt{incx} or \texttt{incy} is equal to zero, an error flag is set and passed to the error handler. For the routine \texttt{SYMV_I}, if \( \texttt{lda} \) is less than one or \( \texttt{lda} \) is less than \( n \), an error flag is set and passed to the error handler. For the routine \texttt{SBMV_I}, if \( \texttt{lda} \) is less than \( k + 1 \) plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:

Symmetric:

```fortran
SUBROUTINE symv_i( a, x, y [, uplo] [, alpha] [, beta] )
```

Symmetric Band:

```fortran
SUBROUTINE sbmv_i( a, x, y [, uplo] [, alpha] [, beta] )
```

Symmetric Packed:

```fortran
SUBROUTINE spmv_i( ap, x, y [, uplo] [, alpha] [, beta] )
```
all:
   TYPE(INTEGER) (<wp>), INTENT(IN) :: a(:,,:), ap(:,), x(:)
   TYPE(INTEGER) (<wp>), INTENT(INOUT) :: y(:)
   TYPE blas_uplo_type, INTENT(IN), OPTIONAL :: uplo
   TYPE(INTEGER) (<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
   x and y have shape (n)
   SY a has shape (n,n)
   SB a has shape (k+1,n), where k = band width
   SP ap has shape (n*(n+1)/2)

• Fortran 77 binding:

Symmetric:
   SUBROUTINE BLAS_XSYMV_I( UPL0, N, ALPHA, A, LDA, X, INCX, BETA, Y,  
                            INCY )
Symmetric Band:
   SUBROUTINE BLAS_XSBMV_I( UPL0, N, K, ALPHA, A, LDA, X, INCX, BETA,  
                            Y, INCY )
Symmetric Packed:
   SUBROUTINE BLAS_XSPMV_I( UPL0, N, ALPHA, AP, X, INCX, BETA, Y, INCY )

all:
   INTEGER INCX, INCY, K, LDA, N, UPL0
   <type> ALPHA( 2 ), BETA( 2 )
   <type> A( 2, LDA, * ) or AP( 2, * ), X( 2, * ),
   $ Y( 2, * )

• C binding:

Symmetric:
   void BLAS_xsymv_i( enum blas_order_type order, enum blas_uplo_type uplo, int n,  
                    <interval> alpha, const <interval_array> a, int lda,  
                    const <interval_array> x, int incx, <interval> beta,  
                    <interval_array> y, int incy );
Symmetric Band:
   void BLAS_xsbmv_i( enum blas_order_type order, enum blas_uplo_type uplo, int n,  
                    int k, <interval> alpha, const <interval_array> a, int lda,  
                    const <interval_array> x, int incx, <interval> beta,  
                    <interval_array> y, int incy );
Symmetric Packed:
   void BLAS_xspmv_i( enum blas_order_type order, enum blas_uplo_type uplo, int n,  
                    <interval> alpha, const <interval_array> ap,  
                    const <interval_array> x, int incx, <interval> beta,  
                    <interval_array> y, int incy );

\{TR,TB,TP\}_MV_J (Interval triangular matrix vector product)  
   \( \mathbf{x} \leftarrow \alpha \mathbf{T} \mathbf{x} \) \( \mathbf{x} \leftarrow \alpha \mathbf{T}^T \mathbf{x} \)
The routines multiply an interval vector $x$ by a general triangular interval matrix $T$ or its transpose, and copies the resulting vector in the vector operand $x$. If $n$ is less than or equal to zero, the routine returns immediately. As described in section 2.5.3, the value $\text{incx}$ less than zero is permitted. However, if $\text{incx}$ is equal to zero, an error flag is set and passed to the error handler. For the routine TRMV, if $\text{l dt}$ is less than one or $\text{l dt}$ is less than $n$, an error flag is set and passed to the error handler. For the routine TBMV, if $\text{l dt}$ is less than $k$ plus one, an error flag is set and passed to the error handler.

The operator argument uplo specifies whether the matrix operand is upper or lower triangular. The operator argument diag specifies whether or not the matrix operand has unit diagonal entries.

- Fortran 95 binding:

  Triangular:
  SUBROUTINE trmv_i( t, x [, uplo] [, transt] [, diag] [, alpha] )
  Triangular Band:
  SUBROUTINE tmbv_i( t, x [, uplo] [, transt] [, diag] [, alpha] )
  Triangular Packed:
  SUBROUTINE tpmv_i( tp, x [, uplo] [, transt] [, diag] [, alpha] )
  all:
  TYPE(INTEGER) (<type>), INTENT(IN) :: t(:,:), tp(:)
  TYPE(INTEGER) (<type>), INTENT(INOUT) :: x(:)
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE(blas_trans_type), INTENT(IN), OPTIONAL :: transt
  TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
  TYPE(INTEGER) (<type>), INTENT(IN), OPTIONAL :: alpha

  where
  $x$ has shape (n)
  $T$ $t$ has shape (n,n)
  $TB$ $t$ has shape (k+1,n) where $k$ = band width
  $TP$ $tp$ has shape (n*(n+1)/2)

- Fortran 77 binding:

  Triangular:
  SUBROUTINE BLAS_xTRMV_I( UPLO, TRANS, DIAG, N, ALPHA, T, LDT, X,
                           $INCR_X$ )
  Triangular Band:
  SUBROUTINE BLAS_xTBMV_I( UPLO, TRANS, DIAG, N, K, ALPHA, T, LDT, X,
                           $INCR_X$ )
  Triangular Packed:
  SUBROUTINE BLAS_xTPMV_I( UPLO, TRANS, DIAG, N, ALPHA, TP, X, INCR_X )
  all:
  INTEGER      DIAG, INCR_X, K, LDA, N, TRANS, UPLO
  <type>        ALPHA( 2 )
  <type>        T( 2, LDA, * ) or TP( 2, * ), X( 2, * )

- C binding:
Triangular:
void BLAS_xtrmv_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag, int n,
<interval> alpha, const <interval_array> t, int ldt,
<interval_array> x, int incx );

Triangular Band:
void BLAS_xtbmv_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag, int n,
<interval> alpha, const <interval_array> t, int ldt,
<interval_array> x, int incx );

Triangular Packed:
void BLAS_xtpmv_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag, int n,
<interval> alpha, const <interval_array> tp,
<interval_array> x, int incx );

{TR,TB,TP}SV_J (Interval triangular solve with a vector) $x \leftarrow \alpha T^{-1}x$, $x \leftarrow \alpha T^{-T}x$

These routines bound one of the systems of equations $x \leftarrow \alpha T^{-1}x$ or $x \leftarrow \alpha T^{-T}x$, where $x$ is an interval vector and the matrix $T$ is a upper or lower triangular (or triangular banded or triangular packed) interval matrix. If $n$ is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler. If ldt is less than one or ldt is less than n, an error flag is set and passed to the error handler.

Advice to users and implementors: Checking for singularity, or near singularity is not specified for these triangular solvers. Users should perform such a test before calling the triangular solver if their applications require such a test.

• Fortran 95 binding:

Triangular:
SUBROUTINE trsv_i( t, x [, uplo] [, transt] [, diag] [, alpha] )

Triangular Band:
SUBROUTINE tbsv_i( t, x [, uplo] [, transt] [, diag] [, alpha] )

Triangular Packed:
SUBROUTINE tpsv_i( tp, x [, uplo] [, transt] [, diag] [, alpha] )

all:
TYPE(INTERVAL) (<wp>), INTENT(IN) :: t(:,,:), tp(:)
TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: x(:)
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_trans_type), INTENT(IN), OPTIONAL :: transt
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL :: alpha

where
x has shape (n)
TR t has shape (n,n)
TB  t has shape (k+1,n) where k = band width
TP  tp has shape (n*(n+1)/2)

- Fortran 77 binding:

Triangular:
SUBROUTINE BLAS_xTRSV_I( UPLO, TRANS, DIAG, N, ALPHA, T, LDT, X, 
   $           INCX )

Triangular Band:
SUBROUTINE BLAS_xTBSV_I( UPLO, TRANS, DIAG, N, K, ALPHA, T, LDT, 
   $           X, INCX )

Triangular Packed:
SUBROUTINE BLAS_xTPSV_I( UPLO, TRANS, DIAG, N, ALPHA, TP, X, INCX )

all:
INTEGER        DIAG, INCX, K, LDT, N, TRANS, UPLO
<type>          ALPHA( 2 )
<type>          T( 2, LDA, * ) or TP( 2, * ), X( 2, * )

- C binding:

Triangular:
void BLAS_xtrsv_i( enum blas_order_type order, enum blas_uplo_type uplo,
   enum blas_trans_type trans, enum blas_diag_type diag, int n,
   const <interval> alpha, const <interval_array> t, int ldt,
   <interval_array> x, int incx );

Triangular Band:
void BLAS_xtbsv_i( enum blas_order_type order, enum blas_uplo_type uplo,
   enum blas_trans_type trans, enum blas_diag_type diag, int n,
   int k, const <interval> alpha, const <interval_array> t,
   int ldt, <interval_array> x, int incx );

Triangular Packed:
void BLAS_xtpsv_i( enum blas_order_type order, enum blas_uplo_type uplo,
   enum blas_trans_type trans, enum blas_diag_type diag, int n,
   const <interval> alpha, const <interval_array> tp,
   <interval_array> x, int incx );

GERJ (Rank one update)  \[ A \leftarrow \alpha xy^T + \beta A \]

This routine performs the operation \[ A \leftarrow \alpha xy^T + \beta A \], where \( \alpha \) and \( \beta \) are intervals, \( x \) and \( y \) are interval vectors, and \( A \) is an interval matrix. This routine returns \( A \) immediately if \( \alpha = [0,0] \) and \( \beta = [1,1] \). If \( m \) or \( n \) is less than or equal to zero, this function returns immediately. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if either incx or incy is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than m, an error flag is set and passed to the error handler.

- Fortran 95 binding:
SUBROUTINE ger_i( a, x, y [, alpha] [, beta] )
TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: a(:,:)
TYPE(INTERVAL) (<wp>), INTENT(IN) ::= x(:,), y(:)
TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL ::= alpha, beta

where
x and y have shape (n)
a has shape (n,n)

• Fortran 77 binding:

SUBROUTINE BLAS_xGER_I( M, N, ALPHA, X, INCX, Y, INCY, BETA, A, LDA )
INTEGER INCX, INCY, LDA, M, N
<type> ALPHA( 2 ), BETA( 2 )
<type> A( 2, LDA, * ), X( 2, * ), Y( 2, * )

• C binding:

void BLAS_xger_i( int m, int n, <interval> alpha, const <interval_array> x,
int incx, const <interval_array> y, int incy, <interval> beta,
<interval_array> a, int lda );

{SY,SP}R1 (Symmetric rank one update)  \[ A \leftarrow \alpha xx^T + \beta A \] with \( A = A^T \)

This routine performs the symmetric update \( A \leftarrow \alpha xy^T + \beta A \), where \( \alpha \) and \( \beta \) are intervals, \( x \) is an interval vector, and \( A \) is a symmetric interval matrix. This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value incx less than zero is permitted. However, if incx is equal to zero, an error flag is set and passed to the error handler. If lda is less than one or lda is less than \( n \), an error flag is set and passed to the error handler.

• Fortran 95 binding:

Symmetric:
SUBROUTINE syr_i( a, x [, uplo] [, alpha] [, beta] )
Symmetric Packed:
SUBROUTINE spr_i( ap, x [, uplo] [, alpha] [, beta] )

all:
TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: a(:,:), ap(:)
TYPE(INTERVAL) (<wp>), INTENT(IN) ::= x(:)
TYPE(blas_uplo_type), OPTIONAL ::= uplo
TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL ::= alpha, beta

where
x has shape (n)
Sy a has shape (n,n)
Sp ap has shape (n*(n+1)/2)

• Fortran 77 binding:
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Symmetric:

```
SUBROUTINE BLAS_xSYR_I( UPLO, N, ALPHA, X, INCX, BETA, A, LDA )
```

Symmetric Packed:

```
SUBROUTINE BLAS_xSPR_I( UPLO, N, ALPHA, X, INCX, BETA, AP )
```

All:

```
INTEGER       INCX, LDA, N, UPLO
<type>        ALPHA( 2 ), BETA( 2 )
<type>        A( 2, LDA, * ) or AP( 2, * ), X( 2, * )
```

- C binding:

```
Symmetric:
void BLAS_xsyr_i( enum blas_order_type order, enum blas_uplo_type uplo, int n,
<interval> alpha, const <interval_array> x, int incx,
<interval> beta, <interval_array> a, int lda );
```

Symmetric Packed:

```
void BLAS_xspr_i( enum blas_order_type order, enum blas_uplo_type uplo, int n,
<interval> alpha, const <interval_array> x, int incx,
<interval> beta, <interval_array> ap );
```

Interval Matrix Operations

```
{GE,GB,SY,SB,SP,TR,TB,TP}_NORM_J (Interval matrix norms)
```

```
r ← ||A||_1, ||A||_F, ||A||_\infty, or ||A||_\max
```

These routines compute the one-norm, Frobenius-norm, infinity-norm, or max-norm of a general
interval matrix \(A\) depending on the value passed as the norm operator argument. This routine
returns immediately with the output scalar \(r\) set to zero if \(m\) (for nonsymmetric matrices) or \(n\) is
less than or equal to zero. For the routine \(GE_{\text{NORM}}J\), if lda is less than one or lda is less than \(m\),
an error flag is set and passed to the error handler. For the routine \(GB_{\text{NORM}}J\), if lda is less than \(k\)
plus ku plus one, an error flag is set and passed to the error handler. For the routines \(SY_{\text{NORM}}J\)
and \(TR_{\text{NORM}}J\), if lda is less than one or lda is less than \(n\), an error flag is set and passed to the
error handler. For the routines \(SB_{\text{NORM}}J\) and \(TB_{\text{NORM}}J\), if lda is less than \(k\) plus one, an error
flag is set and passed to the error handler.

- Fortran 95 binding:

```
General:
REAL (<wp>) FUNCTION ge_norm_i( a [, norm] )
```

General Band:

```
REAL (<wp>) FUNCTION gb_norm_i( a, m, kl [, norm] )
```

Symmetric:

```
REAL (<wp>) FUNCTION sy_norm_i( a [, norm] [, uplo] )
```

Symmetric Band:

```
REAL (<wp>) FUNCTION sb_norm_i( a [, norm] [, uplo] )
```

Symmetric Packed:

```
REAL (<wp>) FUNCTION sp_norm_i( ap [, norm] [, uplo] )
```
Triangular:
    REAL (<<wp>>) FUNCTION tr_norm_i( a [, norm [, uplo] [, diag] )
Triangular Band:
    REAL (<<wp>>) FUNCTION tb_norm_i( a [, norm [, uplo] [, diag] )
Triangular Packed:
    REAL (<<wp>>) FUNCTION tp_norm_i( ap [, norm [, uplo] [, diag] )
all:
    TYPE(INTEGER) (<<wp>>, INTENT(IN)) :: a(:,:) | ap(:)
    INTEGER, INTENT(IN) :: m, kl
    TYPE blas_norm_type, INTENT(IN), OPTIONAL :: norm
    TYPE blas_uplo_type, INTENT(IN), OPTIONAL :: uplo
    TYPE blas_diag_type, INTENT(IN), OPTIONAL :: diag
where
    a has shape (m,n) for general matrix
    (1,n) for general banded matrix (1 > kl)
    (n,n) for symmetric or triangular
    (k+1,n) for symmetric banded, or triangular
    banded (k=band width)
    ap has shape (n*(n+1)/2).

