MATRIX Multiplication with Quantized matrices

Intel® Math Kernel Library (Intel® MKL) Team - Presenter: Murat Efe Guney

Workshop on Batched, Reproducible, and Reduced Precision BLAS

Georgia Tech, Atlanta

February 24, 2017
Acknowledgements

Benoit Jacob - Google*

Greg Henry and Peter Tang - Intel®
Why integer matrix multiplication?

Deep-learning applications rely on integer matrix-matrix multiplication

- GEMMLOWP*: speech/face recognition
- KALDI* speech recognition
- Inference using 8/16bit integer weights
- Open question: training with integers

Intel® Instruction Set Architecture (ISA) extensions for integer matrix operations

- AVX512_4VNNIW: Vector instructions for deep learning enhanced word variable precision

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Quantizing with integers

Integers instead of floating points for lower power/bandwidth and higher throughput

Integers to represent a real number range \([a, b]\)

- real = \text{scale} \times (\text{integer} + \text{offset})

Why do we need an offset?

- Unsigned integers to represent signed real number domains, for e.g., \([-0.5, +1]\)

quantized domain (unsigned 2bits)

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
</table>

real numbers \([-0.5, +1]\)

scale = 1/2

offset = -1
Quantized matrices

The real number range \([a, b]\) is known in advance

- Choose \texttt{scale} and \texttt{offset} values for each matrix accordingly
- Sacrifice dynamic range

Zero point (0) must be represented perfectly in the quantized domain

- Zero-padding of the signal
- \texttt{offset} value represents the zero point

Scope of \texttt{offset} and \texttt{scale}:

- \texttt{offset} is a signed integer typically being the same precision as quantized domain
- \texttt{scale} is an arbitrary real number

\[
\text{real} = \text{scale} \times (\text{integer} + \text{offset})
\]
Quantized matrix operations

Standard GEMM operation:

- $C = \alpha A B + \beta C$

For neural networks, we need a GEMM-like operation:

- Extra requirement: add bias values to rows/columns of the output matrix
- $C_{real} = A_{real} B_{real} + b \nu_n^T$, or $C_{real} = A_{real} B_{real} + v_m b^T$, where $\nu$: vector of all ones; $b$: bias vector

First, let’s look at the $C_{real} = A_{real} B_{real}$ part:

- $c_{scale}(C_{quan} + c_{offset}\nu_m \nu_n^T) = a_{scale}(A_{quan} + a_{offset}\nu_m \nu_k^T)b_{scale}(B_{quan} + b_{offset}\nu_k \nu_n^T)$
- $C_{quan} = \frac{a_{scale}b_{scale}}{c_{scale}}(A_{quan} + a_{offset}\nu_m \nu_k^T)(B_{quan} + b_{offset}\nu_k \nu_n^T) + c_{offset}\nu_m \nu_n^T$ where $(c_{offset} = -c_{offset})$

Next, add the bias term:

- $C_{quan} = \frac{a_{scale}b_{scale}}{c_{scale}}(A_{quan} + a_{offset}\nu_m \nu_k^T)(B_{quan} + b_{offset}\nu_k \nu_n^T) + c_{offset}\nu_m \nu_n^T + (b \nu_n^T$ or $v_m b^T)$
Quantized matrix-matrix multiplication

Finally, add the beta * C term and incorporate the $c_{\text{offset}}$ to bias term

$$c_{\text{quan}} = \frac{a_{\text{scale}}b_{\text{scale}}}{c_{\text{scale}}} (A_{\text{quan}} + a_{\text{offset}} v_m v_k^T) (B_{\text{quan}} + b_{\text{offset}} v_k v_n^T) + \beta C_{\text{quan}} + (b v_n^T \text{ or } v_m b^T)$$

<table>
<thead>
<tr>
<th>Argument</th>
<th>BLAS Type</th>
<th>CBLAS Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>layout</td>
<td>N/A</td>
<td>CBLAS_LAYOUT</td>
<td>Row-major or column-major storage</td>
</tr>
<tr>
<td>transa</td>
<td>char*</td>
<td>CBLAS_TRANSPOSE</td>
<td>op(A)</td>
</tr>
<tr>
<td>transb</td>
<td>char*</td>
<td>CBLAS_TRANSPOSE</td>
<td>op(B)</td>
</tr>
<tr>
<td>biasc</td>
<td>char*</td>
<td>CBLAS_OFFSET</td>
<td>C bias is applied to rows or columns or a fixed offset for the entire matrix</td>
</tr>
<tr>
<td>m</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>First dimension of C matrix (number of rows for column major)</td>
</tr>
<tr>
<td>n</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>Second dimension of C matrix (number of columns for column major)</td>
</tr>
<tr>
<td>k</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>Common dimension of A and B matrices</td>
</tr>
<tr>
<td>alpha</td>
<td>double*</td>
<td>double</td>
<td>Alpha scalar multiplication</td>
</tr>
<tr>
<td>A</td>
<td>MKL_INT16*</td>
<td>MKL_INT16*</td>
<td>Pointer to input matrix A</td>
</tr>
<tr>
<td>lda</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>Leading dimension for A matrix</td>
</tr>
<tr>
<td>lA</td>
<td>MKL_INT16*</td>
<td>MKL_INT16*</td>
<td>Scalar offset value for A matrix</td>
</tr>
<tr>
<td>B</td>
<td>MKL_INT16*</td>
<td>MKL_INT16*</td>
<td>Pointer to input matrix B</td>
</tr>
<tr>
<td>ldb</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>Leading dimension for B</td>
</tr>
<tr>
<td>lB</td>
<td>MKL_INT16*</td>
<td>MKL_INT16*</td>
<td>Scalar offset value for the B matrix</td>
</tr>
<tr>
<td>beta</td>
<td>double*</td>
<td>double</td>
<td>Scalar scaling of the input/output C matrix</td>
</tr>
<tr>
<td>C</td>
<td>MKL_INT32*</td>
<td>MKL_INT32*</td>
<td>Pointer to the C matrix</td>
</tr>
<tr>
<td>ldc</td>
<td>MKL_INT*</td>
<td>MKL_INT</td>
<td>Leading dimension for the C matrix</td>
</tr>
<tr>
<td>bc</td>
<td>MKL_INT32*</td>
<td>MKL_INT32*</td>
<td>Vector storing bias/offsets for C matrix.</td>
</tr>
</tbody>
</table>
Function syntax and naming convention

