User Manual for IIPBF (Infinite Integration of Products of Bessel Functions)

Introduction

This document describes the usage of the IIPBF toolbox. It is meant to be an accompanying guide to the main reference paper:

Ratnanather, J. T., Kim, J. H., Zhang, S., Davis, A. M. J., and Lucas, S. K. 2013. Algorithm XXX: IIPBF, a MATLAB toolbox for infinite integral of products of two Bessel functions. To appear in ACM Transactions on Mathematical Software.

Contents

1. Overall code structure of IIPBF

The current release of the IIPBF toolbox consists a core group of nine *.m files (MATLAB files). The main entry point of the toolbox is the IIPBF.m, called as follows

```
[sol,reterr,evals]=IIPBF(fx,rho,tau,a,b,abserr,relerr,type)
```

Input

fx = function handle, usually a well-behaved function

rho = positive real number

tau = positive real number

a = non negative integer

b = non negative integer

abserr = required minimum absolute error

relerr = required minimum relative error

type = 'JJ', 'JY', or 'YY' (refer to paper)

Output

sol = estimated solution to integral

reterr = predicted error

evals = number of function evaluations.

The directories are structured in the following manner:

+ IIPBF (core folder with code for the toolbox)

    + TESTS (contains all codes needed to generate the results in the paper)

        + BESSELINT (contains the code developed by Van Deun and Cools)

2. Reproducing results from the paper

a. Instructions to reproduce Fig.1 (plot of actual error vs. number of function evaluation for cases 1-18) and Fig. 2 (plot of relative error vs. number of function evaluation for cases 1-18).

Step 1: Navigate to IIPBF --> TESTS.

Step 2: Run testcases_errorplots.m

Note the testcases_errorplots.m script uses a hard code set of parameters for the cases 1-18 listed in Table 1 of the reference paper.

b. Instructions to reproduce Table II (tests the toolbox for cases 11-13 for numerous parameter settings).

Step 1: Navigate to IIPBF --> TESTS.

Step 2a: Run case11tests.m

Step 2b: Run case12tests.m

Step 2c: Run case13tests.m

Note there is one script for each of the cases 11, 12 and 13. The output of the scripts were designed to be incorporated in a LaTeX document

c. Instructions to reproduce Table IV (comparison of IIPBF to BESSELINT of select test cases)

Step 1: Navigate to IIPBF --> TESTS.

Step 2: Run comparetoolboxes

Note the output of the scripts were designed to be incorporated in a LaTeX document

d. Instructions to reproduce Table V and test case 23.

Step 1: Navigate to IIPBF --> TESTS.

Step 2: Run case23tests.m

3. Additional test cases

a. Instructions to test the toolbox for case 24 for numerous parameter settings.

Step 1: Navigate to IIPBF --> TESTS.

Step 2: Run case24tests.m

Note the output of the scripts were designed to be incorporated in a LaTeX document

b. Instructions to test cases 25 to 34 (adapted from Gradshteyn & Ryzhik, Seventh Edition and McPhedran, 1992).

Documentation: Cases25to34.pdf

Step 1: Navigate to IIPBF --> TESTS.

Step 2: Run Cases25to34.m

c. Instructions to reproduce test cases presented at NACONF2013 (adapted from Gradshteyn & Ryzhik, Seventh Edition).

Documentation: RatnanatherNACONF2013.pdf

Step 1: Navigate to IIPBF → TESTS.

Step 2: Run testcases_errorplotsNACONF2013.m to reproduce actual error and estimated error plots for cases 6 & 33-35

Step 3: Run comparetoolboxesNACONF2013.m to generate comparison tables for IIPBF with BESSELINT for cases 33 and 34

Step 4: Run case35testsNACONF2013.m to generate error tables for case 35

Step 5: Uncomment lines 72-74 of IIPBF.m to export the h1 function and first two zeros when running IIPBF

Step 5: Run h1_plotsNACONF2013.m to reproduce plots of the first two zeros of h1 for high/low error examples of case 35 and case 23

4. Suggestions for |a-b| > 5

Generally, IIPBF will work well in cases where |a-b| < 5, due to the simple oscillatory behavior of these functions. However, cases where |a-b| > 5 may result in errors or inaccuracies, especially when rho and/or tau have extreme values. In these cases, it is recommended that asymptotic analysis be used instead. It may also be possible to improve results if these errors occur by increasing the value of the first zero (see first_2_zeros.m).