- Fortran 77 binding:

    General:
        <type> FUNCTION BLAS_xGE_NORM_I( NORM, M, N, A, LDA )
    General Band:
        <type> FUNCTION BLAS_xGB_NORM_I( NORM, M, N, KL, KU, A, LDA )
    Symmetric:
        <type> FUNCTION BLAS_xSY_NORM_I( NORM, UPLO, N, A, LDA )
    Symmetric Band:
        <type> FUNCTION BLAS_xSB_NORM_I( NORM, UPLO, N, K, A, LDA )
    Symmetric Packed:
        <type> FUNCTION BLAS_xSP_NORM_I( NORM, UPLO, N, AP )
    Triangular:
        <type> FUNCTION BLAS_xTR_NORM_I( NORM, UPLO, DIAG, N, A, LDA )
    Triangular Band:
        <type> FUNCTION BLAS_xTB_NORM_I( NORM, UPLO, DIAG, N, K, A, LDA )
    Triangular Packed:
        <type> FUNCTION BLAS_xTP_NORM_I( NORM, UPLO, DIAG, N, AP )
all:
    INTEGER
    DIAG, K, KL, KU, LDA, M, N, NORM, UPLO
    <type>
    A( 2, LDA, * ) or AP( 2, * )

- C binding:

    General:
    void BLAS_xge_norm_i( enum bias_order_type order, enum bias_norm_type norm,
                           int m, int n, const <interval_array> a, int lda,
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```
SCALAR_INOUT r);

General Band:
void BLAS_xgb_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  int m, int n, int kl, int ku, const <interval_array> a,
  int lda, SCALAR_INOUT r);

Symmetric:
void BLAS_xsy_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, int n,
  const <interval_array> a, int lda, SCALAR_INOUT r);

Symmetric Band:
void BLAS_xsb_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, int n, int k,
  const <interval_array> a, int lda, SCALAR_INOUT r);

Symmetric Packed:
void BLAS_xsp_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, int n,
  const <interval_array> a, int lda, SCALAR_INOUT r);

Triangular:
void BLAS_xtr_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, enum blas_diag_type diag,
  int n, const <interval_array> a, int lda,
  SCALAR_INOUT r);

Triangular Band:
void BLAS_xtb_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, enum blas_diag_type diag,
  int n, int k, const <interval_array> a, int lda,
  SCALAR_INOUT r);

Triangular Packed:
void BLAS_xtp_norm_i( enum blas_order_type order, enum blas_norm_type norm,
  enum blas_uplo_type uplo, enum blas_diag_type diag,
  int n, const <interval_array> a, int lda,
  SCALAR_INOUT r);

Advice to implementors: In finite precision floating point arithmetic, an upper bound, preferably the least machine representable upper bound, for the mathematical value should be returned for the norms.

{GE,GB}_DIAG_SCALE_i (Diagonal scaling an interval matrix) \( A \leftarrow DA, AD \) with \( D \) diagonal

These routines scale a general (or banded) interval matrix \( A \) on the left side or the right side by a diagonal interval matrix \( D \). This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. As described in section 2.5.3, the value incd less than zero is permitted. However, if incd is equal to zero, an error flag is set and passed to the error handler. For the routine GE_DIA SCALE_i, if lda is less than one or lda is less than m, an error flag is set and passed to the error handler. For the routine GB_DIA SCALE_i, if lda is less than kl plus ku plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:
General:
  SUBROUTINE ge_diag_scale_i( d, a [, side ] )
General Band:
  SUBROUTINE gb_diag_scale_i( d, a, m, kl [, side] )
all:
  TYPE(INTERVAL) (<wp>), INTENT (IN) :: d(:)
  TYPE(INTERVAL) (<wp>), INTENT (INOUT) :: a(:, :)
  INTEGER, INTENT(IN) :: m, kl
  TYPE blas_side_type, INTENT(IN), OPTIONAL :: side
where
  a has shape (m, n) for general matrix
  (1, n) for general banded matrix (1 > kl)
  d has shape (p) where p = m if side = blas_left_side
         p = n if side = blas_right_side

• Fortran 77 binding:

General:
  SUBROUTINE BLAS_xGE_DIAG_SCALE_I( SIDE, M, N, D, INCID, A, LDA )
General Band:
  SUBROUTINE BLAS_xGB_DIAG_SCALE_I( SIDE, M, N, KL, KU, D, INCID, A,
                                  LDA )
all:
  INTEGER     INCID, KL, KU, LDA, M, N, SIDE
  <type>      A( 2, LDA, * ), D( 2, * )

• C binding:

General:
  void BLAS_xge_diag_scale_i( enum blas_order_type order,
                              enum blas_side_type side, int m, int n,
                              const <interval_array> d, int incd,
                              <interval_array> a, int lda );
General Band:
  void BLAS_xgb_diag_scale_i( enum blas_order_type order,
                              enum blas_side_type side, int m, int n, int kl,
                              int ku, const <interval_array> d, int incd,
                              <interval_array> a, int lda );

{GE,GB}_{LRSCALE}_I (Two-sided diagonal scaling)    \quad A \leftarrow D_L A D_R with D_L, D_R diagonal

These routines scale a general (or banded) interval matrix A on the left side by an interval diagonal matrix D_L and on the right side by an interval diagonal matrix D_R. This routine returns immediately if m or n is less than or equal to zero. As described in section 2.5.3, the value incd or incdu less than zero is permitted. However, if either incdl or incdu is equal to zero, an error flag is set and passed to the error handler. For the routine GE_{LRSCALE}_I, if lda is less than one or lda is
less than \( m \), an error flag is set and passed to the error handler. For the routine `GB\_LRSCALE\_j`, if \( \text{lda} \) is less than \( k_l \) plus \( k_u \) plus one, an error flag is set and passed to the error handler.

- For Fortran 95 binding:

  General:
  
  ```fortran
  SUBROUTINE ge_lrscale_i( dl, dr, a )
  ```

  General Band:
  
  ```fortran
  SUBROUTINE gb_lrscale_i( dl, dr, a, m, kl )
  ```

  all:
  
  ```fortran
  TYPE(INTERVAL) ( <wp> ), INTENT(IN) :: dl(:), dr(:)
  TYPE(INTERVAL) ( <wp> ), INTENT(INOUT) :: a(:,:,)
  INTEGER, INTENT(IN) :: m, kl
  ```

  where
  
  a has shape \((m,n)\) for general matrix
  
  \((1,n)\) for general banded matrix \((1 > kl)\)

  dl has shape \((m)\)

  dr has shape \((n)\)

- For Fortran 77 binding:

  General:
  
  ```fortran
  SUBROUTINE BLAS\_xGE\_LRSCALE\_I( M, N, DL, INCDL, DR, INCDR, A, LDA )
  ```

  General Band:
  
  ```fortran
  SUBROUTINE BLAS\_xGB\_LRSCALE\_I( M, N, KL, KU, DL, INCDL, DR, INCDR, A, LDA )
  ```

  all:
  
  ```fortran
  INTEGER INCDL, INCDR, KL, KU, LDA, M, N
  <type> A( 2, LDA, * ), DL( 2, * ), DR( 2, * )
  ```

- C binding:

  General:
  
  ```c
  void BLAS\_xge\_lrscale\_i( enum blas\_order\_type order, int m, int n,
    const <interval_array> dl, int incdl,
    const <interval_array> dr, int incdr,
    <interval_array> a, int lda );
  ```

  General Band:
  
  ```c
  void BLAS\_xgb\_lrscale\_i( enum blas\_order\_type order, int m, int n,
    int kl, int ku, const <interval_array> dl,
    int incdl, const <interval_array> dr, int incdr,
    <interval_array> a, int lda );
  ```

\{SY\_SB\_SP\}_LRSCALE\_J (Two-sided diagonal scaling) \hspace{1cm} A \leftarrow DAD \text{ with } A = A^T. 
These routines perform a two-sided scaling on a symmetric (or symmetric banded or symmetric packed) interval matrix A by an interval diagonal matrix D. This routine returns immediately if n is less than or equal to zero. As described in section 2.5.3, the value incd less than zero is permitted. However, if incd is equal to zero, an error flag is set and passed to the error handler. For the routines SYLRScaleJ and SPLRScaleJ, if lda is less than one or lda is less than n, an error flag is set and passed to the error handler. For the routine SB.LRScaleJ, if lda is less than kl plus ku plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  Symmetric:
  ```fortran
  SUBROUTINE sy_lrscale_i( d, a [, uplo] )
  ```
  Symmetric Band:
  ```fortran
  SUBROUTINE sb_lrscale_i( d, a [, uplo] )
  ```
  Symmetric Packed:
  ```fortran
  SUBROUTINE sp_lrscale_i( d, ap [, uplo] )
  ```
  all:
  ```fortran
  TYPE(INTERVAL) ( <wp> ), INTENT(IN) :: d(:)
  TYPE(INTERVAL) ( <wp> ), INTENT(INOUT) :: a(:,:) | ap(:)
  TYPE blas_uplo_type, INTENT(IN), OPTIONAL :: uplo
  where
  a has shape (n,n) for symmetric
  (k+1,n) for symmetric banded (k=band width)
  ap has shape (n*(n+1)/2).
  d has shape (n)

- Fortran 77 binding:

  Symmetric:
  ```fortran
  SUBROUTINE BLAS_xSY_LRSCALE_I( UPLO, N, D, INC, A, LDA )
  ```
  Symmetric Band:
  ```fortran
  SUBROUTINE BLAS_xSB_LRSCALE_I( UPLO, N, K, D, INC, A, LDA )
  ```
  Symmetric Packed:
  ```fortran
  SUBROUTINE BLAS_xSP_LRSCALE_I( UPLO, N, D, INC, AP )
  ```
  all:
  ```fortran
  INTEGER INC, K, LDA, N, UPLO
  <type>
  A( 2, LDA, * ) or AP( 2, * ) , D( 2, * )
  ```

- C binding:

  Symmetric:
  ```c
  void BLAS_xsy_lrscale_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          int n, const <interval_array> d, int incd,
                          <interval_array> a, int lda );
  ```
  Symmetric Band:
  ```c
  void BLAS_xsb_lrscale_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          int n, int k, const <interval_array> d, int incd,
```
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<interval_array> a, int lda );

void BLAS_xsp_lrscale_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> d, int incd,
    <interval_array> ap );

{GE,SY,SB,SP}.ACC_I (Matrix accumulation and scale) \( B \leftarrow \alpha A + \beta B, \ B \leftarrow \alpha A^T + \beta B \).

These routines scale an interval matrix \( A \) (or its transpose) and scale an interval matrix \( B \) and accumulate the result in the interval matrix \( B \). Matrices \( A \) (or \( A^T \)) and \( B \) have the same storage format. This routine returns immediately if \( m \) (for nonsymmetric matrices) or \( n \) or \( k \) (for symmetric band matrices) is less than or equal to zero. For the routine GE_ACC_I, if lda is less than one or lda is less than \( m \), an error flag is set and passed to the error handler. For the routine SY_ACC_I, if lda is less than one or lda is less than \( n \), an error flag is set and passed to the error handler. For the routine SB_ACC_I, if lda is less than \( kl \) plus \( ku \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

General:
SUBROUTINE ge_acc_i( a, b [, transa] [, alpha] [, beta] )

Symmetric:
SUBROUTINE sy_acc_i( a, b [ , uplo] [, transa] [, alpha] [, beta] )

Symmetric Band:
SUBROUTINE sb_acc_i( a, b [ , uplo] [, transa] [, alpha] [, beta] )

Symmetric Packed:

all:

TYPE(interval) (<wp>), INTENT(INOUT) :: b(:,:), bp(:)

where
GE a and b have shape (m,n) if transa = blas_no_trans (the default)
    a has shape (n,m) and b has shape (m,n) if transa /= blas_no_trans
SY a and b have shape (n,n)
SB a and b have shape (p+1,n) (p = band width)
SP ap and bp have shape (n*(n+1)/2)

- Fortran 77 binding:

General:
SUBROUTINE BLAS_xGE_ACC_I( TRANS, M, N, ALPHA, A, LDA, BETA, B, LDB )

Symmetric:
SUBROUTINE BLAS_xSY_ACC_I( UPLO, TRANS, N, ALPHA, A, LDA, BETA, B, 1D.
$ LDB )$

Symmetric Band:

```fortran
SUBROUTINE BLAS_xSB_ACC_I( UPLO, TRANS, N, K, ALPHA, A, LDA, BETA,
$ B, LDB )$
```

Symmetric Packed:

```fortran
SUBROUTINE BLAS_xSP_ACC_I( UPLO, TRANS, N, ALPHA, AP, BETA, BP )
```

all:

```
INTEGER K, LDA, LDB, M, N, TRANS, UPLO
 тип >
 тип > A( 2, LDA, * ) or AP( 2, * ), B( 2, LDB, * ) or BP( 2, * )
```

- C binding:

General:

```c
void BLAS_xge_acc_i( enum blas_order_type order, enum blas_trans_type trans,
 int m, int n, <interval> alpha, const <interval_array> a,
 int lda, <interval> beta, <interval_array> b, int ldb );
```

Symmetric:

```c
void BLAS_xsy_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
 enum blas_trans_type trans, int n, <interval> alpha,
 const <interval_array> a, int lda, <interval> beta,
 <interval_array> b, int ldb );
```

Symmetric Band:

```c
void BLAS_xsb_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
 enum blas_trans_type trans, int n, int k, <interval> alpha,
 const <interval_array> a, int lda, <interval> beta,
 <interval_array> b, int ldb );
```

Symmetric Packed:

```c
void BLAS_xsp_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
 enum blas_trans_type trans, int n, <interval> alpha,
 const <interval_array> ap, <interval> beta,
 <interval_array> bp );
```

{GB,TR,TB,TP}_ACC_I (Matrix accumulation and scale)  \[ B \leftarrow \alpha A + \beta B \].

These routines scale interval matrices \( A \) and \( B \) and accumulate the result in the matrix \( B \). Matrices \( A \) and \( B \) have the same storage format. This routine returns immediately if \( m \) or \( k \) or \( k + \)num (for general band matrices) or \( n \) or \( k \) (for triangular band matrices) is less than or equal to zero. For the routine GB_ACC_I, if \( lda \) is less than \( k \) plus \( k + 1 \) plus one, an error flag is set and passed to the error handler. For the routines TR_ACC_I and TP_ACC_I, if \( lda \) is less than one or \( lda \) is less than \( n \), an error flag is set and passed to the error handler. For the routine TB_ACC_I, if \( lda \) is less than \( k + 1 \) plus one, an error flag is set and passed to the error handler.

- Fortran 95 binding:

General Band:
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SUBROUTINE gb_acc_i( a, m, kl, b [, alpha [ , beta ] )
Triangular:
   SUBROUTINE tr_acc_i( a, b [, uplo], [ , diag ] , alpha [ , beta ] )
Triangular Band:
   SUBROUTINE tb_acc_i( a, b [, uplo], [ , diag ] , alpha [ , beta ] )
Triangular Packed:
   SUBROUTINE tp_acc_i( ap, bp [, uplo], [ , diag ] , alpha [ , beta ] )
all:

   TYPE ( INTERVAL ) (<wp> ), INTENT ( IN ) :: a (: , :) | ap (: )
   INTEGER, INTENT ( IN ) :: m, kl
   TYPE ( INTERVAL ) (<wp> ), INTENT ( INOUT ) :: b (: , :) | bp (: )
   TYPE ( INTERVAL ) (<wp> ), INTENT ( IN ), OPTIONAL :: alpha, beta
   TYPE ( blas_uplo_type ), INTENT ( IN ), OPTIONAL :: uplo
   TYPE ( blas_diag_type ), INTENT ( IN ), OPTIONAL :: diag

where
   a and b have shape ( 1 , n ) for general banded matrix ( 1 > kl )
   a and b have shape ( n , n ) for triangular matrix
   a and b have shape ( p+1 , n ) for triangular banded matrix
   ap and bp have shape ( n*(n+1)/2 )

- Fortran 77 binding:

  General Band:
   SUBROUTINE BLAS_xGB_ACC_I( M, N, KL, KU, ALPHA, A, LDA, BETA, B,
                             LDB )
  Triangular:
   SUBROUTINE BLAS_xTR_ACC_I( UPLO, DIAG, N, ALPHA, A, LDA, BETA, B,
                             LDB )
  Triangular Band:
   SUBROUTINE BLAS_xTB_ACC_I( UPLO, DIAG, N, K, ALPHA, A, LDA, BETA,
                             B, LDB )
  Triangular Packed:
   SUBROUTINE BLAS_xTP_ACC_I( UPLO, DIAG, N, ALPHA, AP, BETA, BP )
all:

   INTEGER          DIAG, K, KL, KU, LDA, LDB, M, N, UPLO
   <type>            ALPHA ( 2 ), BETA ( 2 )
   <type>            A ( 2, LDA, * ) or AP ( 2, * ), B ( 2, LDB, * )
   or BP ( 2, * )

- C binding:

  General Band:
  void BLAS_xgb_acc_i( enum blas_order_type order, int m, int n, int kl, int ku,
                    <interval> alpha, <interval_array> a, int lda,
                    <interval> beta, <interval_array> b, int ldb );
  Triangular:
void BLAS_xtr_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
            enum blas_diag_type diag, int n, <interval> alpha,
            <interval_array> a, int lda, <interval> beta,
            <interval_array> b, int ldb );

Triangular Band:
void BLAS_xtb_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
            enum blas_diag_type diag, int n, int k, <interval> alpha,
            <interval_array> a, int lda, <interval> beta,
            <interval_array> b, int ldb );

Triangular Packed:
void BLAS_xtp_acc_i( enum blas_order_type order, enum blas_uplo_type uplo,
            enum blas_diag_type diag, int n, <interval> alpha,
            <interval_array> ap, <interval> beta,
            <interval_array> bp );

{GE,GB,SY,SB,SP,TR,TB,TP}::ADD_I (Matrix add and scale)  C ← αA + βB

These routines scale two interval matrices A and B and store the sum in the matrix C. Matrices
A, B, and C have the same storage format. This routine returns immediately if m or k or k0 (for
general band matrices) or n or k (for symmetric or triangular band matrices) is less than or equal
to zero. For the routine GE_ADD_I, if lda is less than one or less than m, an error flag is set and
passed to the error handler. For the routine GB_ADD_I, if lda is less than k plus k0 plus one, an
error flag is set and passed to the error handler. For the routines SY_ADD_I, TR_ADD_I, SP_ADD_I,
and TP_ADD_I, if lda is less than one or lda is less than n, an error flag is set and passed to the
error handler. For the routines SB_ADD_I and TB_ADD_I, if lda is less than k plus one, an error flag
is set and passed to the error handler.

- Fortran 95 binding:

General:
    SUBROUTINE ge_add_i( a, b, c [, alpha] [, beta] )
General Band:
    SUBROUTINE gb_add_i( a, m, kl, b, c [, alpha] [, beta] )
Symmetric:
    SUBROUTINE sy_add_i( a, b, c [, uplo], [, alpha] [, beta] )
Symmetric Band:
    SUBROUTINE sb_add_i( a, b, c [, uplo], [, alpha] [, beta] )
Symmetric Packed:
    SUBROUTINE sp_add_i( ap, bp, cp [, uplo], [, alpha] [, beta] )
Triangular:
    SUBROUTINE tr_add_i( a, b, c [, uplo], [, diag] [, alpha] [, beta] )
Triangular Band:
    SUBROUTINE tb_add_i( a, b, c [, uplo], [, diag] [, alpha] [, beta] )
Triangular Packed:
    SUBROUTINE tp_add_i( ap, bp, cp [, uplo], [, diag] [, alpha] [, beta] )
all:
    TYPE(INTERVAL) ( <wp> ), INTENT(IN) :: a(:, :) | ap(:, ) | b(:, :) | bp(:, )
C.4. INTERVAL BLAS

```fortran
INTEGER, INTENT(IN) :: m, k1
TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: c(:, :) | cp(:)
TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL :: alpha, beta
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag

where

assuming A, B and C all the same (general, banded or packed) with
the same size.

a, b and c have shape (m,n) for general matrix
(l,n) for general banded matrix (l > k1)
(n,n) for symmetric or triangular
(k+1,n) for symmetric banded or triangular
   banded (k=band width)

ap, bp and cp have shape (n*(n+1)/2).
```

- Fortran 77 binding:

  General:
  ```fortran
  SUBROUTINE BLAS_xGE_ADD_I( M, N, ALPHA, A, LDA, BETA, B, LDB, C, LDC )
  ```

  General Band:
  ```fortran
  SUBROUTINE BLAS_xGB_ADD_I( M, N, KL, KU, ALPHA, A, LDA, BETA, B, LDB, $     C, LDC )
  ```

  Symmetric:
  ```fortran
  SUBROUTINE BLAS_xSY_ADD_I( UPLO, N, ALPHA, A, LDA, BETA, B, LDB, C, $     LDC )
  ```

  Symmetric Band:
  ```fortran
  SUBROUTINE BLAS_xSB_ADD_I( UPLO, N, K, ALPHA, A, LDA, BETA, B, LDB, $     C, LDC )
  ```

  Symmetric Packed:
  ```fortran
  SUBROUTINE BLAS_xSP_ADD_I( UPLO, N, ALPHA, AP, BETA, BP, CP )
  ```

  Triangular:
  ```fortran
  SUBROUTINE BLAS_xtR_ADD_I( UPLO, DIAG, N, ALPHA, A, LDA, BETA, B, $     LDB, C, LDC )
  ```

  Triangular Band:
  ```fortran
  SUBROUTINE BLAS_xtB_ADD_I( UPLO, DIAG, N, K, ALPHA, A, LDA, BETA, B, $     LDB, C, LDC )
  ```

  Triangular Packed:
  ```fortran
  SUBROUTINE BLAS_xtP_ADD_I( UPLO, DIAG, N, ALPHA, AP, BETA, BP, CP )
  ```

all:
```fortran
INTEGER
<type>
ALPHA(2), BETA(2)
<type>
A(2, LDA, *) or AP(2,*) , B(2, LDB, *)
or BP(2,*), C(2, LDC, *) or CP(2,*)
```

- C binding:

General:
void BLAS_xge_add_i( enum blas_order_type order, int m, int n, <interval> alpha,
    const <interval_array> a, int lda, <interval> beta,
    const <interval_array> b, int ldb, <interval_array> c,
    int ldc );

General Band:
void BLAS_xgb_add_i( enum blas_order_type order, int m, int n, int kl, int ku,
    <interval> alpha, const <interval_array> a, int lda,
    <interval> beta, const <interval_array> b, int ldb,
    <interval_array> c, int ldc );

Symmetric:
void BLAS_xsy_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, <interval> alpha, const <interval_array> a, int lda,
    <interval> beta, const <interval_array> b, int ldb,
    <interval_array> c, int ldc );

Symmetric Band:
void BLAS_xsb_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, <interval> alpha, const <interval_array> a, int lda,
    <interval> beta, const <interval_array> b, int ldb,
    <interval_array> c, int ldc );

Symmetric Packed:
void BLAS_xsp_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, <interval> alpha, const <interval_array> ap,
    <interval> beta, const <interval_array> bp,
    <interval_array> cp );

Triangular:
void BLAS_xtr_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n, <interval> alpha,
    const <interval_array> a, int lda, <interval> beta,
    const <interval_array> b, int ldb, <interval_array> c,
    int ldc );

Triangular Band:
void BLAS_xtb_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, enum blas_diag_type diag, int k, <interval> alpha,
    const <interval_array> a, int lda, <interval> beta,
    const <interval_array> b, int ldb, <interval_array> c,
    int ldc );

Triangular Packed:
void BLAS_xtp_add_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, enum blas_diag_type diag, <interval> alpha,
    const <interval_array> ap, <interval> beta,
    const <interval_array> bp, <interval_array> cp );

Interval Matrix-Matrix Operations
In the following specifications, op(X) denotes X or X^T where X is a matrix.

GEMM_I (General interval matrix matrix product) \[ C \leftarrow \alpha \op(A) \op(B) + \beta C. \]
This routine performs a general interval matrix matrix multiply $C \leftarrow \alpha \text{op}(A) \text{op}(B) + \beta C$, where $\alpha$ and $\beta$ are intervals, and $A$, $B$, and $C$ are general interval matrices. This routine returns immediately if $m$ or $n$ or $k$ is less than or equal to zero. If $\text{lda}$ is less than one or less than $m$, or if $\text{ldb}$ is less than one or less than $k$, or if $\text{ldc}$ is less than one or less than $m$, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE gemm_i( a, b, c [, transa] [, transb] [, alpha] [, beta] )
  TYPE INTERVAL (<wp>), INTENT(IN) :: a(:,,:), b(:,:)
  TYPE INTERVAL (<wp>), INTENT(INOUT) :: c(:,:)
  TYPEblas_trans_type, INTENT(IN), OPTIONAL :: transa, transb
  TYPE INTERVAL (<wp>), INTENT(IN), OPTIONAL :: alpha, beta

  where

  c has shape $(m,n)$
  a has shape $(m,k)$ if transa = blas_no_trans (the default)
  $(k,m)$ if transa /= blas_no_trans
  b has shape $(k,n)$ if transb = blas_no_trans (the default)
  $(n,k)$ if transb /= blas_no_trans
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xGEMM_I( TRANSA, TRANSB, M, N, K, ALPHA, A, LDA,
  $ B, LDB, BETA, C, LDC )
  INTEGER K, LDA, LDB, LDC, M, N, TRANSA, TRANSB
  <type> ALPHA( 2 ), BETA( 2 )
  <type> A( 2, LDA, * ), B( 2, LDB, * ),
  $ C( 2, LDC, * )
  ```

- C binding:

  ```c
  void BLAS_xgemm_i( enum blas_order_type order, enum blas_trans_type transa,
  enum blas_trans_type transb, int m, int n, int k,
  <interval> alpha, const <interval_array> a, int lda,
  const <interval_array> b, int ldb, <interval> beta,
  <interval_array> c, int ldc );
  ```

SYMMJ (Symmetric interval matrix matrix product)  
$C \leftarrow \alpha AB + \beta C$ or $C \leftarrow \alpha BA + \beta C$.

These routines perform one of the symmetric interval matrix operations $C \leftarrow \alpha AB + \beta C$ or $C \leftarrow \alpha BA + \beta C$ where $\alpha$ and $\beta$ are intervals, $A$ is a symmetric interval matrix, and $B$ and $C$ are general interval matrices. This routine returns immediately if $m$ or $n$ is less than or equal to zero. For side equal to blas.left_side, and if $\text{lda}$ is less than one or less than $m$, or if $\text{ldb}$ is less than one or less than $m$, or if $\text{ldc}$ is less than one or less than $m$, an error flag is set and passed to the error handler. For side equal to blas.right_side, and if $\text{lda}$ is less than one or less than $n$, or if $\text{ldb}$ is less than one or less than $n$, or if $\text{ldc}$ is less than one or less than $n$, an error flag is set and passed to the error handler.
• Fortran 95 binding:

```fortran
SUBROUTINE symm_i( a, b, c [, side] [, uplo] [, alpha] [, beta] )
TYPE(INTERVAL) ( <wp> ), INTENT(INOUT) :: c(:,:)
TYPE(INTERVAL) ( <wp> ), INTENT(IN) :: a(:,:), b(:,:)
TYPE(blus_side_type), INTENT(IN), OPTIONAL :: side
TYPE(blus_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(INTERVAL) ( <wp> ), INTENT(IN), OPTIONAL :: alpha, beta

where
  c has shape (m,n), b same shape as c
SY  a has shape (m,m) if side = blas_left_side (the default)
     a has shape (n,n) if side /= blas_left_side
```

• Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xSYMm_i( SIDE, UPLO, M, N, ALPHA, A, LDA, B,
                         $                        
                         LDB, BETA, C, LDC )
INTEGER LDA, LDB, LDC, M, N, SIDE, UPLO
<type>   ALPHA( 2 ), BETA( 2 )
<type>   A( 2, LDA, * ), B( 2, LDB, * ),
$          C( 2, LDC, * )
```

• C binding:

```c
void BLAS_xsymm_i( enum blas_order_type order, enum blas_side_type side,
                   enum blas_uplo_type uplo, int m, int n, <interval> alpha,
                   const <interval_array> a, int lda, const <interval_array> b,
                   int ldb, <interval> beta, <interval_array> c, int ldc );
```

TRMM_I (Triangular interval matrix matrix product)

This routine performs one of the interval matrix operations:

\[
B \leftarrow \alpha T B, \ B \leftarrow \alpha T^T B, \ B \leftarrow \alpha BT, \text{ or } B \leftarrow \alpha BT^T
\]

where \( \alpha \) is an interval, \( T \) is a unit, or non-unit, upper or lower triangular interval matrix, and \( B \) is a general interval matrix. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For side equal to blas_left_side, and if ldt is less than or less than m, or if ldb is less than one or less than m, an error flag is set and passed to the error handler. For side equal to blas_right_side, and if ldt is less than one or less than n, or if ldb is less than one or less than m, an error flag is set and passed to the error handler.

• Fortran 95 binding:

```fortran
SUBROUTINE trmm_i( t, b [, side] [, uplo] [, transt] [, diag] &
                   [, alpha ] )
TYPE(INTERVAL) ( <wp> ), INTENT(INOUT) :: b(:,:)
TYPE(INTERVAL) ( <wp> ), INTENT(IN) :: t(:,:)
```
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```
TYPE(blasc_side_type), INTENT(IN), OPTIONAL :: side
TYPE(blasc_trans_type), INTENT(IN), OPTIONAL :: transt
TYPE(blasc_diag_type), INTENT(IN), OPTIONAL :: diag
TYPE(blasc_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE INTERVAL (<wp>), INTENT(IN), OPTIONAL :: alpha, beta

where
b has shape (m,n)
TR t has shape (m,m) if side = blas_left_side (the default)
t has shape (n,n) if side /= blas_left_side
```

- Fortran 77 binding:

```
SUBROUTINE BLAS_xTRMM_I( SIDE, UPLO, TRANST, DIAG, M, N, ALPHA, 
$T, LDT, B, LDB )
INTEGER DIAG, LDA, LDB, M, N, SIDE, TRANST, UPLO
<type> ALPHA( 2 )
<type> T( 2, LDA, * ), B( 2, LDB, * )
```

- C binding:

```
void BLAS_xtrmm_i( enum blas_order_type order, enum blas_side_type side,
enum blas_uplo_type uplo, enum blas_trans_type transt,
enum blas_diag_type diag, int m, int n, <interval> alpha,
const <interval_array> t, int ldt, <interval_array> b, 
int ldb );
```