\[ C = \alpha (\text{op}(A) + a_{\text{offset}} v_m v_k^T)(\text{op}(B) + b_{\text{offset}} v_k v_n^T) + \beta C + (b v_n^T \text{ or } v_m b^T) \]

GEMM_{S,U}{b1}_{S,U}{b2}_{S,U}{b3} (char* transa, char* transb, char* biasc, MKL_INT* m, MKL_INT* n, MKL_INT* k, double* alpha, MKL_[U]INT{b1}* A, MKL_INT{b1}* oa, MKL_INT* lda, MKL_[U]INT{b2}* B, MKL_INT{b2}* ob, MKL_INT* ldb, double* beta, MKL_[U]INT{b3}* C, MKL_INT* ldc, MKL_INT{b3}* bc)

- Offset/bias are same types as the corresponding matrix elements
- alpha and beta are double precision values
- Bias for C (bc) can be a scalar or a vector based on the value of offsetc
  - biasc = “F”, sizeof(bc) = 1
  - biasc = “R”, sizeof(bc) = num_cols(C)
  - biasc = “C”, sizeof(bc) = num_rows(C)
Implementation notes

Results from double-precision multiplications round to the nearest

- \( C_{quan} = \alpha (A_{quan} + a_{offset}v_m v_k^T)(B_{quan} + b_{offset}v_k v_n^T) + \beta C_{quan} + (b v_n^T \text{ or } v_m b^T) \)

- X, Y and Z are the partial results stored in double-precision

- \( C_{quan} = \text{round\_to\_nearest}(X + Y + Z) \);

- Open question: do we need alternative rounding modes (for e.g., stochastic rounding)?

Results may not be identical for \( X + Y + Z \)

- Enforcing the ordering ((X+Y) + Z) provides bitwise identical results
Additional implementation notes

Computation of the X term is susceptible to overflow/underflow

- \( c_{quan} = alpha(A_{quan} + a_{offset} v_m v_k^T)(B_{quan} + b_{offset} v_k v_n^T) + beta c_{quan} + (b v_n^T \text{ or } v_m b^T) \)

- Currently X term is expanded as:
  - \( X = A_{quan}B_{quan} + a_{offset} v_m v_k^T B_{quan} + b_{offset} A_{quan} v_k v_n^T + a_{offset} b_{offset} v_m v_k v_n^T \)
- \( A_{quan}B_{quan} \) is like a regular matrix multiplication with input matrix precision
- This approach allows effectively utilizing input matrix precision for all offset values
- The order of integer addition is important to prevent overflows/underflows

What happens in the event of overflow/underflow?

- Overflow/underflow is highly undesirable for application developers
- Intel® MKL implementations saturate, which may lead to non-reproducible results
Integer GEMM implementations in Intel® MKL

Two variants are available in Intel® MKL 2018 Beta

- **GEMM_S16S16S32**: Input matrices A/B are 16-bit signed integer, input/output matrix C is 32bit signed integer

- **GEMM_S16S16S16**: All matrices A/B/C are 16-bit signed integer

- All scaling factors are double-precision (likely to be changed to single-precision for 16-bit output)

- Internal summation is with at least 32-bit signed integers

- Loosely follows XBLAS naming convention (missing the internal summation precision and abbreviations)

- Fixed-point matrix multiplication is a subset of the functionality (set offset values to 0)

Only saturation variants are implemented

More optimizations are coming for Intel® MKL 2018 Gold release
Future considerations

Rounding modes other than round-to-nearest

- Stochastic rounding may be required for the training (S Gupta et. al, ICML, 2015)

Alternative representations (for e.g., flex-point)

- Adjust scaling/offset values inside integer GEMM

Fuse activation functions with the integer GEMM functionality

- tanh, ReLU, etc…
- Partial results are already in double-precision

A flexible API that allows fusing operations for best performance

- Currently needed: round_function(activation_function(double(GEMM + bias)))
- In the future: f_n(…f_3(f_2(f_1(f_0(GEMM)))))

Scaling factors as fixed-points?

Are FORTRAN interfaces needed?
Summary

Intel® MKL 2018 Beta will provide two GEMM variants for quantized matrices
Bias is fused into the matrix-multiply for improved performance
Saturate instead of over-flowing or under-flowing
Reproducible results due to the integer computations - as long as there is no saturation
Additional variants with different precisions may be introduced based on the hardware support
Operation fusing is important for best performance
FORTRAN APIs are less relevant for the machine learning domain
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