User Documentation

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1. THE SUBPROGRAMS

The subprograms belonging to the present algorithm are classified into the following Groups (A), (B), (C).
Group (A)

—SUBROUTINE bessel(znu, xx, zans, info)
zans: output, the complex value of $J_\nu(x)$.

—SUBROUTINE neumann(znu, xx, zans, info)
zans: output, the complex value of $N_\nu(x)$.

—SUBROUTINE hankel1(znu, xx, zans, info)
zans: output, the complex value of $H_\nu^{(1)}(x)$.

—SUBROUTINE hankel2(znu, xx, zans, info)
zans: output, the complex value of $H_\nu^{(2)}(x)$.

The dummy arguments znu, xx, info have the following common roles.

—znu: input, complex order ν.

—xx: input, nonnegative argument x.

—info: output, integer, information about the output zans.
—info=0: normal output, $e_r \leq 5\times10^3\epsilon_1$, where e_r is the relative error of zans.
—info=5: low accuracy, $5\times10^3\epsilon_1 < e_r \leq 2\times10^7\epsilon_1$.
—info=10: rough accuracy, $2\times10^7\epsilon_1 < e_r$.
—info=20: (1) An overflow occurred. The value of zans is the maximum available real number. (2) The answer zans is indefinite theoretically. For example, $J_i(0)$ is indefinite.
—info=30: out of range, $x < 0$ for example.

Group (B)

—bes_series: this calculates $J_\nu(x)$ with the series expansion in Section 2.2.1.

—neu_series: this calculates $N_\nu(x)$ with the method stated in Section 2.2.2.

—bes_han_dby: this calculates $J_\nu(x)$, $H_\nu^{(1)}(x)$, $H_\nu^{(2)}(x)$ with Debye's expansions in Section 2.2.3.

—bes_olver: this calculates $J_\nu(x)$ with Olver's expansion stated in Section 2.2.4.

—han2_olver: this calculates $H_\nu^{(2)}(x)$ with Olver's expansion in Section 2.2.4.

—bes_recur: this calculates $J_\nu(x)$ with the recurrence method in Section 2.2.5.

—han2_temme: this calculates $H_\nu^{(2)}(x)$ with Temme's algorithm in Section 2.2.6.

Group (C)

—num_region: this determines the region number n that is the suffix n of R_n.

—neu_srs_init: this is invoked by neu_series.

—def_bessel: this is invoked by neu_srs_init.

—sumaabb: this is invoked by bes_olver, han2_olver.

—fzeta: this is invoked by bes_olver, han2_olver.

—aiz: this is invoked by bes_olver, han2_olver; this subprogram calculates the Airy function $\mathrm{Ai}(z)$ and its derivative $\mathrm{Ai}'(z)$, and the subprogram is cited from Algorithm 819 [Gil et al. 2002].

—cdlgam: this is invoked by bes_series, bes_recur; this subprogram calculates $\Gamma(\nu)$, $\log\Gamma(\nu)$ and is cited from Algorithm 421 [Kuki 1972].

—abs2: this calculates a rough absolute value of a complex number.

All the above subprograms are the module subprograms of MODULE mod_bes. The four subroutines of Group (A) can be invoked by a user. If a subroutine of Group (A) is invoked, this subroutine invokes FUNCTION num_region first, determines the region number n and invokes one or two of the subroutines of Group (B) according to n. In addition, subprograms of Group (C) help numerical calculation in subroutines of Group (B).

REFERENCES

Gil, A., Segura, J., and Temme, N. M. 2002. Algorithm 819: AIZ, BIZ: Two Fortran 77 routines for the computation of complex Airy functions. *ACM Trans. Math. Soft. 28,* 3 (Sept.), 325–336.

Kuki, H. 1972. Algorithm 421 Complex gamma function with error control [S14]. *Commun. ACM 15,* 4 (Apr.), 271–272.