TRSM.I (Interval triangular solve)

\[
B \leftarrow \alpha T^{-1} B, B \leftarrow \alpha (T^{-1})^T B, B \leftarrow \alpha BT^{-1} \quad \text{or} \quad B \leftarrow \alpha (T^{-1})^T 
\]

These routines bound one of the matrix equations \( B \leftarrow \alpha T^{-1} B, B \leftarrow \alpha (T^{-1})^T B, B \leftarrow \alpha BT^{-1} \) or \( B \leftarrow \alpha B(T^{-1})^T \) where \( \alpha \) is an interval, \( B \) is a general interval matrix, and \( T \) is a a unit, or non-unit, upper or lower triangular interval matrix. This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. For side equal to blas_left_side, and if ldt is less than one or less than \( m \), or if ldb is less than one or less than \( m \), an error flag is set and passed to the error handler. For side equal to blas_right_side, and if ldt is less than one or less than \( n \), or if ldb is less than one or less than \( n \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```
SUBROUTINE trsm_i( t, b [, side] [, uplo] [, transt] [, diag] [, alpha] )
TYPE INTERVAL (<wp>), INTENT(INOUT) :: b(:, :)
TYPE INTERVAL (<wp>), INTENT(IN) :: t(:, :)
TYPE(blasc_side_type), INTENT(IN), OPTIONAL :: side
TYPE(blasc_trans_type), INTENT(IN), OPTIONAL :: transt
TYPE(blasc_diag_type), INTENT(IN), OPTIONAL :: diag
TYPE(blasc_uplo_type), INTENT(IN), OPTIONAL :: uplo
```
TYPE(INTERVAL) (<wp>), INTENT(IN), OPTIONAL :: alpha, beta
where
b has shape (m,n)
t has shape (m,m) if side = blas_left_side (the default)
t has shape (n,n) if side /= blas_left_side

• Fortran 77 binding:

SUBROUTINE BLAS_xTRSM_I( SIDE, UPLO, TRANSN, DIAG, M, N, ALPHA,
$    ALPHA, T, LDT, B, LDB )
INTEGER DIAG, LDB, LDT, M, N, SIDE, TRANSN, UPLO
<type>
<type> T( 2, LDA, * ), B( 2, LDB, * )

• C binding:

void BLAS_xTRSM_i( enum blas_order_type order, enum blas_side_type side,
    enum blas_uplo_type uplo, enum blas_trans_type transt, 
    enum blas_diag_type diag, int m, int n, <interval> alpha, 
    const <interval_array> t, int ldt, <interval_array> b, 
    int ldb );

Data Movement with Interval Matrices
{GE,GB,SY,SB,SP,TR,TB,TP}.COPY_I (Matrix copy)  \quad B \leftarrow A, B \leftarrow A^T

This routine copies an interval matrix (or its transpose) A and stores the result in an interval matrix B. Matrices A (or \( A^T \)) and B have the same storage format. This routine returns immediately if m (for nonsymmetric matrices), n, k (for symmetric band matrices), or kl or ku (for general band matrices), is less than or equal to zero. For the routine GE_COPY_I, if trans equal to blas_no_trans, and if lda is less than or equal to m, or if ldb is less than or equal to m, an error flag is set and passed to the error handler. For the routine GE_COPY_I, if trans equal to blas_trans, and if lda is less than or equal to m, or if ldb is less than or equal to m, an error flag is set and passed to the error handler. For the routine GB_COPY_I, if lda is less than kl plus ku plus one, or if ldb is less than kl plus ku plus one, an error flag is set and passed to the error handler. For the routines SY_COPY_I and TR_COPY_I, if lda is less than or equal to n, or if ldb is less than or equal to n, an error flag is set and passed to the error handler. For the routines SB_COPY_I and TB_COPY_I, if lda is less than k plus one, or if ldb is less than k plus one, an error flag is set and passed to the error handler.

• Fortran 95 binding:

General:
    SUBROUTINE ge_copy_i( a, b [, , transa] )
General Band:
    SUBROUTINE gb_copy_i( a, kl, b [, , transa] )
Symmetric:
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SUBROUTINE sy_copy_i( a, b [, uplo] )
Symmetric Band:
SUBROUTINE sb_copy_i( a, b [, uplo] )
Symmetric Packed:
SUBROUTINE sp_copy_i( ap, bp [, uplo] )
Triangular:
SUBROUTINE tr_copy_i( a, b [, uplo], [, trans] [, diag] )
Triangular Band:
SUBROUTINE tb_copy_i( a, b [, uplo], [, trans] [, diag] )
Triangular Packed:
SUBROUTINE tp_copy_i( ap, bp [, uplo], [, trans] [, diag] )
all:
TYPE(INTERVAL) (<wp>), INTENT(OUT) :: b(:,:) | bp(:)
TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:) | ap(:)
INTEGER, INTENT(IN) :: k
TYPE blas_trans_type, INTENT(IN), OPTIONAL :: trans
TYPE blas_uplo_type, INTENT(IN), OPTIONAL :: uplo
TYPE blas_diag_type, INTENT(IN), OPTIONAL :: diag
where
If trans = blas_no_trans (the default)
a, b have shape (m,n) for general matrix
(1,n) for general banded matrix (l > k)
(n,n) for symmetric or triangular
(k+1,n) for symmetric banded or triangular
banded (k=band width)
ap and bp have shape (n*(n+1)/2).

If trans \neq blas_no_trans
a has shape (m,n) and b has shape (n,m) for general matrix
(1,n) and b has shape (1,m) for general banded matrix (l>kl)
a and b have shape (n,n) for symmetric or triangular
(k+1,n) for symmetric banded or triangular
banded (k=band width)
ap and bp have shape (n*(n+1)/2).

• Fortran 77 binding:

General:
SUBROUTINE BLAS_xGE_COPY_I( TRANS, M, N, A, LDA, B, LDB )
General Band:
SUBROUTINE BLAS_xGB_COPY_I( TRANS, M, N, KL, KU, A, LDA, B, LDB )
Symmetric:
SUBROUTINE BLAS_xSY_COPY_I( UPLO, N, A, LDA, B, LDB )
Symmetric Band:
SUBROUTINE BLAS_xSB_COPY_I( UPLO, N, K, A, LDA, B, LDB )
Symmetric Packed:
SUBROUTINE BLAS_xSP_COPY_I( UPLO, N, AP, BP )
Triangular:
SUBROUTINE BLAS_xTR_COPY_I( UPLO, TRANS, DIAG, N, A, LDA, B, LDB )
Triangular Band:
  SUBROUTINE BLAS_xTB_COPY_I( UPLO, TRANS, DIAG, N, K, A, LDA, B, LDB )
Triangular Packed:
  SUBROUTINE BLAS_xTP_COPY_I( UPLO, TRANS, DIAG, N, AP, BP )
all:
  INTEGER DIAG, LDA, LDB, N, K, KL, KU, TRANS, UPLO
  <type> A( 2, LDA, * ) or AP( 2, * ), B( 2, LDB, * )
  or BP( 2, * )

• C binding:

  General:
  void BLAS_xge_copy_i( enum blas_order_type order, enum blas_trans_type transa,
    int m, int n, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  General Band:
  void BLAS_xgb_copy_i( enum blas_order_type order, enum blas_trans_type transa,
    int m, int n, int kl, int ku, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  Symmetric:
  void BLAS_xsy_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  Symmetric Band:
  void BLAS_xsb_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  Symmetric Packed:
  void BLAS_xsp_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> ap, <interval_array> bp );
  Triangular:
  void BLAS_xtr_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  Triangular Band:
  void BLAS_xtb_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, int k, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
  Triangular Packed:
  void BLAS_xtp_copy_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, const <interval_array> ap, <interval_array> bp );
GE_TRANS_I (Matrix transposition) \[ A \leftarrow A^T \]

This routine performs the transposition of a square interval matrix \( A \) and overwrites the matrix \( A \). This routine returns immediately if \( n \) is less than or equal to zero. If \( lda \) is less than one or less than \( n \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE ge_trans_i( a )
  TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: a(:,:)
  where
  a has shape (n,n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xGE_TRANS_I( N, A, LDA )
  INTEGER LDA, N
  <type> A( 2, LDA, * )
```

- C binding:

```c
void BLAS_xge_trans_i( int n, <interval_array> a, int lda );
```

GE_PERMUTE_I (Permute an interval matrix ) \[ A \leftarrow PA \text{ or } A \leftarrow AP \]

This routine permutes the rows or columns of an interval matrix \( A \) by the permutation matrix \( P \). This routine returns immediately if \( m \) or \( n \) is less than or equal to zero. As described in section 2.5.3, the value incp less than zero is permitted. However, if incp is equal to zero, an error flag is set and passed to the error handler. If \( lda \) is less than one or less than \( m \), an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
SUBROUTINE ge_permute_i( p, a [, side] )
  TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: a(:,:)
  INTEGER, INTENT(IN) :: p(:)
  TYPE blas_side_type, INTENT(IN), OPTIONAL :: side
  where
  a has shape (m,n)
  p has shape (k) where k = m if side = blas_left_side
  \[ k = n \text{ if side = blas_right_side} \]
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xGE_PERMUTE_I( SIDE, M, N, P, INCP, A, LDA )
  INTEGER INCP, LDA, M, N, SIDE
  INTEGER P( * )
  <type> A( 2, LDA, * )
```
The value of `INCP` may be positive or negative. A negative value of `INCP` applies the permutation in the opposite direction.

- **C binding:**

```c
void BLAS_xge_permute_i( enum blas_order_type order, enum blas_side_type side, 
  int m, int n, const int *p, int incp, 
  <interval_array> a, int lda );
```

The value of `incp` may be positive or negative. A negative value of `incp` applies the permutation in the opposite direction.

**Set Operations Involving Interval Vectors**

ENCV_I (Checks if an interval vector is enclosed in another interval vector)  True if \( x \subseteq y \)

This routine checks if an interval vector \( x \) is enclosed in another interval vector \( y \). We say that an interval vector \( x \) is enclosed in \( y \), denoted as \( x \subseteq y \), if and only if \( y_i \leq x_i \leq y_i \forall i \). This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value `incx` or `incy` less than zero is permitted. However, if `incx` or `incy` is equal to zero, an error flag is set and passed to the error handler.

- **Fortran 95 binding:**

```fortran
LOGICAL FUNCTION encv_i(x, y) 
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:,), y(:,)
  where 
  x and y have shape (n)
```

- **Fortran 77 binding:**

```fortran
LOGICAL FUNCTION BLAS_xENCV_I( N, X, INCX, Y, INCY ) 
  INTEGER N, INCX, INCY 
  <type> X( 2, * ), Y( 2, * )
```

- **C binding:**

```c
int BLAS_xencv_i( int n, const <interval_array> x, int incx, 
  const <interval_array> y, int incy );
```

INTERIORV_I (If an interval vector is in the interior of another interval vector)  True if \( x \subset y \)

This routine checks if an interval vector \( x \) is enclosed in the interior of another interval vector \( y \). We say that an interval vector \( x \) is enclosed in the interior of \( y \), denoted as \( x \subset y \), if and only if \( y_i < x_i < y_i \forall i \). This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value `incx` or `incy` less than zero is permitted. However, if `incx` or `incy` is equal to zero, an error flag is set and passed to the error handler.
C.4. INTERVAL BLAS

- Fortran 95 binding:

```fortran
LOGICAL FUNCTION interiorv_i(x, y)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:), y(:)
  where
  x and y have shape (n)
```

- Fortran 77 binding:

```fortran
LOGICAL FUNCTION BLAS_xINTERIORV_I( N, X, INCX, Y, INCY )
  INTEGER N, INCX, INCY
  <type> X( 2, * ), Y( 2, * )
```

- C binding:

```c
int BLAS_xinteriorv_i( int n, const <interval_array> x, int incx,
  const <interval_array> y, int incy );
```

---

DISJ_V_I (Checks if two interval vectors disjoint) True if \( x \cap y = \emptyset \)

This routine checks if two interval vectors \( x \) and \( y \) are disjoint, which means that \( x_i \cap y_i = \emptyset \) for some \( i \). This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if incx or incy is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

```fortran
LOGICAL FUNCTION disjv_i(x, y)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:), y(:)
  where
  x and y have shape (n)
```

- Fortran 77 binding:

```fortran
LOGICAL FUNCTION BLAS_xDISJ_V_I( N, X, INCX, Y, INCY )
  INTEGER N, INCX, INCY
  <type> X( 2, * ), Y( 2, * )
```

- C binding:

```c
int BLAS_xdisjv_i( int n, const <interval_array> x, int incx,
  const <interval_array> y, int incy );
```
INTERV_I (Intersection of an interval vector with another) \[ y \leftarrow x \cap y. \]

This routine finds the intersection of two interval vectors \( x \) and \( y \), and stores the result in \( y \). This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) less than zero is permitted. However, if \( \text{incx} \) or \( \text{incy} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE interv_i( x, y )
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: y(:)
  where
  x and y have shape (n)
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_XINTERV_I( N, X, INCX, Y, INCY )
  INTEGER N, INCX, INCY
  <type> X( 2, * ), Y( 2, * )
  ```

- C binding:

  ```c
  void BLAS_xinterv_i( int n, const <interval_array> x, int incx,
  <interval_array> y, int incy );
  ```

WINTERV_I (Intersection of two interval vectors) \[ z \leftarrow x \cap y. \]

This routine finds the intersection of two interval vectors \( x \) and \( y \), and stores the result in another interval vector \( z \). This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value \( \text{incx} \) or \( \text{incy} \) or \( \text{incz} \) less than zero is permitted. However, if \( \text{incx} \), \( \text{incy} \), or \( \text{incz} \) is equal to zero, an error flag is set and passed to the error handler.

- Fortran 95 binding:

  ```fortran
  SUBROUTINE winterv_i(x, y, z )
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:), y(:)
  TYPE(INTERVAL) (<wp>), INTENT(OUT) :: z(:)
  where
  x, y and z have shape (n)
  ```

- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_XWINTERV_I( N, X, INCX, Y, INCY, Z, INCZ )
  INTEGER SIDE, LDA, M, N
  INTEGER N, INCX, INCY, INCZ
  <type> X( 2, * ), Y( 2,* ), Z( 2, * )
  ```
\begin{itemize}
\item C binding:
\begin{verbatim}
void BLAS_xwinterv_i( int n, const <interval_array> x, int incx,
    const <interval_array> y, int incy,
    <interval_array> z, int incz );
\end{verbatim}
\end{itemize}

\textbf{HULLV\_I} (Convex hull of an interval vector with another)  $y \leftarrow$ a convex set which contains $x \cup y$

This routine computes a convex set which contains both interval vectors $x$ and $y$, and overwrites the input interval vector $y$ with the result. This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value incx or incy less than zero is permitted. However, if incx or incy is equal to zero, an error flag is set and passed to the error handler.

\begin{itemize}
\item Fortran 95 binding:
\begin{verbatim}
SUBROUTINE hullv_i( x, y )
    TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
    TYPE(INTERVAL) (<wp>), INTENT(INOUT) :: y(:)
where
    x and y have shape (n)
\end{verbatim}
\end{itemize}

\begin{itemize}
\item Fortran 77 binding:
\begin{verbatim}
SUBROUTINE BLAS_xHULLV_I( N, X, INCX, Y, INCY )
    INTEGER \quad N, INCX, INCY
    <type> \quad X( 2, * ), Y( 2, * )
\end{verbatim}
\end{itemize}

\begin{itemize}
\item C binding:
\begin{verbatim}
void BLAS_xhullv_i( int n, const <interval_array> x, int incx,
    <interval_array> y, int incy );
\end{verbatim}
\end{itemize}

\textbf{WHULLV\_I} (Convex hull of two interval vectors)  $z \leftarrow$ a convex set which contains $x \cup y$

This routine finds a convex hull of two interval vectors $x$ and $y$, and stores the result in another interval vector $z$. This routine returns immediately if \( n \) is less than or equal to zero. As described in section 2.5.3, the value incx, incy, or incz less than zero is permitted. However, if incx or incy or incz is equal to zero, an error flag is set and passed to the error handler.

\begin{itemize}
\item Fortran 95 binding:
\begin{verbatim}
SUBROUTINE whullv_i( x, y, z )
    TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:), y(:)
    TYPE(INTERVAL) (<wp>), INTENT(OUT) :: z(:)
where
    x, y and z have shape (n)
\end{verbatim}
\end{itemize}
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- Fortran 77 binding:

  ```fortran
  SUBROUTINE BLAS_xwhullv_i( N, X, INCX, Y, INCY, Z, INCZ )
  INTEGER N, INCX, INCY, INCZ
  <type> X( 2, * ), Y( 2, * ), Z( 2, * )
  ```

- C binding:

  ```c
  void BLAS_xwhullv_i( int n, const <interval_array> x, int incx,
                      const <interval_array> y, int incy, <interval_array> z,
                      int incz );
  ```

Set Operations Involving Interval Matrices

\{ \text{GE,GB,SY,SB,SP,TR,TB,TP} \} \_\text{ENCMI} \text{J} \ (\text{If an interval matrix is enclosed in another}) \quad \text{True if } A \subseteq B

This routine checks if an interval matrix $A$ is enclosed in another interval matrix $B$. We say that an interval matrix $A$ is enclosed in another interval matrix $B$, denoted as $A \subseteq B$, if and only if $a_{i,j} \subseteq b_{i,j} \ \forall i$ and $\forall j$. Matrices $A$ and $B$ have the same storage format.

- Fortran 95 binding:

  ```fortran
  LOGICAL FUNCTION ge_encm_i( a, b )
  LOGICAL FUNCTION gb_encm_i( a, m, kl, b )
  LOGICAL FUNCTION sy_encm_i( a, b [, uplo] )
  LOGICAL FUNCTION sb_encm_i( a, b [, uplo] )
  LOGICAL FUNCTION sp_encm_i( ap, bp [, uplo] )
  LOGICAL FUNCTION tr_encm_i( a, b [, uplo] [, diag] )
  LOGICAL FUNCTION tb_encm_i( a, b [, uplo] [, diag] )
  LOGICAL FUNCTION tp_encm_i( ap, bp [, uplo], [, diag] )
  ```

all:

```fortran
TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), ap(:,:), b(:,:), bp(:)
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
where
  a and b have shape (m, n) for general matrix
```
(1, n) for general banded matrix (l > kl)
(n, n) for symmetric or triangular
(p+1, n) for symmetric banded or triangular
banded (p = band width)
ap and bp have shape (n*(n+1)/2)

- Fortran 77 binding:

General:

    LOGICAL FUNCTION BLAS_xGE_ENCM_I( M, N, A, LDA, B, LDB)

General Band:

    LOGICAL FUNCTION BLAS_xGB_ENCM_I( M, N, KL, KU, A, LDA, B, LDB )

Symmetric:

    LOGICAL FUNCTION BLAS_xSY_ENCM_I( N, A, LDA, B, LDB )

Symmetric Band:

    LOGICAL FUNCTION BLAS_xSB_ENCM_I( N, K, A, LDA, B, LDB )

Symmetric Packed:

    LOGICAL FUNCTION BLAS_xSP_ENCM_I( N, AP, BP )

triangular:

    LOGICAL FUNCTION BLAS_xTR_ENCM_I( N, A, LDA, B, LDB )

triangular Band:

    LOGICAL FUNCTION BLAS_xTB_ENCM_I( N, K, A, LDA, B, LDB )

Triangular Packed:

    LOGICAL FUNCTION BLAS_xTP_ENCM_I( N, AP, BP )

all:

    INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
    <type> A( 2, LDA, *) or AP( 2, * ), B( 2, LDA, * )
or BP( 2, * )

- C binding:

General:

    int BLAS_xge_encm_i( enum blas_order_type order, int m, int n,
    const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

General Band:

    int BLAS_xgb_encm_i( enum blas_order_type order, int m, int n, int kl,
    int ku, const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

Symmetric:

    int BLAS_xsy_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

Symmetric Band:

    int BLAS_xsb_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, const <interval_array> a, int lda,
    const <interval_array> b, int ldb );
Symmetric Packed:
int BLAS_xsp_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> ap,
const <interval_array> bp );

Triangular:
int BLAS_xtr_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> a, int lda,
const <interval_array> b, int ldb );

Triangular Band:
int BLAS_xtb_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n, int k,
const <interval_array> a, int lda,
const <interval_array> b, int ldb );

Triangular Packed:
int BLAS_xtp_encm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> ap, <interval_array> bp );

{GE,GB,SY,SB,SP,TR,TB,TP}.INTERIORM.J (If an interval matrix is in the interior of another interval matrix)
True if \( A \subset B \)

This routine checks if an interval matrix \( A \) is enclosed in the interior of another interval matrix \( B \). We say that an interval matrix \( A \) is enclosed in the interior of an interval matrix \( B \), if and only if \( a_{i,j} \subset b_{i,j} \ \forall i \text{ and } \forall j \). Matrices \( A \) and \( B \) have the same storage format.

- Fortran 95 binding:

General:

```
LOGICAL FUNCTION ge_interiorm_i( a, b )
```

General Band:

```
LOGICAL FUNCTION gb_interiorm_i( a, m, kl, b )
```

Symmetric:

```
LOGICAL FUNCTION sy_interiorm_i( a, b [, uplo] )
```

Symmetric Band:

```
LOGICAL FUNCTION sb_interiorm_i( a, b [, uplo] )
```

Symmetric Packed:

```
LOGICAL FUNCTION sp_interiorm_i( ap, bp [, uplo] )
```

Triangular:

```
LOGICAL FUNCTION tr_interiorm_i( a, b [, uplo] [, diag ])
```

Triangular Band:

```
LOGICAL FUNCTION tb_interiorm_i( a, b [, uplo] [, diag ])
```

Triangular Packed:

```
LOGICAL FUNCTION tp_interiorm_i( ap, bp [, uplo], [, diag ])
```

all:

```
TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:) | ap(:,), b(:,:) | bp(:,)
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
```
C.4. INTERVAL BLAS

```fortran
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
where
  a and b have shape (m, n) for general matrix
  (l, n) for general banded matrix (l > kl)
  (n, n) for symmetric or triangular
  (p+1, n) for symmetric banded or triangular
    banded (p = band width)
  ap and bp have shape (n*(n+1)/2)

• Fortran 77 binding:

  General:
  LOGICAL FUNCTION BLAS_xGE_INNERORM_I( M, N, A, LDA, B, LDB )
  General Band:
  LOGICAL FUNCTION BLAS_xGB_INNERORM_I( M, N, KL, KU, A, LDA, B, LDB )
  Symmetric:
  LOGICAL FUNCTION BLAS_xSY_INNERORM_I( N, A, LDA, B, LDB )
  Symmetric Band:
  LOGICAL FUNCTION BLAS_xSB_INNERORM_I( N, K, A, LDA, B, LDB )
  Symmetric Packed:
  LOGICAL FUNCTION BLAS_xSP_INNERORM_I( N, AP, BP )
  Triangular:
  LOGICAL FUNCTION BLAS_xTR_INNERORM_I( N, A, LDA, B, LDB )
  Triangular Band:
  LOGICAL FUNCTION BLAS_xTB_INNERORM_I( N, K, A, LDA, B, LDB )
  Triangular Packed:
  LOGICAL FUNCTION BLAS_xTP_INNERORM_I( N, AP, BP )
all:
  INTEGER          UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
  <type>          A( 2, LDA, * ) or AP( 2, * ), B( 2, LDA, * )
```

• C binding:

```c
int BLAS_xge_interiorm_i( enum blas_order_type order, int m, int n,
  const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xgb_interiorm_i( enum blas_order_type order, int m, int n, int kl,
  int ku, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xsy_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,
  int n, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xsb_interiorm_i( enum blas_order_type order, int m, int n, int kl,
  int ku, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xsp_interiorm_i( int uplo, enum blas_diagonal_type diag,
  const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xtb_interiorm_i( int uplo, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
int BLAS_xtp_interiorm_i( int uplo, enum blas_diagonal_type diag,
  const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
```
int BLAS_xsb_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,  
  int n, int k, const <interval_array> a, int lda,  
  const <interval_array> b, int ldb );

Symmetric Packed:
int BLAS_xsp_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,  
  int n, const <interval_array> a, int lda,  
  const <interval_array> bp );

Triangular:
int BLAS_xtr_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,  
  enum blas_diag_type diag, int n,  
  const <interval_array> a, int lda,  
  const <interval_array> b, int ldb );

Triangular Band:
int BLAS_xtb_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,  
  enum blas_diag_type diag, int n, int k,  
  const <interval_array> a, int lda,  
  const <interval_array> b, int ldb );

Triangular Packed:
int BLAS_xtp_interiorm_i( enum blas_order_type order, enum blas_uplo_type uplo,  
  enum blas_diag_type diag, int n,  
  const <interval_array> ap, <interval_array> bp );

\{GE,GB,SY,SB,SP,TR,TB,TP\}_DISJM_I (If two interval matrices disjoint) \quad \text{True if } \mathbf{A} \cap \mathbf{B} = \emptyset

This routine checks if two interval matrices \( \mathbf{A} \) and \( \mathbf{B} \) disjoint, which means that if for some \( i,j \), \( a_{i,j} \cap b_{i,j} = \emptyset \). Matrices \( \mathbf{A} \) and \( \mathbf{B} \) have the same storage format.

- Fortran 95 binding:
  
  General:
  \[
  \text{LOGICAL FUNCTION ge_disjm_i( a, b )}
  \]
  General Band:
  \[
  \text{LOGICAL FUNCTION gb_disjm_i( a, m, k1, b )}
  \]
  Symmetric:
  \[
  \text{LOGICAL FUNCTION sy_disjm_i( a, b [], uplo )}
  \]
  Symmetric Band:
  \[
  \text{LOGICAL FUNCTION sb_disjm_i( a, b [], uplo )}
  \]
  Symmetric Packed:
  \[
  \text{LOGICAL FUNCTION sp_disjm_i( ap, bp [], uplo )}
  \]
  Triangular:
  \[
  \text{LOGICAL FUNCTION tr_disjm_i( a, b [], uplo [], diag )}
  \]
  Triangular Band:
  \[
  \text{LOGICAL FUNCTION tb_disjm_i( a, b [], uplo [], diag )}
  \]
  Triangular Packed:
  \[
  \text{LOGICAL FUNCTION tp_disjm_i( ap, bp [], uplo [], [], diag )}
  \]
  all:
  \[
  \text{TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,::) | ap::), b(:,::) | bp::)
  \]
C.4. INTERVAL BLAS

```
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
where
a and b have shape (m, n) for general matrix
(1, n) for general banded matrix (1 > k1)
(n, n) for symmetric or triangular
(p+1, n) for symmetric banded or triangular
    banded (p = band width)
ap and bp have shape (n*(n+1)/2)

- Fortran 77 binding:

  General:
  LOGICAL FUNCTION BLAS_xGE_DISJM_I( M, N, A, LDA, B, LDB )
  General Band:
  LOGICAL FUNCTION BLAS_xGB_DISJM_I( M, N, KL, KU, A, LDA, B, LDB )
  Symmetric:
  LOGICAL FUNCTION BLAS_xSY_DISJM_I( N, A, LDA, B, LDB )
  Symmetric Band:
  LOGICAL FUNCTION BLAS_xSB_DISJM_I( N, K, A, LDA, B, LDB )
  Symmetric Packed:
  LOGICAL FUNCTION BLAS_xSP_DISJM_I( N, AP, BP )
  Triangular:
  LOGICAL FUNCTION BLAS_xTR_DISJM_I( N, A, LDA, B, LDB )
  Triangular Band:
  LOGICAL FUNCTION BLAS_xTB_DISJM_I( N, K, A, LDA, B, LDB )
  Triangular Packed:
  LOGICAL FUNCTION BLAS_xTP_DISJM_I( N, AP, BP )
all:
  INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
  <type> A( 2, LDA, * ) or AP( 2, * ), B( 2, LDA, * )
  $ or BP( 2, * )

- C binding:

  General:
  int BLAS_xge_disjm_i( enum blas_order_type order, int m, int n,
  const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
  General Band:
  int BLAS_xgb_disjm_i( enum blas_order_type order, int m, int n, int kl,
  int ku, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
  Symmetric:
  int BLAS_xsy_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
  int n, const <interval_array> a, int lda,
  const <interval_array> b, int ldb );
```
Symmetric Band:
int BLAS_xsb_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

Symmetric Packed:
int BLAS_xsp_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> ap,
    const <interval_array> bp );

Triangular:
int BLAS_xtr_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n,
    const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

Triangular Band:
int BLAS_xtb_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n, int k,
    const <interval_array> a, int lda,
    const <interval_array> b, int ldb );

Triangular Packed:
int BLAS_xtp_disjm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n,
    const <interval_array> ap, <interval_array> bp );

\{GE,GB,SY,SB,SP,TR,TB,TP\}\_INTERM_I (Elementwise intersection of two interval matrices)
\( B \leftarrow A \cap B \)

This routine finds the elementwise intersection of two interval matrices \( A \) and \( B \), and stores the result in \( B \). Matrices \( A \) and \( B \) have the same storage format.

- Fortran 95 binding:

  General:
  
  SUBROUTINE ge_interm_i( a, b )

  General Band:
  
  SUBROUTINE gb_interm_i( a, m, kl, b )

  Symmetric:
  
  SUBROUTINE sy_interm_i( a, b [, uplo ] )

  Symmetric Band:
  
  SUBROUTINE sb_interm_i( a, b [, uplo ] )

  Symmetric Packed:
  
  SUBROUTINE sp_interm_i( ap, bp [, uplo ] )

  Triangular:
  
  SUBROUTINE tr_interm_i( a, b [, uplo ] [, diag ] )

  Triangular Band:
  
  SUBROUTINE tb_interm_i( a, b [, uplo ] [, diag ] )

  Triangular Packed:
  
  SUBROUTINE tp_interm_i( ap, bp [, uplo ], [, diag ] )
C.4. INTERVAL BLAS

all:

    TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:, :) | ap(:, :), b(:, :) | bp(:, :)

    TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo

    TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag

where

    a and b have shape (m, n) for general matrix
    (1, n) for general banded matrix (1 > kl)
    (n, n) for symmetric or triangular
    (p+1, n) for symmetric banded or triangular
          banded (p = band width)

    ap and bp have shape (n*(n+1)/2)

• Fortran 77 binding:

General:

    SUBROUTINE BLAS_xGE_INTERM_I( M, N, A, LDA, B, LDB )

General Band:

    SUBROUTINE BLAS_xGB_INTERM_I( M, N, KL, KU, A, LDA, B, LDB )

Symmetric:

    SUBROUTINE BLAS_xSY_INTERM_I( N, A, LDA, B, LDB )

Symmetric Band:

    SUBROUTINE BLAS_xSB_INTERM_I( N, K, A, LDA, B, LDB )

Symmetric Packed:

    SUBROUTINE BLAS_xSP_INTERM_I( N, AP, BP )

Triangular:

    SUBROUTINE BLAS_xTR_INTERM_I( N, A, LDA, B, LDB )

Triangular Band:

    SUBROUTINE BLAS_xTB_INTERM_I( N, K, A, LDA, B, LDB )

Triangular Packed:

    SUBROUTINE BLAS_xTP_INTERM_I( N, AP, BP )

all:

    INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
    <type> A( 2, LDA, * ) or AP( 2, * ), B( 2, LDA, * )
          $ or BP( 2, * )

• C binding:

General:

    void BLAS_xge_interm_i( enum bias_order_type order, int m, int n,
                            const <interval_array> a, int lda,
                            <interval_array> b, int ldb );

General Band:

    void BLAS_xgb_interm_i( enum bias_order_type order, int m, int n, int kl,
                            int ku, const <interval_array> a, int lda,
                            <interval_array> b, int ldb );

Symmetric:

    void BLAS_xsy_interm_i( enum bias_order_type order, enum bias_uplo_type uplo,
int n, const <interval_array> a, int lda,
<interval_array> b, int ldb );

Symmetric Band:
void BLAS_xsb_interm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, int k, const <interval_array> a, int lda,
<interval_array> b, int ldb );

Symmetric Packed:
void BLAS_xsp_interm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> ap,
<interval_array> bp );

Triangular:
void BLAS_xtx_interm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> a, int lda,
<interval_array> b, int ldb );

Triangular Band:
void BLAS_xtb_interm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n, int k,
const <interval_array> a, int lda,
<interval_array> b, int ldb );

Triangular Packed:
void BLAS_xtx_interm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> ap, <interval_array> bp );

GE_WINTERM_I (Intersection of two interval matrices)

This routine finds the intersection of two interval matrices A and B, and stores the result in another
interval matrix C. Matrices A, B and C have the same storage format.

- Fortran 95 binding:

  General:
  SUBROUTINE ge_winterm_i( a, b, c )

  General Band:
  SUBROUTINE gb_winterm_i( a, m, kl, b, c )

  Symmetric:
  SUBROUTINE sy_winterm_i( a, b, c [, uplo] )

  Symmetric Band:
  SUBROUTINE sb_winterm_i( a, b, c [, uplo] )

  Symmetric Packed:
  SUBROUTINE sp_winterm_i( ap, bp, cp [, uplo] )

  Triangular:
  SUBROUTINE tr_winterm_i( a, b, c [, uplo] [, diag] )

  Triangular Band:
  SUBROUTINE tb_winterm_i( a, b, c [, uplo] [, diag] )

  Triangular Packed:
C.4. INTERVAL BLAS

SUBROUTINE tp_wintem_i( ap, bp, cp [, uplo], [, diag] )
all:
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), b(:,:),
  c(:,:), ap(:,), bp(:,), cp(:)
  TYPE blas_uplo_type, INTENT(IN), OPTIONAL :: uplo
  TYPE blas_diag_type, INTENT(IN), OPTIONAL :: diag
where
  a, b and c have shape (m, n) for general matrix
  (l, n) for general banded matrix (l > kl)
  (n, n) for symmetric or triangular
  (p+1, n) for symmetric banded or triangular
  banded (p = band width)
  ap, bp and cp have shape (n*(n+1)/2)

  • Fortran 77 binding:

    General:
      SUBROUTINE BLAS_xGE_WINTERM_I( M, N, A, LDA, B, LDB, C, LDC )
    General Band:
      SUBROUTINE BLAS_xGB_WINTERM_I( M, N, KL, KU, A, LDA, B, LDB,
      C, LDC )
    Symmetric:
      SUBROUTINE BLAS_xSY_WINTERM_I( N, A, LDA, B, LDB, C, LDC )
    Symmetric Band:
      SUBROUTINE BLAS_xSB_WINTERM_I( N, K, A, LDA, B, LDB, C, LDC )
    Symmetric Packed:
      SUBROUTINE BLAS_xSP_WINTERM_I( N, AP, BP, CP )
    Triangular:
      SUBROUTINE BLAS_xTR_WINTERM_I( N, A, LDA, B, LDB, C, LDC )
    Triangular Band:
      SUBROUTINE BLAS_xTB_WINTERM_I( N, K, A, LDA, B, LDB, C, LDC )
    Triangular Packed:
      SUBROUTINE BLAS_xTP_WINTERM_I( N, AP, BP, CP )
all:
  INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB,
  C, LDC
  <type> A( 2, LDA, * ) or AP( 2, * ), B( 2, LDA, * )
  or BP(2,*), C(2, LDC, * ) or CP(2,*)

  • C binding:

    General:
      void BLAS_xge_wintem_i( enum blas_order_type order, int m, int n,
      const <interval_array> a, int lda,
      const <interval_array> b, int ldb,
      <interval_array> c, int ldc );

    General Band:

void BLAS_xgb_winterm_i( enum blas_order_type order, int m, int n, int kl, int ku, const <interval_array> a, int lda, const <interval_array> b, int ldb, <interval_array> c, int ldc );

Symmetric:
void BLAS_xsy_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, int n, const <interval_array> a, int lda, const <interval_array> b, int ldb, <interval_array> c, int ldc );

Symmetric Band:
void BLAS_xsb_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, int n, int k, const <interval_array> a, int lda, const <interval_array> b, int ldb, <interval_array> c, int ldc );

Symmetric Packed:
void BLAS_xsp_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, int n, const <interval_array> ap, const <interval_array> bp, <interval_array> cp );

Triangular:
void BLAS_xtr_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, enum blas_diag_type diag, int n, const <interval_array> a, int lda, const <interval_array> b, int ldb, <interval_array> c, int ldc );

Triangular Band:
void BLAS_xtb_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, enum blas_diag_type diag, int n, int k, const <interval_array> a, int lda, const <interval_array> b, int ldb, <interval_array> c, int ldc );

Triangular Packed:
void BLAS_xtp_winterm_i( enum blas_order_type order, enum blas_uplo_type uplo, enum blas_diag_type diag, int n, const <interval_array> ap, <interval_array> bp, <interval_array> cp );

{GE,GB,SY,SB,SP,TR,TB,TP}.HULLM_J (Convex hull of an interval matrix with another) \( \mathbf{B} \leftarrow \) the convex hull contains \( \mathbf{A} \cup \mathbf{B} \)

This routine finds an interval matrix which contains both interval matrices \( \mathbf{A} \) and \( \mathbf{B} \), and stores the result in \( \mathbf{B} \). Matrices \( \mathbf{A} \) and \( \mathbf{B} \) have the same storage format.

- Fortran 95 binding:

  General:
  ```fortran
  SUBROUTINE ge_hullm_i( a, b )
  ```

  General Band:
C.4. INTERVAL BLAS

SUBROUTINE gb_hullm_i( a, m, kl, b )
Symmetric:
   SUBROUTINE sy_hullm_i( a, b [, uplo] )
Symmetric Band:
   SUBROUTINE sb_hullm_i( a, b [, uplo] )
Symmetric Packed:
   SUBROUTINE sp_hullm_i( ap, bp [, uplo] )
Triangular:
   SUBROUTINE tr_hullm_i( a, b [, uplo [, diag] )
Triangular Band:
   SUBROUTINE tb_hullm_i( a, b [, uplo [, diag] )
Triangular Packed:
   SUBROUTINE tp_hullm_i( ap, bp [, uplo [, [, diag] )

all:
   TYPE(INTERVAL) ( <wp > ), INTENT(IN) :: a(:,:), b(:,:), ap(:), bp(:)
   TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
   TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag

where
   a and b have shape (m, n) for general matrix
   (1, n) for general banded matrix (1 > kl)
   (n, n) for symmetric or triangular
   (p+1, n) for symmetric banded or triangular
   banded (p = band width)
   ap and bp have shape (n*(n+1)/2)

• Fortran 77 binding:

   General:
   SUBROUTINE BLAS_xGE_HULLM_I( M, N, A, LDA, B, LDB )
   General Band:
   SUBROUTINE BLAS_xGB_HULLM_I( M, N, KL, KU, A, LDA, B, LDB )
   Symmetric:
   SUBROUTINE BLAS_xSY_HULLM_I( N, A, LDA, B, LDB )
   Symmetric Band:
   SUBROUTINE BLAS_xSB_HULLM_I( N, K, A, LDA, B, LDB )
   Symmetric Packed:
   SUBROUTINE BLAS_xSP_HULLM_I( N, AP, BP )
   Triangular:
   SUBROUTINE BLAS_xTR_HULLM_I( N, A, LDA, B, LDB )
   Triangular Band:
   SUBROUTINE BLAS_xTB_HULLM_I( N, K, A, LDA, B, LDB )
   Triangular Packed:
   SUBROUTINE BLAS_xTP_HULLM_I( N, AP, BP )

all:
   INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
   <type>
   A( 2, LDA, * ) or AP( 2, * ) or BP( 2, * )
• C binding:

General:

```c
void BLAS_xge_hullm_i( enum blas_order_type order, int m, int n,
    const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

General Band:

```c
void BLAS_xgb_hullm_i( enum blas_order_type order, int m, int n, int kl,
    int ku, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

Symmetric:

```c
void BLAS_xsy_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

Symmetric Band:

```c
void BLAS_xsb_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, int k, const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

Symmetric Packed:

```c
void BLAS_xsp_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> ap,
    <interval_array> bp );
```

Triangular:

```c
void BLAS_xtr_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n,
    const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

Triangular Band:

```c
void BLAS_xtb_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n, int k,
    const <interval_array> a, int lda,
    <interval_array> b, int ldb );
```

Triangular Packed:

```c
void BLAS_xtp_hullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_diag_type diag, int n,
    const <interval_array> ap, <interval_array> bp );
```

---

`(GE,GB,SY,SB,SP,TR,TB,TP)_WHULLM_I (Convex hull of two interval matrices)

C ← the convex hull contains A ∪ B

This routine finds the convex set which contains both interval matrices A and B, and stores the
result in an interval matrix C. Matrices A,B and C have the same storage format.

• Fortran 95 binding:

General:

```fortran
SUBROUTINE ge_whullm_i( a, b, c )
```
General Band:
SUBROUTINE gb_whullm_i( a, m, kl, b, c )

Symmetric:
SUBROUTINE sy_whullm_i( a, b, c [, uplo] )
Symmetric Band:
SUBROUTINE sb_whullm_i( a, b, c [, uplo] )
Symmetric Packed:
SUBROUTINE sp_whullm_i( ap, bp, cp [, uplo] )
Triangular:
SUBROUTINE tr_whullm_i( a, b, c [, uplo] [, diag] )
Triangular Band:
SUBROUTINE tb_whullm_i( a, b, c [, uplo] [, diag] )
Triangular Packed:
SUBROUTINE tp_whullm_i( ap, bp, cp [, uplo], [, diag] )

all:
TYPE INTERVAL (<wp>), INTENT (IN) :: a(:,:), b(:,:),

   c(:,:), ap(:,), bp(:,), cp(:)

   TYPE blas_uplo_type, INTENT (IN), OPTIONAL :: uplo
   TYPE blas_diag_type, INTENT (IN), OPTIONAL :: diag
where
   a, b and c have shape (m, n) for general matrix
   (1, n) for general banded matrix (1 > kl)
   (n, n) for symmetric or triangular
   (p+1, n) for symmetric banded or triangular
   banded (p = band width)

   ap, bp and cp have shape (n*(n+1)/2)

- Fortran 77 binding:

General:
SUBROUTINE BLAS_xGE_WHULLM_I( M, N, A, LDA, B, LDB, C, LDC )
General Band:
SUBROUTINE BLAS_xGB_WHULLM_I( M, N, KL, KU, A, LDA, B, LDB,

   C, LDC )
Symmetric:
SUBROUTINE BLAS_xSY_WHULLM_I( N, A, LDA, B, LDB, C, LDC )
Symmetric Band:
SUBROUTINE BLAS_xSB_WHULLM_I( N, K, A, LDA, B, LDB, C, LDC )
Symmetric Packed:
SUBROUTINE BLAS_xSP_WHULLM_I( N, AP, BP, CP )
Triangular:
SUBROUTINE BLAS_xTR_WHULLM_I( N, A, LDA, B, LDB, C, LDC )
Triangular Band:
SUBROUTINE BLAS_xTB_WHULLM_I( N, K, A, LDA, B, LDB, C, LDC )
Triangular Packed:
SUBROUTINE BLAS_xTP_WHULLM_I( N, AP, BP, CP )
all:
   INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, B, LDB
C, LDC
A(2, LDA, *) or AP(2, *), B(2, LDA, *) or BP(2, *)
C(2, LDC, *) or CP(2, *)

• C binding:

General:
void BLAS_xge_whullm_i( enum blas_order_type order, int m, int n,
const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

General Band:
void BLAS_xgb_whullm_i( enum blas_order_type order, int m, int n, int kl,
int ku, const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

Symmetric:
void BLAS_xsy_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

Symmetric Band:
void BLAS_xsb_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, int k, const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

Symmetric Packed:
void BLAS_xsp_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> ap,
const <interval_array> bp, <interval_array> cp );

Triangular:
void BLAS_xtr_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

Triangular Band:
void BLAS_xtb_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n, int k,
const <interval_array> a, int lda,
const <interval_array> b, int ldb,
<interval_array> c, int ldc );

Triangular Packed:
void BLAS_xtp_whullm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_diag_type diag, int n,
const <interval_array> ap, <interval_array> bp,
<interval_array> cp );
Utility Functions Involving Interval Vectors

**EMPTYELEV.J** (Empty entry & location) \( k \leftarrow x_k = \emptyset \); or \(-1\)

This routine checks if an interval vector, \( x \), contains an empty interval entry. If \( x \) contains empty interval entries, then the routine returns the smallest offset or index \( k \) such that \( x_k = [\text{NaN-empty}, \text{NaN-empty}] \). Otherwise, the routine returns \(-1\).

- Fortran 95 binding:

```fortran
INTEGER FUNCTION emptyelev_i( x )
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  where
  x has shape (n)
```

- Fortran 77 binding:

```fortran
INTEGER FUNCTION BLAS_xEMPTYELEV_I( N, X, INCX )
  INTEGER N, INCX
  <type> X( 2, * )
```

- C binding:

```c
int BLAS_xemptyelev_i( int n, const <interval_array> x, int incx);
```

**INFV.J** (The left endpoint of an interval vector) \( v \leftarrow x \)

This routine finds the real vector \( v \) such that \( v_i = x_i \) \( \forall i \).

- Fortran 95 binding:

```fortran
SUBROUTINE infv_i( x, v )
  REAL (<wp>), INTENT(OUT) :: v(:)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  where
  v and x have shape (n)
```

- Fortran 77 binding:

```fortran
SUBROUTINE BLAS_xINFV_I( N, X, INCX, V )
  INTEGER N, INCX
  <type> X( 2, * ), V( * )
```

- C binding:
void BLAS_xinfv_i( int n, const <interval_array> x, int incx, RARRAY v );

SUPV_i (The right endpoint of an interval vector)   \( v \leftarrow x \)

This routine finds the real vector \( v \) such that \( v_i = x_i \forall i \).

- Fortran 95 binding:

```fortran
  SUBROUTINE supv_i( x, v )
  REAL (<wp>), INTENT(OUT) :: v(:)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  where
  v and x have shape (n)
```

- Fortran 77 binding:

```fortran
  SUBROUTINE BLAS_xSUPV_I( N, X, INCX, V )
  INTEGER N, INCX
  <type> X( 2, * ), V( * )
```

- C binding:

```c
void BLAS_xsupv_i( int n, const <interval_array> x, int incx, RARRAY v );
```

MIDV_i (The approximate midpoint of an interval vector)   \( v \leftarrow (x + y)/2 \)

This routine finds the real vector \( v \) such that \( v_i = \frac{x_i + y_i}{2} \forall i \).

- Fortran 95 binding:

```fortran
  SUBROUTINE midv_i( x, v )
  REAL (<wp>), INTENT(OUT) :: v(:)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  where
  v and x have shape (n)
```

- Fortran 77 binding:

```fortran
  SUBROUTINE BLAS_xMIDV_I( N, X, INCX, V )
  INTEGER N, INCX
  <type> X( 2, * ), V( * )
```

- C binding:
void BLAS_xmidv_i( int n, const <interval_array> x, int incx, RARRAY v );

WIDTHV_I (The elementwise width of an interval vector)  
\[ v \leftarrow x - x \]

This routine finds the real vector \( v \) such that \( v_i = x_i - x_i \forall i \).

- Fortran 95 binding:
  
  SUBROUTINE widthv_i( x, v )
  REAL (<wp>), INTENT(OUT) :: v(:)
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: x(:)
  where
  \( v \) and \( x \) have shape (n)

- Fortran 77 binding:
  
  SUBROUTINE BLAS_xWIDTHV_I( N, X, INCX, V )
  INTEGER N, INCX
  <type> X( 2, * ), V( * )

- C binding:
  
  void BLAS_xwidthv_i( int n, const <interval_array> x, int incx, RARRAY v );

CONSTRUCTV_I (Constructs an interval vector from two floating point vectors)  
\[ x \leftarrow [\min\{u, v\}, \max\{u, v\}] \]

This routine constructs an interval vector \( x \) from two floating point vectors \( u \) and \( v \) such that \( x_i \) contains the interval \([\min\{u_i, v_i\}, \max\{u_i, v_i\}] \forall i \). By letting \( u = v \), the routine constructs an interval vector from a single floating point vector.

- Fortran 95 binding:
  
  SUBROUTINE constructv_i( x, u, v )
  REAL (<wp>), INTENT(IN) :: u(:), v(:)
  TYPE(INTERVAL) (<wp>), INTENT(OUT) :: x(:)
  where
  \( u, v \) and \( x \) have shape (n)

- Fortran 77 binding:
  
  SUBROUTINE BLAS_xCONSTRUCTV_I( N, U, INCU, V, INCV, X, INCX )
  INTEGER N, INCU, INCV, INCX
  <type> X( 2, * ), U( * ), V( * )

- C binding:
  
  void BLAS_xconstructv_i( int n, RARRAY u, int incu, RARRAY v, int incv,
  <interval_array> x, int incx );
Utility Functions Involving Interval Matrices

\{GE,GB,SY,SB,SP,TR,TB,TP\} _EMPTYELEM_I (Empty entry & location) \( l \leftarrow a_{i,j} = \emptyset \) or \( -1 \)

This routine checks if an interval matrix, \( A \), contains an empty interval entry. If \( A \) contains empty interval entries, then the routine returns the smallest offset or index \( l \) (according to the first index) such that \( a_{i,j} = [\text{NaN,empty,NaN,empty}] \). Otherwise, it returns \( -1 \).

- **Fortran 95 binding:**

  General:
  ```fortran
  INTEGER FUNCTION ge_emptyelem_i( a )
  GENERAL Band:
  INTEGER FUNCTION gb_emptyelem_i( a, m, kl )
  Symmetric:
  INTEGER FUNCTION sy_emptyelem_i( a [, uplo] )
  Symmetric Band:
  INTEGER FUNCTION sb_emptyelem_i( a, kl [, uplo] )
  Symmetric Packed:
  INTEGER FUNCTION sp_emptyelem_i( ap [, uplo] )
  Triangular:
  INTEGER FUNCTION tr_emptyelem_i( a [, uplo] [, diag] )
  Triangular Band:
  INTEGER FUNCTION tb_emptyelem_i( a, kl [, uplo] [, diag] )
  Triangular Packed:
  INTEGER FUNCTION tp_emptyelem_i( ap, cp [, uplo], [, diag] )
  all:
  TYPE(INTEGER) (<wp>), INTENT(IN) :: a(:,:), ap(:)
  INTEGER, INTENT(OUT) :: i, j
  INTEGER, INTENT(IN) :: kl
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
  where
  a has shape \((m, n)\) for general matrix
  \((l, n)\) for general banded matrix \((l > kl)\)
  \((n, n)\) for symmetric or triangular
  \((p+1, n)\) for symmetric banded or triangular
  banded \((p = \text{band width})\)
  ap has shape \((n*(n+1)/2)\)
  ```

- **Fortran 77 binding:**

  General:
  ```fortran
  INTEGER FUNCTION BLAS_xGE_EMPTYELEM_I( M, N, A, LDA, B, LDB )
  General Band:
  INTEGER FUNCTION BLAS_xGB_EMPTYELEM_I( M, N, KL, KU, A, LDA )
  Symmetric:
  INTEGER FUNCTION BLAS_xSY_EMPTYELEM_I( UPLO, N, A, LDA )
  ```
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Symmetric Band:

```c
INTEGER FUNCTION BLAS_xSB_EMPTYELEM_I( UPLO, N, K, A, LDA )
```

Symmetric Packed:

```c
INTEGER FUNCTION BLAS_xSP_EMPTYELEM_I( UPLO, N, AP )
```

Triangular:

```c
INTEGER FUNCTION BLAS_xTR_EMPTYELEM_I( UPLO, TRANS, DIAG, N, A, LDA )
```

Triangular Band:

```c
INTEGER FUNCTION BLAS_xTB_EMPTYELEM_I( UPLO, TRANS, DIAG, N, K, A, LDA )
```

Triangular Packed:

```c
INTEGER FUNCTION BLAS_xTP_EMPTYELEM_I( UPLO, TRANS, DIAG, N, AP )
```

all:

```c
INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, I, J
<type> A( 2, LDA, * ) or AP( 2, * )
```

• C binding:

General:

```c
int BLAS_xge_emptyelem_i( enum blas_order_type order, int m, int n,
const <interval_array> a, int lda );
```

General Band:

```c
int BLAS_xgb_emptyelem_i( enum blas_order_type order, int m, int n, int kl,
int ku, const <interval_array> a, int lda, int i,
int j);
```

Symmetric:

```c
int BLAS_xsy_emptyelem_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> a, int lda );
```

Symmetric Band:

```c
int BLAS_xsb_emptyelem_i( enum blas_order_type order, int n, int k,
const <interval_array> a, int lda );
```

Symmetric Packed:

```c
int BLAS_xsp_emptyelem_i( enum blas_order_type order, int n,
const <interval_array> ap );
```

Triangular:

```c
int BLAS_xtr_emptyelem_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> a, int lda );
```

Triangular Band:

```c
int BLAS_xtb_emptyelem_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, int k, const <interval_array> a, int lda );
```

Triangular Packed:

```c
int BLAS_xtp_emptyelem_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> ap, int i, int j);
```

\{GE,GB,SY,SB,SP,TR,TR,TB,TP\}I \text{NFM}\_J (Left endpoint of an interval matrix) \quad C \leftarrow \underline{A}
This routine finds the real matrix $C$ such that $c_{i,j} = a_{i,j} \forall i$ and $\forall j$, where $A = \{a_{i,j}\}$ is a general (or general banded, or symmetric, or symmetric banded, symmetric packed, or triangular, triangular banded, triangular packed) interval matrix.

- Fortran 95 binding:

  General:
  ```fortran
  SUBROUTINE ge_infm_i( a, c )
  SUBROUTINE gb_infm_i( a, m, kl, c )
  SUBROUTINE sy_infm_i( a, c [, uplo] )
  SUBROUTINE sb_infm_i( a, kl, c [, uplo] )
  SUBROUTINE sp_infm_i( ap, cp [, uplo] )
  SUBROUTINE tr_infm_i( a, c [, uplo] [, diag] )
  SUBROUTINE tb_infm_i( a, kl, c [, uplo] [, diag] )
  SUBROUTINE tp_infm_i( ap, cp [, uplo], [, diag] )
  ```

  all:
  ```fortran
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), ap(:)
  REAL (<wp>), INTENT(OUT) :: c(:,:), cp(:)
  INTEGER, INTENT(IN) :: kl
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
  ```

  where
  - $a$ and $c$ have shape
    - $(m, n)$ for general matrix
    - $(1, n)$ for general banded matrix ($1 > kl$)
    - $(n, n)$ for symmetric or triangular
    - $(p+1, n)$ for symmetric banded or triangular
      banded ($p = $band width)
  - $ap$ and $cp$ have shape $(n*(n+1)/2)$

- Fortran 77 binding:

  General:
  ```fortran
  SUBROUTINE BLAS_xGE_INFM_I( M, N, A, LDA C, LDC )
  SUBROUTINE BLAS_xGB_INFM_I( M, N, KL, KU, A, LDA, C, LDC )
  SUBROUTINE BLAS_xSY_INFM_I( UPLO, N, A, LDA, C, LDC )
  SUBROUTINE BLAS_xSB_INFM_I( UPLO, N, K, A, LDA, C, LDC )
  ```

  Symmetric Packed:
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SUBROUTINE BLAS_xSP_INFMI( UPLO, N, AP, CP )
Triangular:
SUBROUTINE BLAS_xTR_INFMI( UPLO, TRANS, DIAG, N, A, LDA, C, LDC )
Triangular Band:
SUBROUTINE BLAS_xTB_INFMI( UPLO, TRANS, DIAG, N, K, A, LDA, C, LDC )
Triangular Packed:
SUBROUTINE BLAS_xTP_INFMI( UPLO, TRANS, DIAG, N, AP, CP )
all:
INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, LDC
ctype A( 2, LDA, * ) or AP( 2, * ), C( LDC, * ) or CP( * )

- C binding:

General:
void BLAS_xge_infmi( enum blas_order_type order, int m, int n,
const <interval_array> a, int lda, RARRAY c, int ldc );
General Band:
void BLAS_xgb_infmi( enum blas_order_type order, int m, int n, int kl, int ku,
const <interval_array> a, int lda, RARRAY c, int ldc );
Symmetric:
void BLAS_xsy_infmi( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> a, int lda, RARRAY c, int ldc );
Symmetric Band:
void BLAS_xsb_infmi( enum blas_order_type order, int n, int k,
const <interval_array> a, int lda, RARRAY c, int ldc );
Symmetric Packed:
void BLAS_xsp_infmi( enum blas_order_type order, int n,
const <interval_array> ap, RARRAY cp );
Triangular:
void BLAS_xtr_infmi( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> a, int lda, RARRAY c, int ldc );
Triangular Band:
void BLAS_xtb_infmi( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, int k, const <interval_array> a, int lda, RARRAY c, int ldc );
Triangular Packed:
void BLAS_xtp_infmi( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> ap, RARRAY cp );
This routine finds the real matrix $C$ such that $c_{i,j} = \overline{a_{i,j}} \forall i$ and $\forall j$, where $A = \{a_{i,j}\}$ is a general (or general banded, or symmetric, or symmetric banded, symmetric packed, or triangular, triangular banded, triangular packed) interval matrix.

- **Fortran 95 binding:**

  General:
  
  ```fortran
  SUBROUTINE ge_supm_i( a, c )
  ```

  General Band:
  
  ```fortran
  SUBROUTINE gb_supm_i( a, m, kl, c )
  ```

  Symmetric:
  
  ```fortran
  SUBROUTINE sy_supm_i( a, c [, uplo] )
  ```

  Symmetric Band:
  
  ```fortran
  SUBROUTINE sb_supm_i( a, kl, c [, uplo] )
  ```

  Symmetric Packed:
  
  ```fortran
  SUBROUTINE sp_supm_i( ap, cp [, uplo] )
  ```

  Triangular:
  
  ```fortran
  SUBROUTINE tr_supm_i( a, c [, uplo] [, diag] )
  ```

  Triangular Band:
  
  ```fortran
  SUBROUTINE tb_supm_i( a, kl, c [, uplo] [, diag] )
  ```

  Triangular Packed:
  
  ```fortran
  SUBROUTINE tp_supm_i( ap, cp [, uplo], [, diag] )
  ```

- **Fortran 77 binding:**

  ```fortran
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), ap(:)
  REAL (<wp>), INTENT(OUT) :: c(:,:), cp(:)
  INTEGER, INTENT(IN) :: kl
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
  ```

  Where
  
  - $a$ and $c$ have shape
    - $(m, n)$ for general matrix
    - $(1, n)$ for general banded matrix ($1 > k_l$)
    - $(n, n)$ for symmetric or triangular
    - $(p+1, n)$ for symmetric banded or triangular
      banded ($p = \text{band width}$)
  
  - $ap$ and $cp$ have shape $(n*(n+1)/2)$
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SUBROUTINE BLAS_xSP_SUPM_I( UPLO, N, AP, CP )
Triangular:
    SUBROUTINE BLAS_xTR_SUPM_I( UPLO, TRANS, DIAG, N, A, LDA, C, LDC )
Triangular Band:
    SUBROUTINE BLAS_xTB_SUPM_I( UPLO, TRANS, DIAG, N, K, A, LDA, C, LDC )
Triangular Packed:
    SUBROUTINE BLAS_xTP_SUPM_I( UPLO, TRANS, DIAG, N, AP, CP )
all:
    INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, LDC
    <type> A( 2, LDA, * ) or AP( 2, * ), C( LDC, * ) or CP( * )

- C binding:

General:
    void BLAS_xge_supm_i( enum blas_order_type order, int m, int n,
    const <interval_array> a, int lda, RARRAY c, int ldc );
General Band:
    void BLAS_xgb_supm_i( enum blas_order_type order, int m, int n, int kl, int ku,
    const <interval_array> a, int lda, RARRAY c, int ldc );
Symmetric:
    void BLAS_xsy_supm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    int n, const <interval_array> a, int lda,
    RARRAY c, int ldc );
Symmetric Band:
    void BLAS_xsb_supm_i( enum blas_order_type order, int n, int k,
    const <interval_array> a, int lda, RARRAY c, int ldc );
Symmetric Packed:
    void BLAS_xsp_supm_i( enum blas_order_type order, int n,
    const <interval_array> ap, RARRAY cp );
Triangular:
    void BLAS_xtr_supm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, const <interval_array> a, int lda,
    RARRAY c, int ldc );
Triangular Band:
    void BLAS_xtb_supm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, int k, const <interval_array> a, int lda,
    RARRAY c, int ldc );
Triangular Packed:
    void BLAS_xtp_supm_i( enum blas_order_type order, enum blas_uplo_type uplo,
    enum blas_trans_type trans, enum blas_diag_type diag,
    int n, const <interval_array> ap, RARRAY cp );

{GE,GB,SY,SB,SP,TR,TB,TP}_MIDM_I (Midpoint matrix of an interval matrix) $C \leftarrow (A + B)/2$
This routine finds the real matrix \( C \) such that \( BLAS_{i,j} = \frac{a_{i,j} + a_{j,i}}{2} \) \( \forall i \) and \( \forall j \), where \( \mathbf{A} = \{a_{i,j}\} \) is a general (or general banded, or symmetric, or symmetric banded, symmetric packed, or triangular, triangular banded, triangular packed) interval matrix.

- Fortran 95 binding:

  General:
  
  ```fortran
  SUBROUTINE ge_midm_i( a, c )
  SUBROUTINE gb_midm_i( a, m, kl, c )
  SUBROUTINE sy_midm_i( a, c [, uplo] )
  SUBROUTINE sb_midm_i( a, kl, c [, uplo] )
  SUBROUTINE sp_midm_i( ap, cp [, uplo] )
  SUBROUTINE tr_midm_i( a, c [, uplo] [, diag] )
  SUBROUTINE tb_midm_i( a, kl, c [, uplo] [, diag] )
  SUBROUTINE tp_midm_i( ap, cp [, uplo], [, diag] )
  ```

  all:

  ```fortran
  TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), ap(:)
  REAL(<wp>), INTENT(OUT) :: c(:,:), cp(:)
  INTEGER, INTENT(IN) :: kl
  TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
  ```

  where

  a and c have shape
  
  \((m, n)\) for general matrix

  \((1, n)\) for general banded matrix \((1 > kl)\)

  \((n, n)\) for symmetric or triangular

  \((p+1, n)\) for symmetric banded or triangular

  banded \((p = \text{band width})\)

  ap and cp have shape \((n*(n+1)/2)\)

- Fortran 77 binding:

  General:
  
  ```fortran
  SUBROUTINE BLAS_xGE_MIDM_I( M, N, A, LDA, C, LDC )
  SUBROUTINE BLAS_xGB_MIDM_I( M, N, KL, KU, A, LDA, C, LDC )
  SUBROUTINE BLAS_xSY_MIDM_I( UPLO, N, A, LDA, C, LDC )
  SUBROUTINE BLAS_xSB_MIDM_I( UPLO, N, K, A, LDA, C, LDC )
  ```
Symmetric Packed:
SUBROUTINE BLAS_xSP_MIDM_I( UPLO, N, AP, CP )

Triangular:
SUBROUTINE BLAS_xTR_MIDM_I( UPLO, TRANS, DIAG, N, A, LDA, C, LDC )

Triangular Band:
SUBROUTINE BLAS_xTB_MIDM_I( UPLO, TRANS, DIAG, N, K, A, LDA, C, LDC )

Triangular Packed:
SUBROUTINE BLAS_xTP_MIDM_I( UPLO, TRANS, DIAG, N, AP, CP )

all:
INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, LDC
[type] A( 2, LDA, * ) or AP( 2, * ), C( 2, LDA, * ) or CP( 2, * )

• C binding:

General:
void BLAS_xge_midm_i( enum blas_order_type order, int m, int n,
const <interval_array> a, int lda, RARRAY c, int ldc );

General Band:
void BLAS_xgb_midm_i( enum blas_order_type order, int m, int n, int kl, int ku,
const <interval_array> a, int lda, RARRAY c, int ldc );

Symmetric:
void BLAS_xsy_midm_i( enum blas_order_type order, enum blas_uplo_type uplo,
int n, const <interval_array> a, int lda, RARRAY c, int ldc );

Symmetric Band:
void BLAS_xsb_midm_i( enum blas_order_type order, int n, int k,
const <interval_array> a, int lda, RARRAY c, int ldc );

Symmetric Packed:
void BLAS_xsp_midm_i( enum blas_order_type order, int n,
const <interval_array> ap, RARRAY cp );

Triangular:
void BLAS_xtr_midm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> a, int lda,
RARRAY c, int ldc );

Triangular Band:
void BLAS_xtb_midm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, int k, const <interval_array> a, int lda,
RARRAY c, int ldc );

Triangular Packed:
void BLAS_xtp_midm_i( enum blas_order_type order, enum blas_uplo_type uplo,
enum blas_trans_type trans, enum blas_diag_type diag,
int n, const <interval_array> ap, RARRAY cp );
{GE,GB,SY,SB,SP,TR,TB,TP}.WIDTH.J (Elementwise width of an interval matrix) \( C \leftarrow A - A \)

This routine finds the real matrix \( C \) such that \( c_{i,j} = \overline{a}_{i,j} - \underline{a}_{i,j} \) \( \forall \) \( i \) and \( \forall j \), where \( A = \{a_{i,j}\} \) is a general (or general banded, or symmetric, or symmetric banded, symmetric packed, or triangular, triangular banded, triangular packed) interval matrix.

- Fortran 95 binding:

General:
```
SUBROUTINE ge_widthm_i( a, c )
```
General Band:
```
SUBROUTINE gb_widthm_i( a, m, kl, c )
```
Symmetric:
```
SUBROUTINE sy_widthm_i( a, c [, uplo] )
```
Symmetric Band:
```
SUBROUTINE sb_widthm_i( a, kl, c [, uplo] )
```
Symmetric Packed:
```
SUBROUTINE sp_widthm_i( ap, cp [, uplo] )
```
Triangular:
```
SUBROUTINE tr_widthm_i( a, c [, uplo] [, diag] )
```
Triangular Band:
```
SUBROUTINE tb_widthm_i( a, kl, c [, uplo] [, diag] )
```
Triangular Packed:
```
SUBROUTINE tp_widthm_i( ap, cp [, uplo], [, diag] )
```
all:
```
TYPE(INTERVAL) (<wp>), INTENT(IN) :: a(:,:), ap(:)
REAL (<wp>), INTENT(OUT) :: c(:,:), cp(:)
INTEGER, INTENT(IN) :: kl
TYPE(blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
TYPE(blas_diag_type), INTENT(IN), OPTIONAL :: diag
```
where
- \( a \) and \( c \) have shape
  - \((m, n)\) for general matrix
  - \((1, n)\) for general banded matrix \((1 > kl)\)
  - \((n, n)\) for symmetric or triangular
  - \((p+1, n)\) for symmetric banded or triangular banded \((p = \text{band width})\)
- \( \text{ap} \) and \( \text{cp} \) have shape \( (n*(n+1)/2) \)

- Fortran 77 binding:

General:
```
SUBROUTINE BLAS_xGE_WIDTHM_I( M, N, A, LDA, C, LDC )
```
General Band:
```
SUBROUTINE BLAS_xGB_WIDTHM_I( M, N, KL, KU, A, LDA, C, LDC )
```
Symmetric:
```
SUBROUTINE BLAS_xSY_WIDTHM_I( UPLO, N, A, LDA, C, LDC )
```
Symmetric Band:
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SUBROUTINE BLAS_xSB_WIDTHM_I( UPLO, N, K, A, LDA, C, LDC )

Symmetric Packed:
  SUBROUTINE BLAS_xSP_WIDTHM_I( UPLO, N, AP, CP )

Triangular:
  SUBROUTINE BLAS_xTR_WIDTHM_I( UPLO, TRANS, DIAG, N, A, LDA, C, LDC )

Triangular Band:
  SUBROUTINE BLAS_xTB_WIDTHM_I( UPLO, TRANS, DIAG, N, K, A, LDA, C, LDC )

Triangular Packed:
  SUBROUTINE BLAS_xTP_WIDTHM_I( UPLO, TRANS, DIAG, N, AP, CP )

all:
  INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, LDC
  <type> A( 2, LDA, * ) or AP( 2, * ), C( LDC, * ) or CP( * )

• C binding:

  General:
  void BLAS_xge_widthm_i( enum blas_order_type order, int m, int n,
                           const <interval_array> a, int lda, RARRAY c,
                           int ldc );

  General Band:
  void BLAS_xgb_widthm_i( enum blas_order_type order, int m, int n, int kl,
                           int ku, const <interval_array> a, int lda, RARRAY c,
                           int ldc );

  Symmetric:
  void BLAS_xsy_widthm_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          int n, const <interval_array> a, int lda,
                          RARRAY c, int ldc );

  Symmetric Band:
  void BLAS_xsb_widthm_i( enum blas_order_type order, int n, int k,
                          const <interval_array> a, int lda, RARRAY c, int ldc );

  Symmetric Packed:
  void BLAS_xsp_widthm_i( enum blas_order_type order, int n,
                          const <interval_array> ap, RARRAY cp );

  Triangular:
  void BLAS_xtr_widthm_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          enum blas_trans_type trans, enum blas_diag_type diag,
                          int n, const <interval_array> a, int lda,
                          RARRAY c, int ldc );

  Triangular Band:
  void BLAS_xtb_widthm_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          enum blas_trans_type trans, enum blas_diag_type diag,
                          int n, int k, const <interval_array> a, int lda,
                          RARRAY c, int ldc );

  Triangular Packed:
  void BLAS_xtp_widthm_i( enum blas_order_type order, enum blas_uplo_type uplo,
                          enum blas_trans_type trans, enum blas_diag_type diag,
                          int n, const <interval_array> ap, RARRAY cp );
{GE,GB,SY,SB,SP,TR,TB,TP}_CONSTRUCTM_I (Constructs an interval matrix from two floating
point matrices)

This routine constructs an interval matrix from two floating point matrices $B$ and $C$ such that

$$a_{i,j} = [\min\{b_{i,j}, BLAS_{i,j}\}, \max\{b_{i,j}, BLAS_{i,j}\}] \forall i \in \{0,1,\ldots, m - 1\} \text{ and } \forall j \in \{0,1,\ldots, n - 1\}.$$

Both floating point matrices $B$ and $C$ have the same storage format.

- Fortran 95 binding:

  General:
  ```fortran
  SUBROUTINE ge_constructm_i( a, b, c )
  END
  ```

  General Band:
  ```fortran
  SUBROUTINE gb_constructm_i( a, b, m, kl, c )
  END
  ```

  Symmetric:
  ```fortran
  SUBROUTINE sy_constructm_i( a, b, c [, uplo] )
  END
  ```

  Symmetric Band:
  ```fortran
  SUBROUTINE sb_constructm_i( a, b, kl, c [, uplo] )
  END
  ```

  Symmetric Packed:
  ```fortran
  SUBROUTINE sp_constructm_i( ap, bp, cp [, uplo] )
  END
  ```

  Triangular:
  ```fortran
  SUBROUTINE tr_constructm_i( a, b, c [, uplo] [, diag] )
  END
  ```

  Triangular Band:
  ```fortran
  SUBROUTINE tb_constructm_i( a, b, kl, c [, uplo] [, diag] )
  END
  ```

  Triangular Packed:
  ```fortran
  SUBROUTINE tp_constructm_i( ap, bp, cp [, uplo], [, diag] )
  END
  ```

  all:
  ```fortran
  TYPE(INTERVAL) (<wp>), INTENT(OUT) :: a(:,,:), ap(:)
  REAL (<wp>), INTENT(IN) :: b(:,,:), c(:,,:), cp(:)
  INTEGER, INTENT(IN) :: kl
  TYPE blas_uplo_type), INTENT(IN), OPTIONAL :: uplo
  TYPE blas_diag_type), INTENT(IN), OPTIONAL :: diag
  ```

where

- $a$, $b$ and $c$ have shape
  - $(m, n)$ for general matrix
  - $(l, n)$ for general banded matrix ($l > kl$)
  - $(n, n)$ for symmetric or triangular
  - $(p+1, n)$ for symmetric banded or triangular
    banded ($p = $ band width)

- $ap$ and $cp$ have shape $(n*(n+1)/2)$

- Fortran 77 binding:

  General:
  ```fortran
  SUBROUTINE BLAS_xGE_CONSTRUCTM_I( M, N, A, LDA, B, LDB, C, LDC )
  ```

  General Band:
  ```fortran
  SUBROUTINE BLAS_xGB_CONSTRUCTM_I( M, N, KL, KU, A, LDA, B, LDB, C, LDC )
  ```
C.4. INTERVAL BLAS

Symmetric:
  SUBROUTINE BLAS_xSY_CONSTRUCTM_I( UPLO, N, A, LDA, B, LDB, C, LDC )
Symmetric Band:
  SUBROUTINE BLAS_xSB_CONSTRUCTM_I( UPLO, N, K, A, LDA, B, LDB, C, LDC )
Symmetric Packed:
  SUBROUTINE BLAS_xSP_CONSTRUCTM_I( UPLO, N, AP, BP, CP )
Triangular:
  SUBROUTINE BLAS_xTR_CONSTRUCTM_I( UPLO, TRANS, DIAG, N, A, LDA,
    $ B, LDB, C, LDC )
Triangular Band:
  SUBROUTINE BLAS_xTB_CONSTRUCTM_I( UPLO, TRANS, DIAG, N, K, A, LDA,
    $ B, LDB, C, LDC )
Triangular Packed:
  SUBROUTINE BLAS_xTP_CONSTRUCTM_I( UPLO, TRANS, DIAG, N, AP, BP, CP )

all:
  INTEGER UPLO, TRANS, DIAG, M, N, K, KL, KU, LDA, LDC
  <type> A( 2, LDA, * ) or AP( 2, * ), B( LDC, * )
    $ or BP( * ), C( LDC, * ) or CP( * ),

• C binding:

  void BLAS_xge_constructm_i( enum blas_order_type order, int m, int n,
    <interval_array> a, int lda, RARRAY b, int ldb
    RARRAY c, int ldc );

  void BLAS_xgb_constructm_i( enum blas_order_type order, int m, int n,
    int kl, int ku, <interval_array> a, int lda, RARRAY c, int ldc );

  void BLAS_xsy_constructm_i( enum blas_order_type order,
    enum blas_uplo_type uplo, int n,
    <interval_array> a, int lda, RARRAY b,
    int ldb, RARRAY c, int ldc );

  void BLAS_xsb_constructm_i( enum blas_order_type order, int n, int k,
    <interval_array> a, int lda, RARRAY b, int ldb,
    RARRAY c, int ldc );

  void BLAS_xsp_constructm_i( enum blas_order_type order, int n,
    <interval_array> ap, RARRAY bp, RARRAY cp );

  void BLAS_xtr_constructm_i( enum blas_order_type order,
    enum blas_uplo_type uplo,
    enum blas_trans_type trans,
    enum blas DIAG_type diag, int n,
    <interval_array> a, int lda, RARRAY b,
    int ldb, RARRAY c, int ldc );
Triangular Band:
void BLAS_xtb_constructm_i( enum blas_order_type order,
    enum blas_uplo_type uplo,
    enum blas_trans_type trans,
    enum blas_diag_type diag, int n, int k,
    <interval_array> a, int lda, RARRAY b,
    int ldb, RARRAY c, int ldc );

Triangular Packed:
void BLAS_xtp_constructm_i( enum blas_order_type order,
    enum blas_uplo_type uplo,
    enum blas_trans_type trans,
    enum blas_diag_type diag, int n,
    <interval_array> ap, RARRAY bp, RARRAY cp );

Environmental Enquiry

FPINFO_I (Environmental enquiry)

This routine queries for machine-specific floating point characteristics. Refer to section 1.6 for a
list of all possible return values of this routine, and sections A.4, A.5, and A.6, for their respective
language dependent representations in Fortran 95, Fortran 77, and C.

- Fortran 95 binding:

  REAL(<wp>) FUNCTION fpinfo_i( cmach, prec )
  TYPE (blas_cmach_type), INTENT(IN) :: cmach
  <type>(<wp>), INTENT(IN) :: prec

- Fortran 77 binding:

  <rtype> FUNCTION BLAS_xFPINFO_I( CMACH )
  INTEGER CMACH

- C binding:

  <rtype> BLAS_xfpinfo_i( enum blas_cmach_type cmach );
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